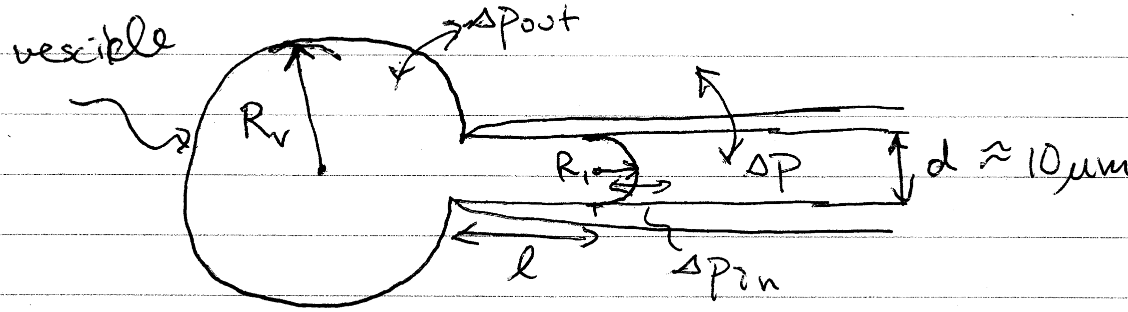
Applications:

How do we measure the energies of membrane deformations?

Micropipette Experiments:

- use a tiny pipepette to apply pressures to lipid membranes to see what deformations result.



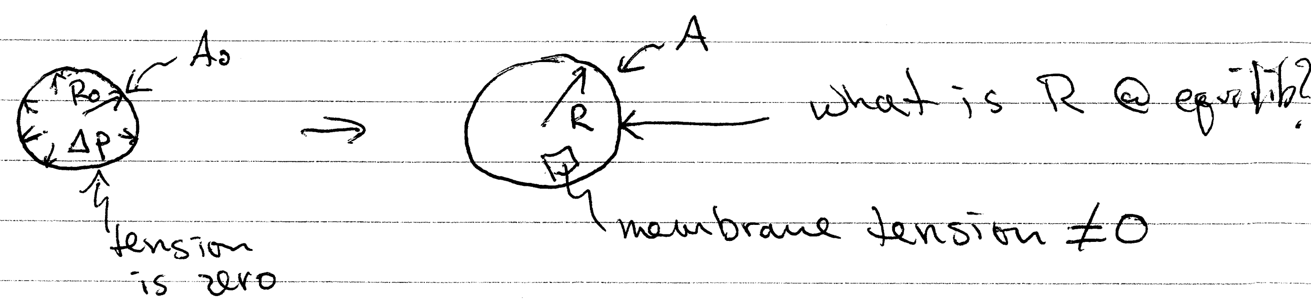
Relating the geometry to the pressure:

$\Delta p_{out} \equiv$ difference in pressure between inside and outside of vesicle

$\Delta p_{in} \equiv$ diff in pressure between inside & outside of bleb

$$\Delta p = \Delta p_{in} - \Delta p_{out}$$

Some physics: What radius does a soap bubble take?



Energy:

$$G_{stretch}(R) = \frac{K_A}{2} \frac{(A(R) - A_0)^2}{A_0} \quad \text{(for a sphere)}$$

$$G_{work}(R) = -\Delta p \left(\frac{4}{3}\pi R^3\right)$$

so

$$G_{tot}(R) = G_{stretch}(R) + G_{work}(R)$$

Equilibrium R from $\partial G / \partial R = 0$

$$\frac{\partial G_{tot}}{\partial R} = K_A \frac{(A - A_0)}{A_0} \frac{dA}{dR} - 4\pi \Delta p R^2$$

$$= \underbrace{K_A \frac{(A - A_0)}{A_0}}_{= \text{ tension}, \tau} (8\pi R) - 4\pi R^2 \Delta p$$

$$= 8\pi R \tau - 4\pi R^2 \Delta p = 0$$

so

$$\Delta p = \frac{2\tau}{R} \quad \left(\text{or } R = \frac{2\tau}{\Delta p}\right)$$

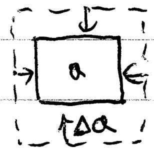
$\equiv$ Laplace - Young relation

Aside: Tension:

- Tension is the isotropic force / length that all surfaces exert when stretched.

