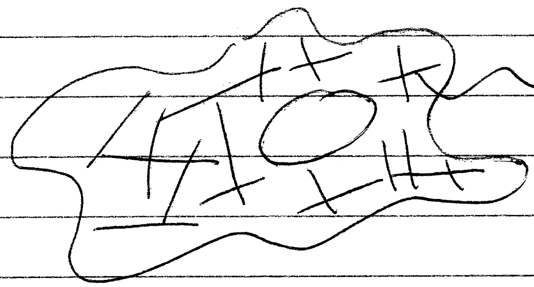
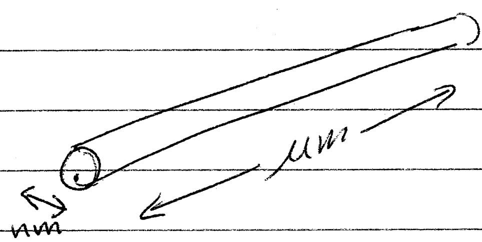
Cellular Architecture: Biomolecular Beams

- Many biomolecules are filamentous: microtubules, actin, collagen, hair-cell bundles, DNA, flagellum

- 

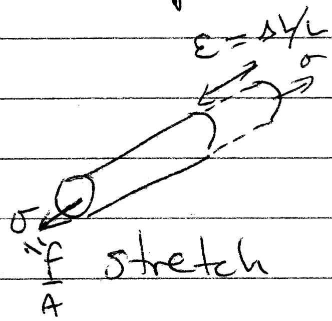
cytoskeleton ≡ network of actin filaments

beam ≡ longitudinal length >> lateral length

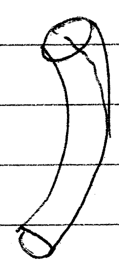


e.g. actin filament

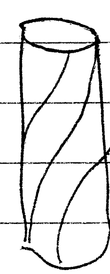
- Deforming beams



$\sigma = E\varepsilon$



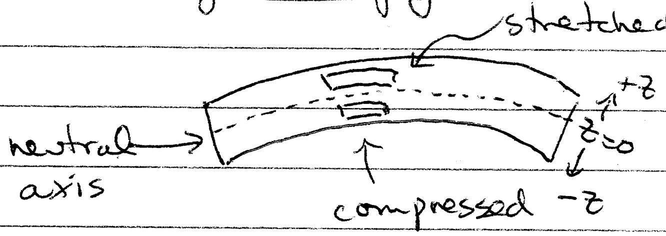
bend



twist

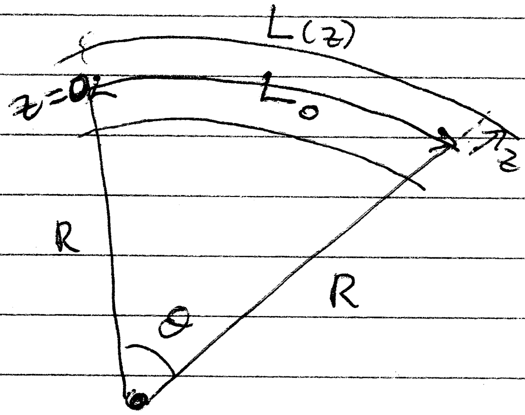
what is the energy cost for these deformations?

Bending Energy



- The region above the neutral axis is stretched and below is compressed

- Locally, a bent region will fall on a circular arc of radius R ~~for~~



- What is $\Delta L(z) = L(z) - L_0$?

$$L_0 = R\theta \Rightarrow \theta = \frac{L_0}{R}$$

and

$$L(z) = (R+z)\theta = (R+z)\frac{L_0}{R}$$

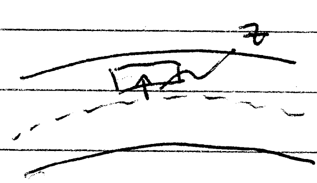
so

$$\Delta L(z) = (R+z)\frac{L_0}{R} - L_0 = \frac{z L_0}{R}$$

so

strain, $\varepsilon = \frac{\Delta L(z)}{L_0} = \frac{z}{R}$

Energetic cost for small volume element

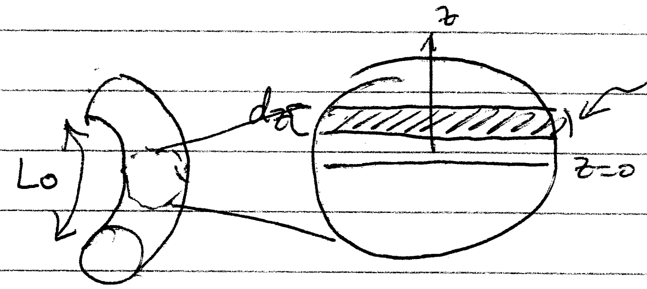


$$w(z) = \frac{1}{2} E \varepsilon(z)^2 = \frac{1}{2} E \left(\frac{\Delta L(z)}{L_0}\right)^2$$

energy density

$$= \frac{1}{2} E \frac{z^2}{R}$$

Now need to integrate this:

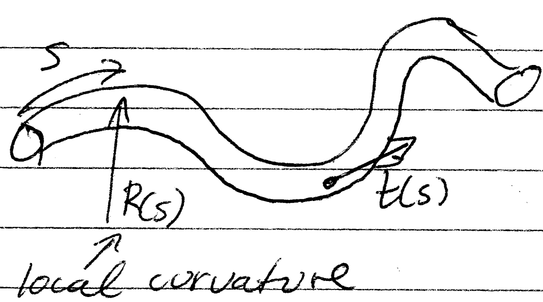


so

$$E_{bend} = L_0 \int dA \, \frac{E}{2R^2} z^2$$

or $$E_{bend} = \frac{L_0 E I}{2R^2} \; ; \quad I = \underbrace{\int dA \, z^2}_{\text{moment of material}}$$

