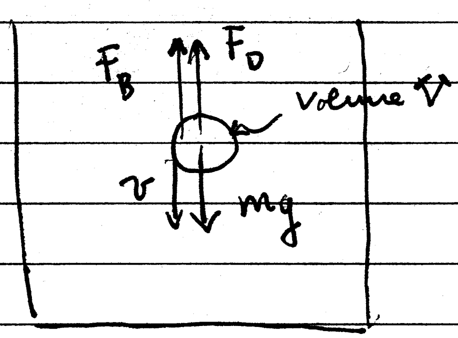
Phys 347 Topic 4 Swimming in the Nanoworld

- Outline:
  - previously we showed that diffusion was the main way that molecules move around inside cells
  - Does water @ the nano-scale behave the same way as we experience it?
  - Do bacteria swim the same way as fish?
  - In order to answer these questions we need to work out some physics of fluids.
  - This will lead us into discussing viscosity — something which you may not have seen yet

Sedimentation



- Consider a particle drifting down in a fluid, with a velocity v
- There are 3 forces:

mass of liquid $= \rho V$

gravity: $F_g = mg$ boyancy: $F_B = m_w g$ Drag: $F_D = cv$

@ equilibrium $F = 0$, so $mg - m_w g - cv = 0$

So *drift velocity*: $v = \frac{(m - m_w)g}{c} = \frac{m_{net}\, g}{c}$

where $m_{net} = m - \rho V$

- So will all solutions get unmixed? Ans: no, depends on the comparison between the thermal motion & gravitational energy.

- For settling; $m_{net}\, g\, z \approx kT$ or $z = \frac{kT}{m_{net}\, g}$

e.g.: for myoglobin, $m = 17000$ g/mol & m_{net} (in H_2O) $\approx 0.25\, m$

this gives $z = 59$ m!

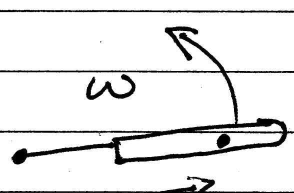
so in a 4 cm test tube $\frac{c(4\,\text{cm})}{c(0)} \approx e^{-\frac{mgh}{kT}} \approx e^{-0.04/59}$

$= 0.999$

∴ myoglobin solution remains well mixed.

Q? How do we separate out proteins from solution?

$z = \frac{kT}{m_{net}\, g}$ can increase g by using a centrifuge



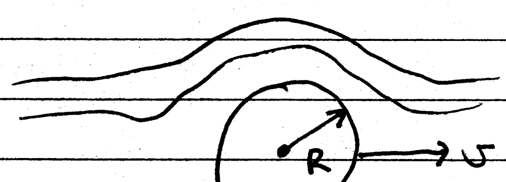
$g' \sim r\omega^2 \sim 10^6\, \text{m/s}^2 \approx 10^5\, g$!

Drift Velocity:

$$v_D = \frac{m_{net}\, g}{c}$$

- Clearly v_D depends on liquid "thickness" and the size of the particle. Where do these appear in the above? Ans. in the drag coefficient, c.

Stoke's formula: $\boxed{c = 6\pi \eta R}$ $\qquad [\eta] = \text{Pa}\cdot\text{s}$



viscosity, η

- Stoke's formula is valid for a sphere in laminar flow

- It depends on size of object, R & viscosity, η.

#'s: For water, $\eta_w = 10^{-3}$ kg/m/s $= 10^{-3}$ Pa·s

~~Drift velocity in model:~~

~~$m = 70$ kg, $R = 0.25$ m $\rightarrow c = 0.0023$~~

~~$m_{net} = \ldots \frac{1}{1.06}\ 70$ kg $= 66$ kg~~

~~$m_{net} = 4$ kg~~

What is viscosity?

- Do mixing experiment.

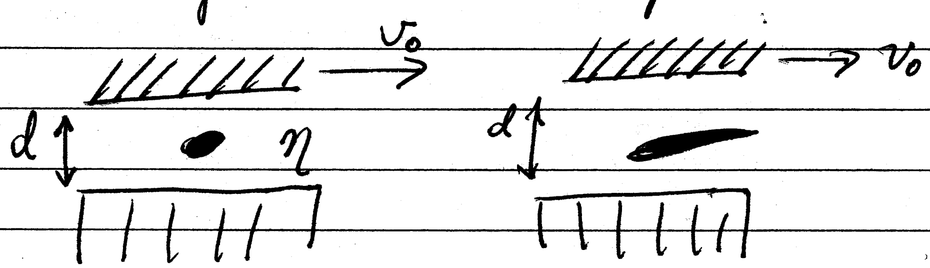
Find: it's hard to mix viscous liquids

- the ink drop experienced "laminar flow"
- cream in coffee experiences "turbulent flow"

2 regimes: laminar vs turbulent

- We'll see that these behaviours are determined by the viscosity but also the size ~~of~~ of the particle.