Just like for rods where we had $\underset{\text{stress}}{\sigma} = E \underset{\text{strain}}{\varepsilon}$

Now



$$\underset{\text{tension}}{\tau} = K_A \left(\frac{\Delta a}{a}\right) \quad \left[\frac{N}{m}\right]$$

- Let's apply the Laplace-Young relation to the micropipette expt, to see how to measure K_A of a membrane.

- For the vesicle with radius, R_v, L-Y gives,

$$\Delta P_{out} = \frac{2\tau}{R_v}$$

- For the extrusion in the micropipette:

$$\Delta P_{in} = \frac{2\tau}{R_1}$$

- Using $\Delta P = \Delta P_{in} - \Delta P_{out}$ gives,

$$\Delta P = \frac{2\tau}{R_1} - \frac{2\tau}{R_v}$$

Solving for the tension, τ gives:

$$\tau = \frac{\Delta p}{2} \frac{R_1}{1 - \frac{R_1}{R_v}} \equiv y\text{-axis}$$

- Thus in expt, we control Δp, & for each Δp can measure R_1 and R_v to find the tension.
- Can relate this to K_A via, $\tau = K_A (\Delta a/a)$
- Now Δa is the area change due to extrusion in pipette, so

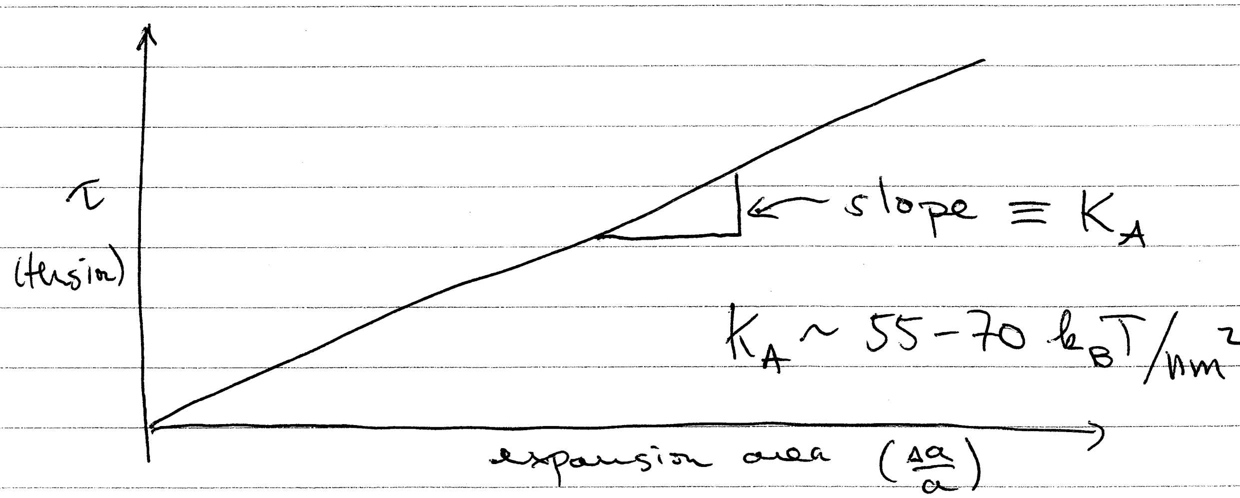
$$\Delta a = 2\pi R_1 \ell + 2\pi R_1^2$$

so

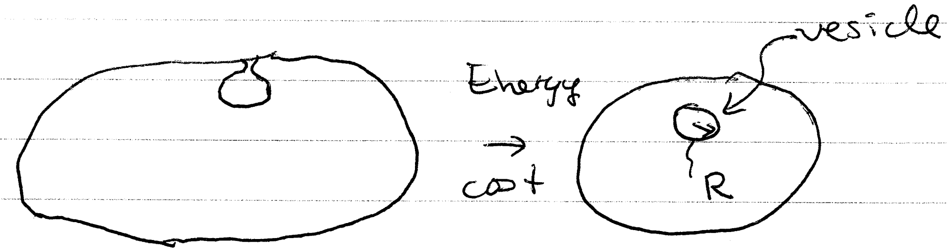
$$\frac{\Delta a}{a} = \frac{2\pi R_1 \ell + 2\pi R_1^2}{\underset{\substack{\uparrow \\ \text{assume } R_v \\ \text{doesn't change}}}{4\pi R_v^2}} = \frac{R_1^2 (1 + \ell/R_1)}{2 R_v^2}$$

so

$$\tau = K_A \underbrace{\left[\frac{R_1^2 (1 + \ell/R_1)}{2 R_v^2} \right]}_{x\text{-axis}}$$