More generally, what if beam has an arbitrary bent shape



$$\Rightarrow \boxed{E_{bend} = \frac{K_{eff}}{2} \int_0^L ds \, \frac{1}{R(s)^2}}$$

where $K_{eff} = E \cdot I \equiv$ material constant

- If we have $\vec{t}(s) \equiv$ tangent vector to the curve then

$$K(s) \equiv \frac{1}{R(s)} = \left| \frac{d\vec{t}}{ds} \right| \equiv \text{curvature}$$

Connection to persistence length

- The bend degree of freedom will have $k_B T$ of energy due to thermal kicks.
- Persistence length $\equiv$ length @ which polymer loops back so $L \approx R = \xi_p$

so $\quad k_B T = \dfrac{EIL}{2R^2} \approx \dfrac{EI}{2\xi_p}$ (estimate)

or

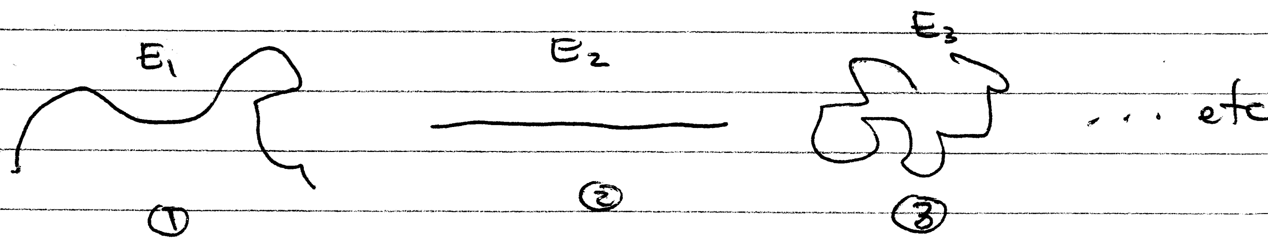
$$\boxed{\xi_p = \frac{EI}{k_B T}}$$ (more sophisticated calculation)

- gets smaller @ higher temperature (makes sense)
- gets larger as E gets bigger (makes sense)

also $\quad EI = \xi_p k_B T = K_{eff}$

Worm-like Chain:

- We can now compute the energy of a given bent shape
- What is the most likely shape?



Need to calculate the partition function

$$Z = \sum_{\text{shapes}} \exp\left(-\frac{E^i_{bend}}{k_B T}\right) = \sum_{\text{shapes}} \exp\left(-\frac{\xi_p}{2}\int_0^L \left|\frac{d\vec{t}(s)}{ds}\right| ds\right)$$

Now there are an infinite # of shapes so $\sum_{\text{shapes}} \rightarrow \int$

so

$$Z = \int \mathcal{D}\vec{t}(s) \exp\left(-\frac{\xi_p}{2}\int_0^L \left|\frac{d\vec{t}}{ds}\right|^2 ds\right)$$

- This includes the competition between entropy and energy, because some values of the energy will have many corresponding shapes $\Rightarrow$ high entropy
- The $\int \mathcal{D}\vec{t}(s)$ is a Feynman path integral
- Now imagine pulling on the polymer with a force, $F\hat{k}$
- so the polymer does work $-F\int_0^L t_z\, ds$ for a given shape
- What is $\langle z \rangle$ for a given force F?

$$\langle z \rangle = \frac{1}{Z}\int \mathcal{D}\vec{t}(s)\ z \exp\left(-\frac{\xi_p}{2}\int_0^L \left|\frac{dt}{ds}\right|^2 ds + \frac{F}{k_B T}\int_0^L t_z\, ds\right)$$

$$\left(= \not{\int} \sum z\, p(z) \quad \text{just fancier way of writing it} \right)$$

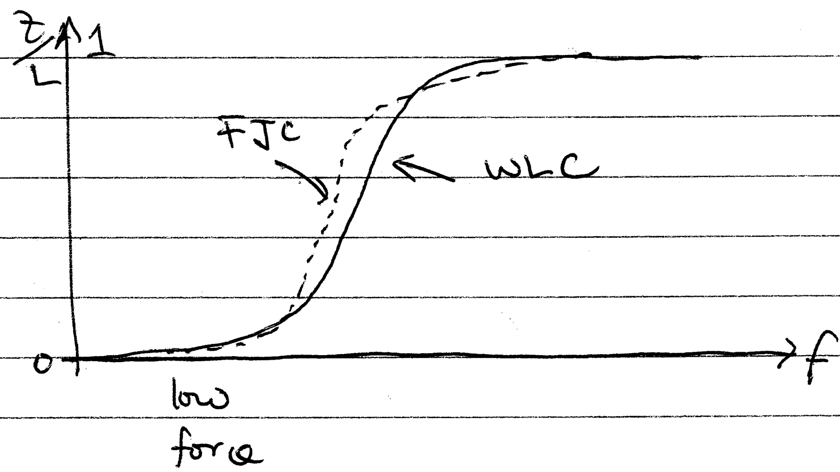
To evaluate this requires some fancy math.

In the low force limit: $f\xi_p \ll 1 \Rightarrow f\xi_p = \frac{3}{2}\frac{\langle z\rangle}{L}$

which is the same as FJC.

- An interpolation formula (Marko & Siggia) results

$$f\xi_p \approx \frac{z}{L} + \frac{1}{4\left(1-\left(\frac{z}{L}\right)\right)^2} - \frac{1}{4}$$



- Data from optical tweezers can be fit with WLC to find $\underline{\xi_p}$ & $\underline{L}$