- Viscosity $\equiv$ shear drag force



- Consider the drag force experienced by sliding a plate of area A @ speed v_0 over a liquid with viscosity η.

- What does the drag force, f depend on?

$f \propto A$; $f \propto v_0$; $f \propto \frac{1}{d}$ ← bigger d → less drag

So $$\boxed{f = \eta \frac{v_0 A}{d}}$$ (opposes motion)

- This allows us to define & measure viscosity
- So 2 physical properties that can specify a fluid: viscosity, η & denisty, ρ
- Dimensional analysis: using just ρ & η we can define a quantity that has units of force.

$$\boxed{f_c = \frac{\eta^2}{\rho}}$$

Proof: $[f] = \left[\frac{M}{LT}\right]^2 / \left[M/L^3\right] = \frac{M^{\cancel{2}}}{\cancel{L^2}T^2}\frac{L^{\cancel{3}}}{\cancel{M}} = \frac{ML}{T^2} = N$

- Can create a dimensionless number to characterize "thickness":

$\frac{f}{f_c} \ll 1 \rightarrow$ thick ~~[illegible]~~ laminar flow

$\frac{f}{f_c} \gg 1 \rightarrow$ turbulent flow

#'s	ρ (kg/m³)	η (Pa·s)	f_c (N)
air	1	2×10^{-5}	4×10^{-10}
water	1000	0.0009	8×10^{-10}
corn syrup	1000	5	0.03

Water: water will be viscous for forces $\sim 8\times10^{-10}$ N = 1 nN

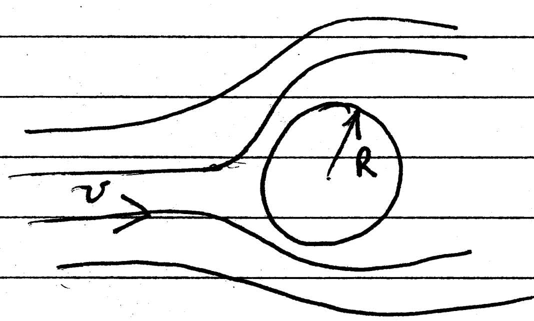
Biology: For a cell, forces ~ 1 pN → Water is Viscous!

- Thus for cells, water appears to be very viscous! This is very different than how we experience H_2O.

Reynold's Number:

- Previous section showed that the character of a fluid depends on the size of the external force & changed @ $f_c = \eta^2/\rho$

- We can say more though. Consider an object in a fluid — will the flow be laminar or turbulent?



- The liquid experiences 2 forces

① inertial force due to colliding @ sphere
② viscous drag force

Idea: Ⓐ if inertial force > viscous force → turbulent

Ⓑ if viscous force > inertial force → laminar

$$\frac{\text{inertial force}}{\text{viscous force}} \equiv \boxed{R = \frac{v R \rho}{\eta}}$$

≡ Reynold's number

So when an object of size R moves through a fluid with speed v, if $\mathcal{R} > 1 \rightarrow$ turbulence & $\mathcal{R} < 1 \rightarrow$ laminar

e.g.: Swimming whale: $R \sim 10\,m$ & $v \approx 10\,m/s$

$$\text{so} \quad \mathcal{R} = \frac{(10\,m/s)(10\,m)(1000\,kg/m^3)}{(0.001\,Pa \cdot s)} = 10^8 \; ! \gg 1$$

$\rightarrow$ turbulence

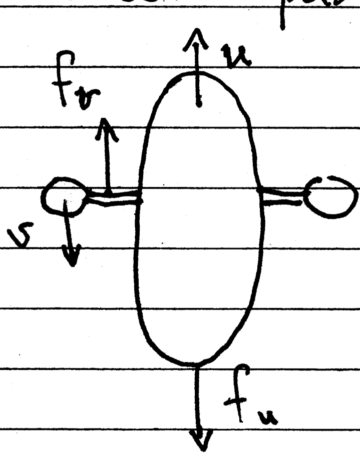
Swimming bacteria: $R = 10^{-6}\,m$ $\quad v = 30 \times 10^{-6}\,m/s$

$$\mathcal{R} = \frac{(30 \times 10^{-6})(10^{-6})(1000)}{(0.001)} = 3 \times 10^{-5} \ll 1$$

$\rightarrow$ laminar & dominated by friction.

How to swim?

