Allometry: Lecture 1



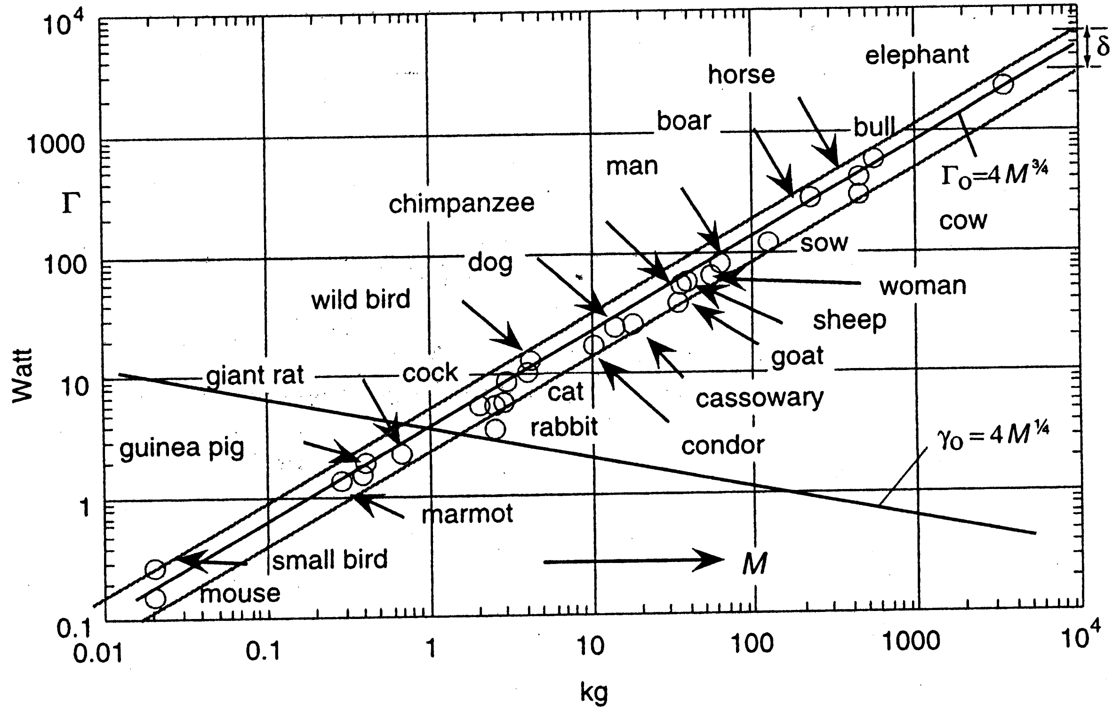


**Fig. 1.7.** The metabolic rate $\Gamma$, "Mouse to elephant curve", and specific metabolic rate $\gamma_0 = \Gamma_0 / M$. Data adopted from Schmidt Nielsen [1984]

where $M$ is measured in kg, and the constant $a = 3.6 \approx 4$ has the dimension Watt/kg$^{3/4}$ to give $\Gamma_0$ the unit Watt. Active animals exceed the resting metabolic rate $\Gamma_0$ by a factor $b$ in the range $2 \leq b \leq 15$. One would expect that the metabolic rate is a linear function of the body mass ($\alpha = 1$), however the numerical value of the metabolic exponent $\alpha$ is not easy to explain.

Initially it was thought [see for instance K. Schmidt-Nielsen 1984] that $\Gamma_0$ should scale with the exponent $\alpha = 2/3$, which arises if the metabolic power production just compensates the heat losses of the warm-blooded body conducting heat through its surface. The exponent 0.75 can be derived from physical models that look at the mechanical power loss of the fluid flow in the blood system due to the branching of arteries into smaller and smaller vessels [see Spatz, 1991, West et al. 1997 and Bejan 2000]. However, it seems that neither of these models is com-

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In allometric relations the quantity $f$ cannot be expressed as a fixed percentage of the body mass for small and large animals. Compare for instance the brain mass of a small rodent of $M = 0.1$ kg and a water buffalo at $M = 1\,000$ kg. The rodent