Making Vesicles Costs Energy



- The energy cost is associated with bending the flat membrane into a sphere with $K_1 = K_2 = \frac{1}{R}$

so

$$F_{vesicle} = \frac{K_B}{2} \int \left(\frac{2}{R}\right)^2 dA$$

constant for spherical vesicle

$$= \frac{K_B}{2} \left(\frac{2}{R}\right)^2 4\pi R^2 = 8\pi K_B$$

so

$$F_{vesicle} = 8\pi K_B \quad \text{(indep of radius!)}$$

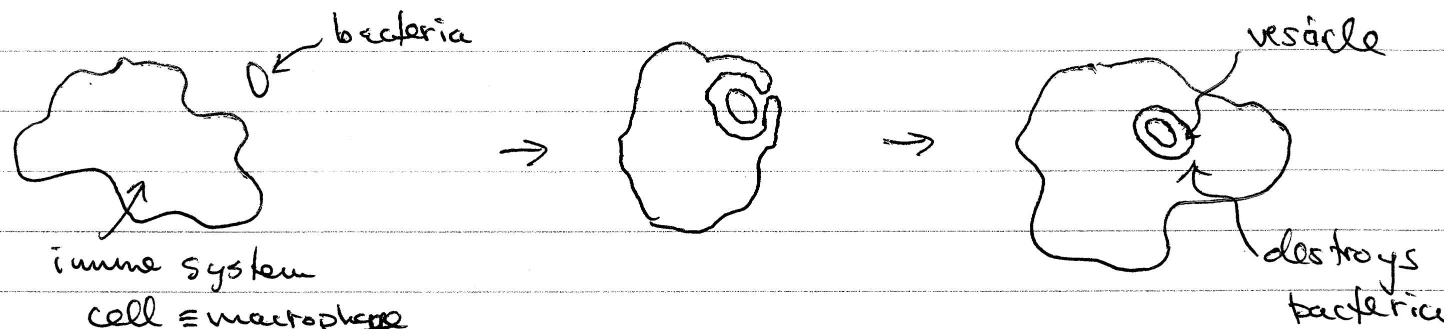
- Thus no matter what the vesicle size, there is a fixed cost to the cell.

- For most membranes, $K_B \sim 10-20\, k_B T$, so $F_{vesicle} \approx 250-500\, k_B T \sim 25$ ATP molecules.

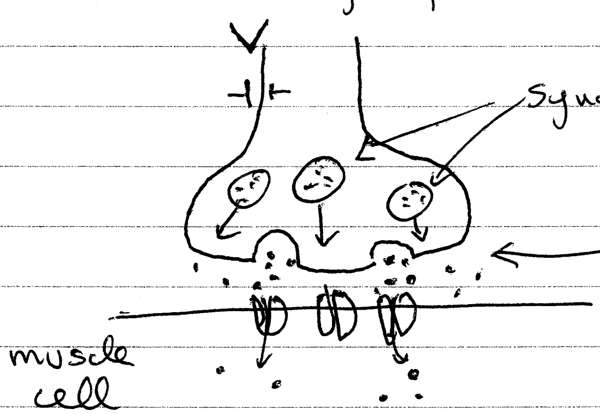
- There are active proteins which use ATP to form vesicles and hence overcome the high energy cost of formation

What do vesicles do?

- Vescicles are used by cells to transport molecules within cells and outside of cells
- Trap object (e.g. bacterium by macrophage)



- Neurons & synapses



synaptic vesicles (contain molecule which activates receptors on target cell)

vesicles fuse to membrane and dump contents into synaptic cleft.

Shapes of Cells:

- There is a huge variety in cell shape
- Provides ability to specialize function and hence gives a competative advantage.
- Can organize into communities to make new function (ie tissue

What about entropy?

- So far we have only been considering the mechanical energy associated with deforming a ~~[illegible]~~ membrane.
- For a given mechanical energy there may be a variety of different shapes $\Rightarrow$ an entropy.
- Considering all these shapes (just like in the worm-like chain) makes the problem very challenging.
- For the micropipette expt, the force exerted by the pipette stretches out all the membrane fluctuations ($\equiv$ entropy), so we are OK just looking at the mechanical energy.

Now onto diffusion...