- how do bacteria swim in such a frictional liquid when the flow is laminar?
- Consider pushing with paddle motions



- move paddles relative to body with speed v
- liquid pushes on paddle with force $f_v \approx c_v(u - v)$
- drag on bacteria $f_u = c_u u$

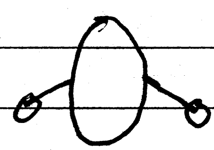
- Since moving with constant velocity $\Rightarrow$ $f_v = f_u$

$$\Rightarrow \quad u = \frac{c_1 v}{(c_1 + c_2)}$$

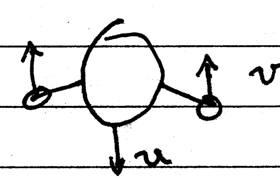
- distance moved: $\Delta x = u \cdot \Delta t$

- However, need to return paddles on back stroke along the same path. Thus the bacteria ends up pushing back

after Δt



back stroke

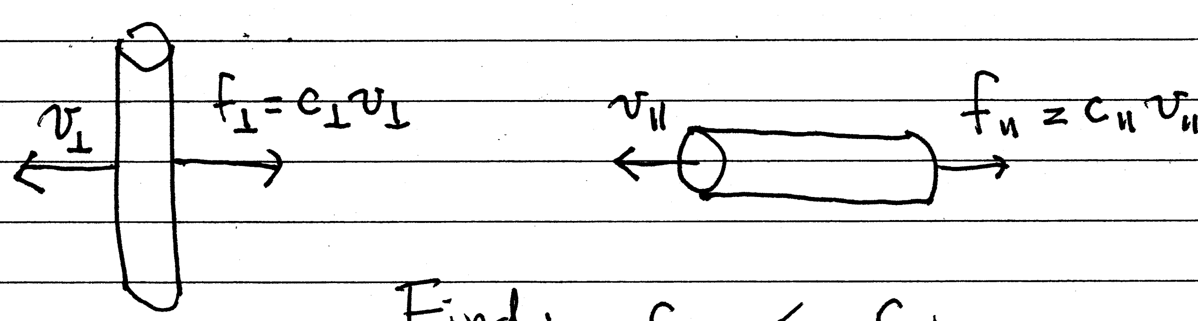


- Since motion is reciprocal $\rightarrow$ $\Delta x' = -\Delta x$ & the bacteria doesn't go anywhere!

- To move/swim, motion can <u>not</u> be reciprocal.

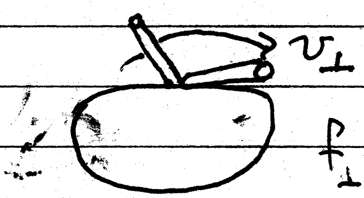
- <u>Key:</u> viscous drag coefficient depends on shape! Therefore must change the shape of paddle on power stroke vs. back stroke.

<u>e.g.</u> Rod



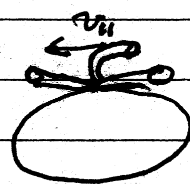
<u>Find:</u> $c_\parallel < c_\perp$

Propulsion by Cilia



$f_\perp \sim c_\perp v_\perp$

Power

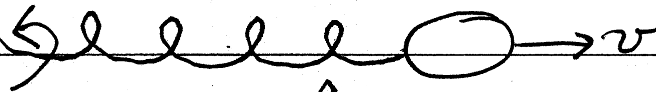


$f_\parallel \sim c_\parallel v_\parallel$

backstroke

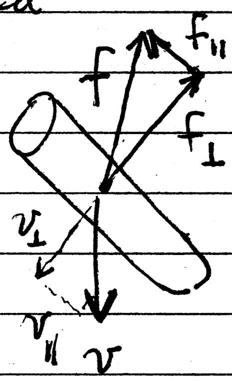
and $f_\perp > f_\parallel$ → net force forward!

Swimming E coli



How does helical flagellum develop force?

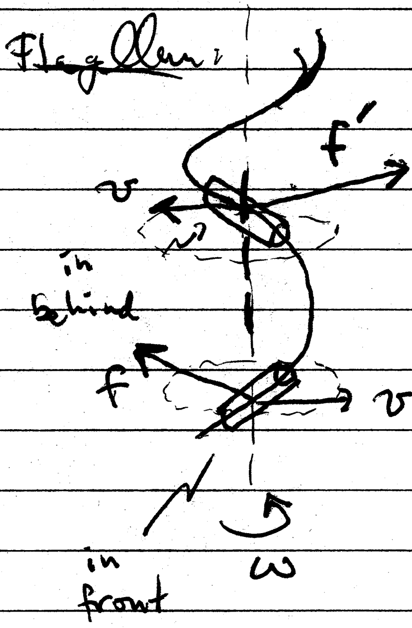
First consider the drag on a rod which is being pulled



