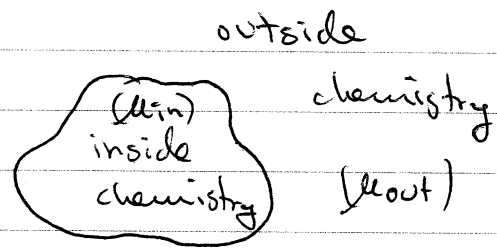


# • Packaging of Cells: Membrane Physics

- Why do cells have membranes? what are the properties of membranes?

Why?



- cell can regulate its internal chemistry if it is packaged

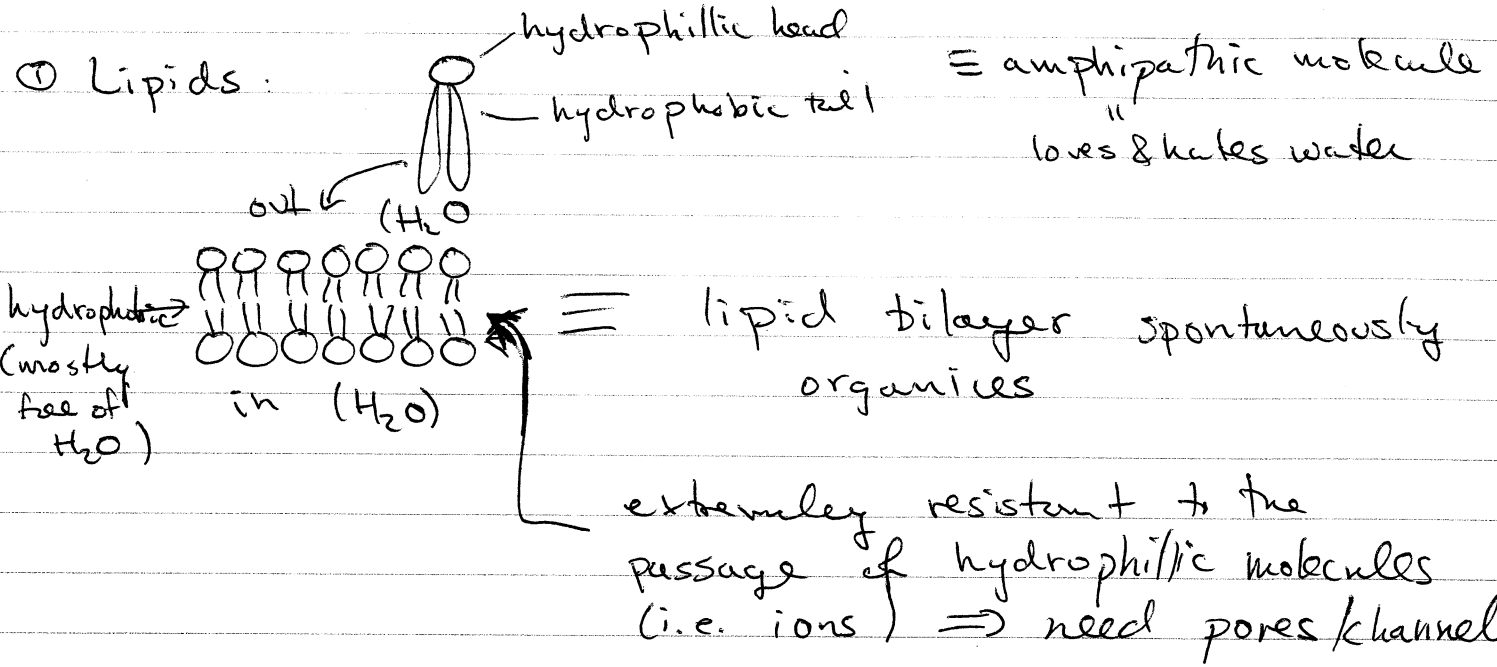
• differences in chemical concentrations between inside & outside  $\Rightarrow$  differences in chemical potential  $\Rightarrow$  WORK

- Protection from bad chemicals - regulated transport
- Can organic molecules/receptors on surface to receive the signals that the cell cares about.