**Table 1.2.** Allometric parameters for mammals. Most data are data adapted from Vogel [1988], more extensive allometric data are given by Schmidt-Nielsen [1984]

| parameter | factor $a$ | exponent $\alpha$ |
|---|---|---|
| body surface in $m^2$ | 0.11 | 0.65 |
| brain mass (man) in kg | 0.085 | 0.66 |
| brain mass (non primates) in kg | 0.01 | 0.7 |
| breathing frequency in Hz | 0.892 | −0.26 |
| cost of transport (running) in J/m · k | 7 | −0.33 |
| cost of transport(swimming) in J/m · kg | 0.6 | −0.33 |
| effective lung volume in $m^3$ | $5.67 \cdot 10^{-5}$ | 1.03 |
| frequence of heartbeat in Hz | 4.02 | −0.25 |
| heart mass in kg | $5.8 \cdot 10^{-3}$ | 0.97 |
| life time in years | 11.89 | 0.20 |
| metabolic rate in W | 4.1 | 0.75 |
| muscle mass in kg | 0.45 | 1.0 |
| skeletal mass (cetaceans) in kg | 0.137 | 1.02 |
| skeletal mass (terrestrial) in kg | 0.068 | 1.08 |
| speed of flying in m/s | 15 | 1/6 |
| speed of walking in m/s | 0.5 | 1/6 |

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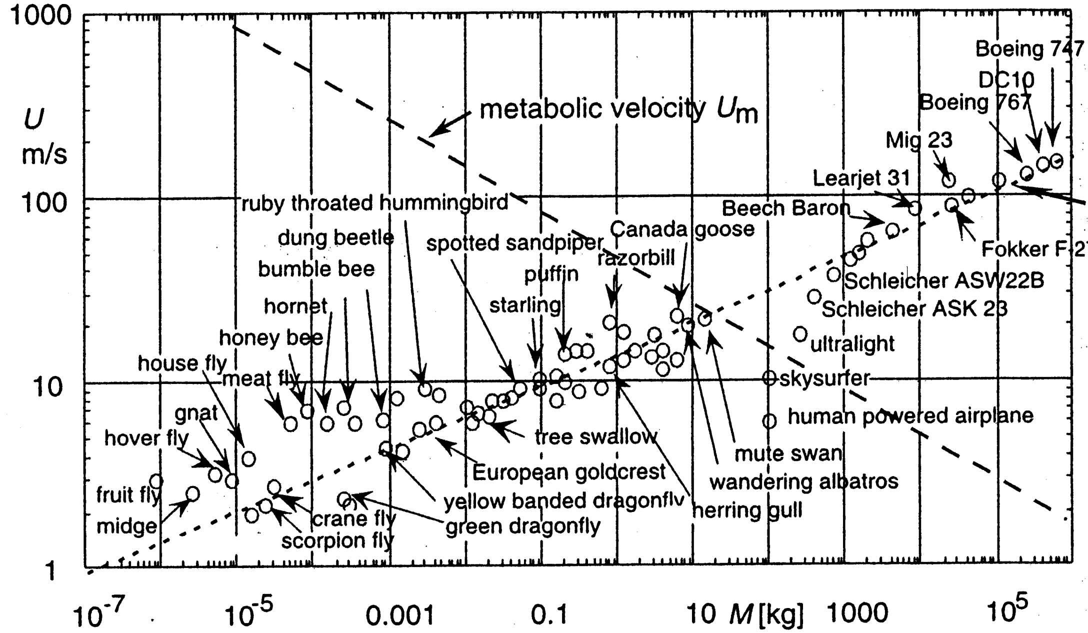


**Fig. 6.20.** The great flight diagram. Data adopted from Tennekes [1997]

scorpion fly to a Boeing 747. Sky surfers and ultra lights, do not fit in because they use updrafts to stay aloft.

The Gossamer Albatross, built by the team of McReady, crossed the English Channel to win the Kremer Prize. It was powered by a cyclist, who produced a steady 125 W of mechanical power. The ultralight flew very close to the water surface to utilize the ground effect. Halfway across the Channel it encountered a tanker, and in order to avoid a collision the aviator pulled the plane up and over the ship. This required so much extra effort that the cyclist did not have enough energy left to fly the plane up the beach to the waiting dignitaries. Instead he crash-landed the Gossamer Albatross into the surf of the French coast.

Birds have to overcome the drag resistance $F_D = \frac{1}{2} C_D A_f \rho U^2$, which depends on the front surface area $A_f$, the speed $U$, and the drag coefficient $C_D$. If they fly the distance $d$ they will expend the energy

$$\Delta E = F_D d\,. \tag{6.24}$$