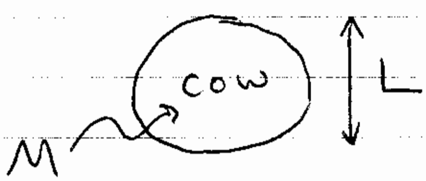
Notes 1 P347: Biophysics

- What does physics have to say about the sizes of animals & there shapes?
- Q: What is the biggest a mammal can be?
- Q: How small can a bird be?
- Q: How heavy can a bird be and still fly?

To answer these questions we need to look at how biological processes scale with the size of an animal (or equivalently the mass)

Approximation: The Spherical Cow



mass $\equiv M$
size $\equiv L$

now $M \propto L^3$ or $L \propto M^{1/3}$

Animal's surface area: $A \propto L^2 \propto (M^{1/3})^2 = M^{2/3}$

Our First scaling relation: $A \propto M^{2/3}$

- Such scaling laws are called "allometric"
- Allometry is the study of an animal's physical attributes as a function of it's size.

Allometric scaling: $\boxed{f = a M^{\alpha}}$

- This is a power law. For surface area $\alpha = 2/3$. The constant 'a' can be determined from data.
- In figure 1: the metabolic rate as a function of M is shown

$$\Gamma = a M^{3/4} \quad (\text{Watt})$$

and

$$a = 3.6 \ W/kg^{+3/4}$$

- Table 1.2 gives other exponents.

<u>Benefits of larger bodies:</u>

$\Gamma = \dfrac{\Delta E}{\Delta t}$ and assume ΔE come from burning fat. so $\Delta E \propto M$

time taken: $\Delta t = \dfrac{\Delta E}{\Gamma} \propto \dfrac{M}{M^{3/4}} = M^{1/4}$

so larger animals can go longer without eating.

- Heat loss: heat loss $\propto A \propto M^{2/3}$

  but surface area/unit mass $= \frac{A}{M} = M^{-1/3}$

- So large animals will lose less heat when it's cold. e.g. Deep diving whales.

Other interesting relations:

- frequency of heart beats: $f_H = 4.02\, M^{-0.25}$ Hz
- life time in years: $t_L = 11.89\, M^{+0.2}$
- Total # of heart beats:

$$N_H = t_L \cdot f_H \cdot (31.5 \times 10^6 \tfrac{s}{yr})$$

$$= (1.5 \times 10^9)\, \cancel{M^{-0.05}}^{\,1}$$

$$N_H = 1.5 \times 10^9 \text{ beats} \cong \text{constant.}$$

[Aside: note that $f_H \propto M^{-1/4}$

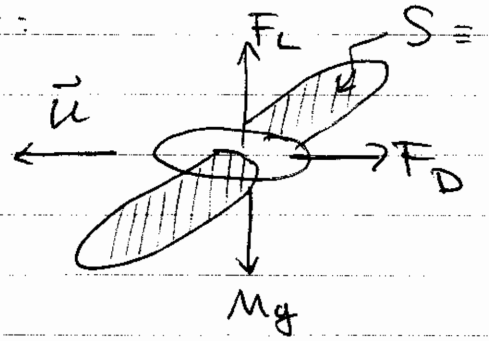
if the heart were behaving as a simple harmonic oscillator, $f_H \propto \sqrt{M}$]

Derivation of Scaling Laws:

- the scaling law shown in fig 1, and the numbers in table 1.2 are empirical
- some of the scaling relationships can be derived from first principles.

E.g.: Max size of flying birds:

Flight:



$S \equiv$ surface area shaded

Lift force:

$$F_L = \frac{1}{2} c_L \rho_{air} S u^2$$

- In order to just support the weight:

$$W = Mg = F_L \propto S u^2$$

so

$$W/S \propto u^2 \quad (1)$$

But the surface area $S \propto L^2 \propto W^{2/3}$

so

$$\frac{W}{S} \propto \frac{W}{W^{2/3}} = W^{1/3} \quad (2)$$

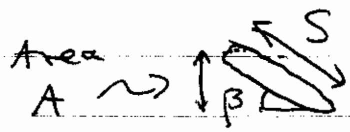
- ~~combining~~ equating (1) & (2):

$$u^2 \propto W^{1/3} \quad \text{or} \quad \boxed{u \propto W^{1/6}}$$

See fig 2 for empirical data. For the data $\alpha = 1/6$ which agrees with our derivation.

What about the power required?

- Bird must work against the drag force.



$$F_D = \frac{1}{2} C_D A \rho_{air} u^2$$

or

$$F_D = F_L \frac{A c_D}{S c_L} = (Mg)\left(\frac{A c_D}{S c_L}\right) \text{ @ min } \bar{u}$$

so Power, $P = u \cdot F_D = u_m M g \left(\frac{A c_D}{S c_L}\right)$ ①

- this power must be supplied by metabolism which is only $\eta \sim 0.25$ efficient

so $P = \eta \Gamma = \eta \alpha M^{3/4}$ ②

- equating ① & ② & solving for u_m:

$$\Rightarrow u_m \propto M^{-1/4}$$

- So the flight velocity decreases with body mass.

- On fig 2, $u \propto M^{1/6}$ & $u_m \propto M^{-1/4}$ are plotted showing that the mute swan ~15kg is as big as a bird can be.