Mathematics of Sums:

Notation:

\[ x_1 + x_2 + x_3 + \ldots + x_N = \sum_{i=1}^{N} x_i \]

dummy index

Adding or subtracting:

\[ \sum_{i} x_i + \sum_{j} x_j = 2 \sum_{i} x_i \]

\( (x_1 + x_2 + x_3 + \ldots) + (x_1 + x_2 + \ldots) \)

\[ \sum_{i} x_i^2 - \sum_{i} x_i = \sum_{i} (x_i^2 - x_i) \]

Sum of constants:

\[ \sum_{i=1}^{N} c = (c+c+\ldots) = Nc \]

a constant

Product of sums:

\[ (\sum_{i} x_i)(\sum_{j} x_j) = \sum_{i} \sum_{j} x_i x_j \]

IT'S NOT \[ \sum_{i} x_i^2 \]

\( (x_1 + x_2 + x_3 + \ldots)(x_1 + x_2 + \ldots) = (x_1^2 + x_1 x_2 + x_1 x_3 + \ldots x_2^2 + x_2 x_1 + \ldots) \)

\[ = \sum_{i} x_i^2 + \sum_{i<j} x_i x_j \]

↑ sum over \( i \& j \) with \( j \) not equal to \( i \)