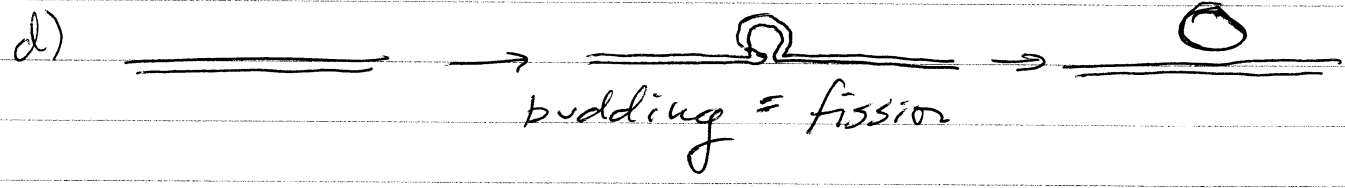
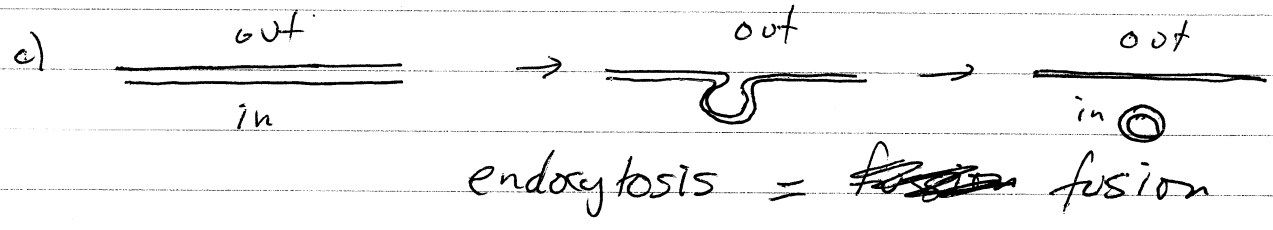
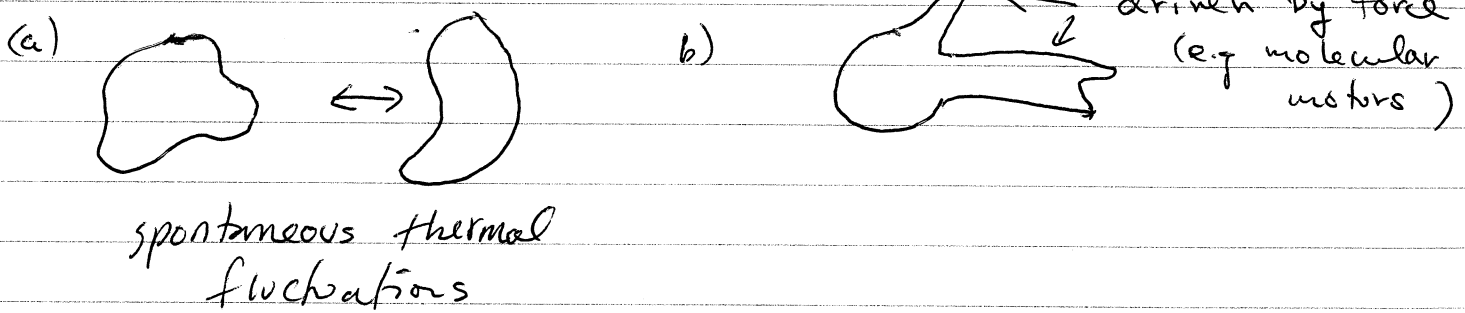
## Properties:

- malleable (can change topologies)
- flexible
- semi-permeable

# Membrane Composition

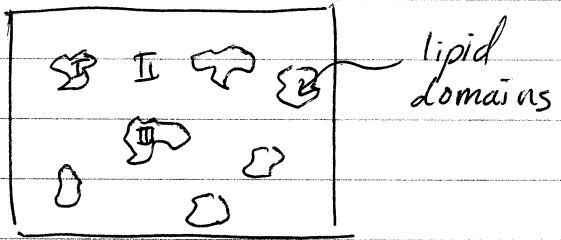


# Membrane Shapes:



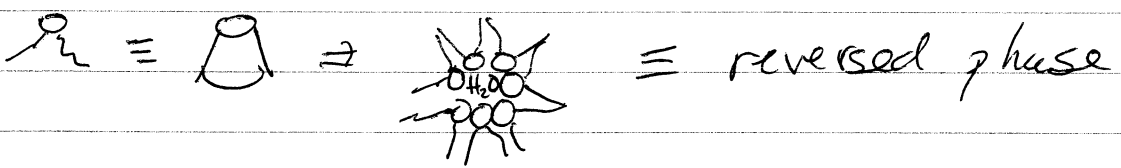
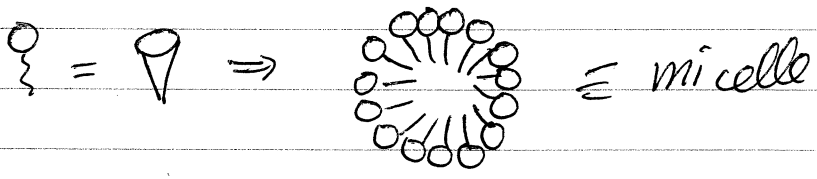
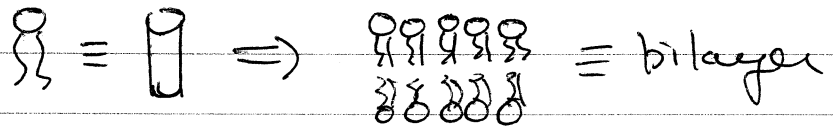
Membrane is fluid with complex phases:

- membranes are made up of different types of lipids, cholesterol, and many types of proteins.
- The different types of lipids often separate into different domains that have different fluid properties



• small domains  $\equiv$  "rafts" and are thought to be important for organizing receptors

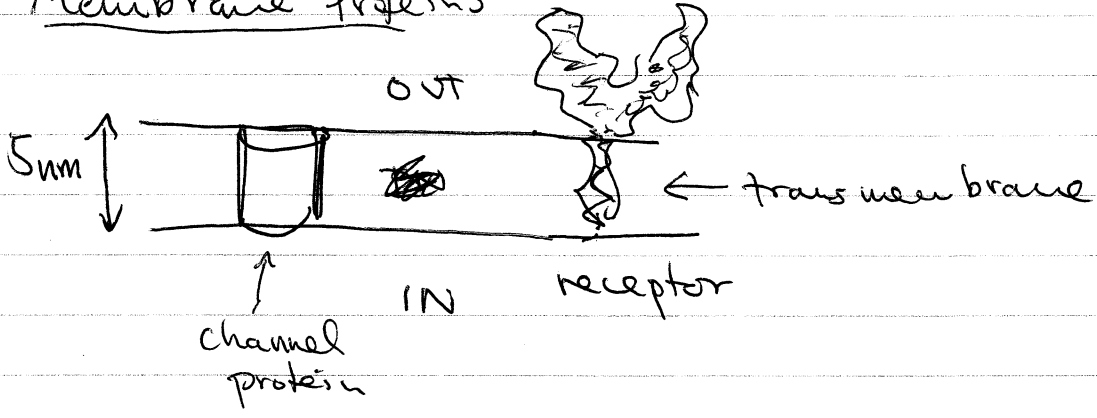
Lipid shape / geometry affects membrane curvature



• Combining different shapes  $\Rightarrow$  different structural properties.