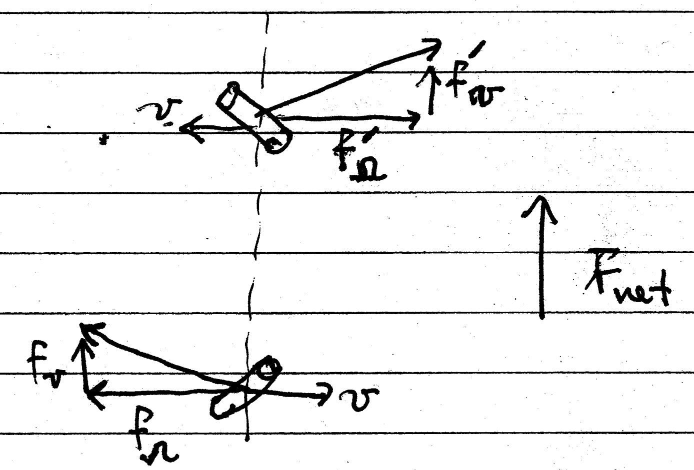
• Because $\frac{f_\perp}{f_\parallel} = \frac{c_\perp v_\perp}{c_\parallel v_\parallel} \neq \frac{v_\perp}{v_\parallel}$

Key: so $\vec{f}$ is not in the same direction as $\vec{v}$!

What implication does this have for a helical flagellum?



$\Rightarrow$

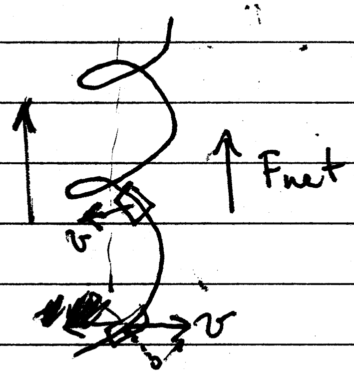


- Forces: $\perp$ to axis: $\vec{f}_\Omega = -\vec{f}_{\Omega'}$ so forces cancel

  $\parallel$ to axis: $\vec{f}_v = \vec{f}'_v$ so forces ADD!

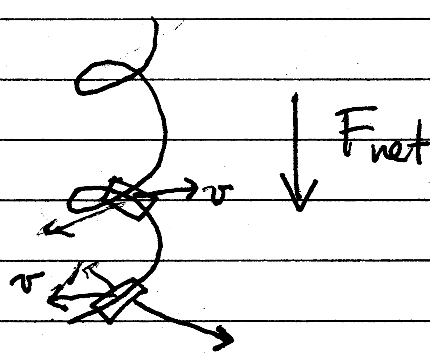
  $\therefore$ there is a NET force $\parallel$ to helix axis.

- This net force propels bacteria



RH helix

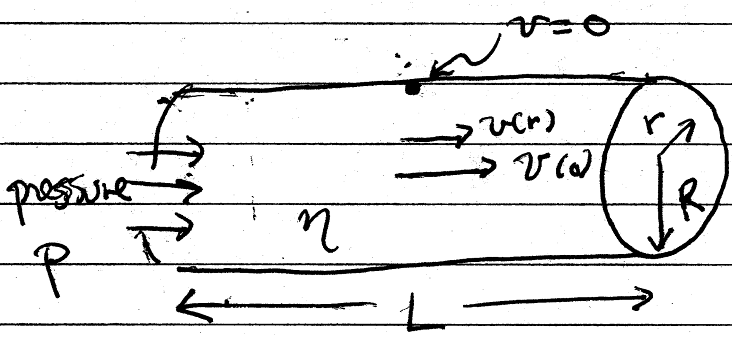
CCW



RH helix

CW

How does viscosity affect flow in a pipe?



- The walls drag the fluid as does the viscosity of the fluid.

Solving fluid dynamics $\Rightarrow$ $v(r) = \dfrac{(R^2 - r^2)\,p}{4L\eta}$

makes sense:

$p\uparrow \rightarrow v\uparrow$ more pressure

$L\uparrow \rightarrow v\downarrow$ longer pipe

$\eta\uparrow \rightarrow v\downarrow$ thicker liquid

Flow rate:

$$Q = \int_0^R 2\pi r\,dr\,v(r) = \frac{\pi R^4}{8L\eta}\,p$$

or

$$\boxed{Q = \frac{p}{Z}}$$ where $Z = \dfrac{8L\eta}{\pi R^4} \equiv$ resistance

(Ohm's Law for pipes)

- Interesting fact: $2\times R \rightarrow 8\times$ the flow

e.g.: 2 Pipes $\rightarrow$ $2Q$ but one pipe @ $2\times$ Area $\rightarrow$ $4Q$!