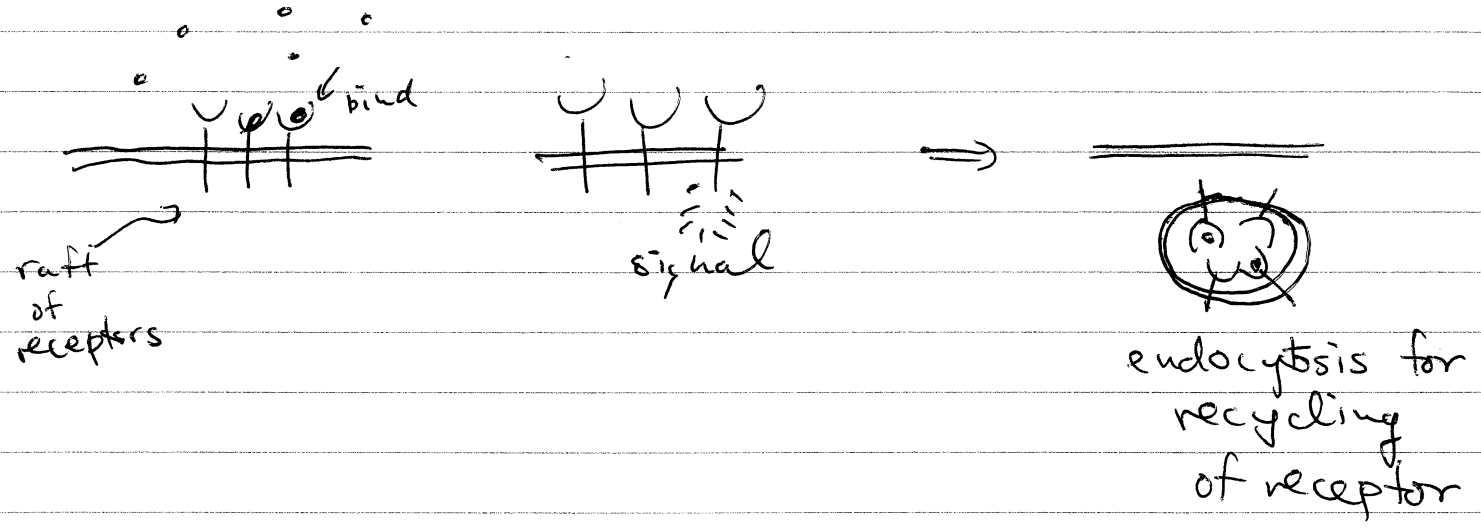
$\Rightarrow$  Different cell types have different lipid compositions.

# Membrane Proteins



(very hydrophobic, typically  $\alpha$ -helices & hard to determine the structure of)

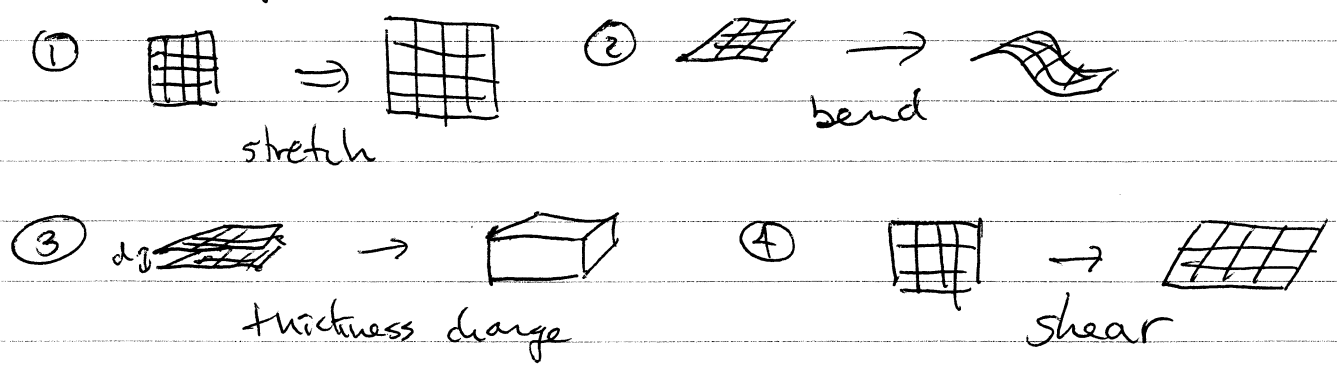
# Signal Reception:



# Membrane Energetics

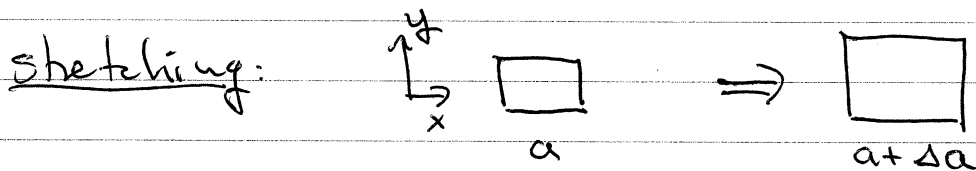
• It costs energy to deform a membrane

## 4 changes

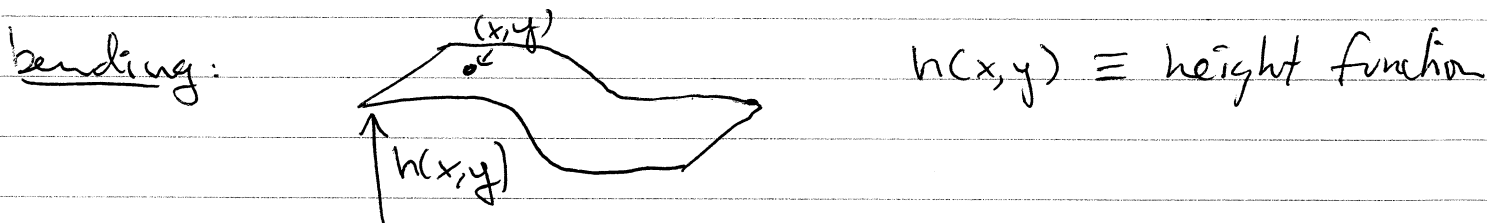


- ① stretching  $\rightarrow$  micropipette, tentril formation
- ② bending  $\rightarrow$  action of motors on membrane motion
- ③ thickness  $\rightarrow$  ion channels
- ④ shear  $\rightarrow$  red blood cell spectrin network

Quantifying the 4 modes



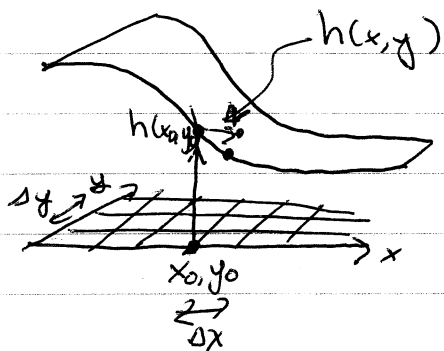
•  $\Delta a(x,y)$  gives area change @  $(x,y)$  :-  
membrane



• for small deformations near  $x,y$ , the height function will be assumed to be parabolic

$$h(x,y) = h(x_0, y_0) + \frac{\partial h}{\partial x} \Delta x + \frac{\partial h}{\partial y} \Delta y + \frac{1}{2} \left( \frac{\partial^2 h}{\partial x^2} \Delta x^2 + 2 \frac{\partial^2 h}{\partial x \partial y} \Delta x \Delta y + \frac{\partial^2 h}{\partial y^2} \Delta y^2 \right)$$

Interpret:



where  $x = x_0 + \Delta x$

$y = y_0 + \Delta y$

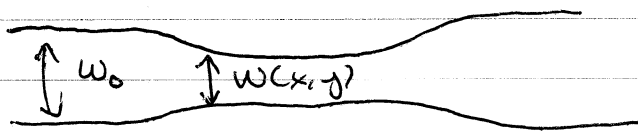
where  $\frac{\partial^2 h}{\partial x_i \partial x_j} \equiv$  curvature matrix

$$= \begin{pmatrix} K_{xx} & K_{xy} \\ K_{xy} & K_{yy} \end{pmatrix}$$

• eigenvalues/vectors  $\rightarrow$  give the principle curvatures & directions =  $K_1$  &  $K_2$

• for a sphere the 2 principle curvatures  $\equiv \frac{1}{R}$   
where  $R \equiv$  radius of sphere.

Thickness:



•  $w(x,y)$  is the thickness of membrane @  $(x,y)$

Shear



$\theta \equiv$  shear angle

## Free energy of membrane deformations

$$F_{\text{stretch}} = \frac{K_A}{2} \int \left( \frac{\Delta a}{a} \right)^2 dA$$

$$F_{\text{bend}} = \frac{K_b}{2} \int dA (K_1(x,y) + K_2(x,y))^2$$

$$F_{\text{thickness}} = \frac{K_t}{2} \int dA \left( \frac{w(x,y) - w_0}{w_0} \right)^2$$