Policy and Welfare Effects of Within-Period Commitment

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Abstract

I study the implications of different institutional frameworks for the conduct of fiscal policy, under the assumption that the government cannot commit to future policy choices. The environments analyzed vary on whether the government is endowed with the ability to commit to beginning-of-period policy announcements or not. If it cannot, then there are two variants, depending on which actions private agents take before observing the government's policy choice. How the three possible cases rank in terms of tax rates and welfare varies substantially with the economy's fundamentals and whether depreciation is tax deductible or not. More generally, I find that regimes with higher tax rates do not necessarily imply lower welfare. I also find that making depreciation not tax-deductible typically involves a welfare loss. Within the context of the environments studied in this paper, I find that there are only small gains from modifying the way fiscal policy is conducted in modern developed economies. Furthermore, some reforms may lead to large welfare losses.

Keywords: fiscal policy; time-consistency; lack of commitment; Markov-perfect equilibrium.

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1 Introduction

In modern developed economies, fiscal budgets are typically proposed and approved on an annual basis. Although there is some ex-post discretion in the form of supplemental appropriation and emergency bills, fiscal policy is mainly determined at the beginning of each fiscal year. This suggests the presence of institutions that endow the government with the ability to commit to policy announcements within a period. In economies where the government does not have access to lump-sum taxes and cannot commit to policy choices in future periods, one may wonder about the implications on welfare and policy of within-period commitment.

In a recent paper, Ortigueira (2006) adopts the notions of commitment suggested by Cohen and Michel (1988) and compares an environment in which the government is able to commit to policy choices within a period—as in Klein, Krusell and Ríos-Rull (2008)—with one in which the government lacks this intra-period commitment and thus, moves simultaneously with private agents—as in Turnovsky and Brock (1980) and Judd (1998). Ortigueira concludes that an economy where the government is endowed with within-period commitment features higher tax rates and lower welfare, and attributes this result to the different way in which governments internalize policy distortions across fiscal regimes. This analysis raises some questions and concerns which need to be addressed before we start deriving implications on institutional design.

First, how are policy and welfare related to economic fundamentals? It seems particularly important to understand how the size of government—which determines the size of policy distortions—affects the relative behavior of policy under the different regimes. In a related point, small institutional modifications to the environment—e.g., making capital depreciation not tax deductible—may alter the findings significantly.[1] Second, for the case with simultaneous actions (i.e., without within-period commitment), the quantitative results Ortigueira reports apply only to the case where households choose consumption, and savings are determined residually. The alternative specification—where instead consumption is determined residually—may offer higher welfare in some settings. Furthermore, it is conceivable that under certain circumstances, the two variants of simultaneous actions have very different implications for policy and welfare. If this is the case, it is important to identify these circumstances, as they would affect the type of institutional reform that we would recommend. Third, due to the potential existence of multiple equilibria, we need to ensure that the equilibria we compare correspond to the same class, i.e., survive the same set of refinements.[2]

In this paper, I revisit the question of how the institutional design of fiscal policy affects tax rates and welfare, and address the concerns listed above. I start by defining a Markov-perfect equilibrium[3] for each of the following three environments with different within-period timing of actions: government moves first; simultaneous actions with consumption as residual; and simultaneous actions with investment as residual. All these cases assume the government cannot commit to policy choices in *future* periods. The difference lies in whether the government is endowed with within-

[1]See Guo and Lansing (1997) and Guo and Lansing (1999) for an analysis of the effects of the depreciation allowance in a standard Ramsey model of optimal taxation. See also Pecorino (1993) and Stokey and Rebelo (1995) for how tax structure affects growth.

[2]Among others, Krusell and Smith (2003) and Martin (2009*c*) report coexistence of differentiable and non-differentiable Markov-perfect equilibria. Ortigueira and Pereira (2008) report multiplicity of differentiable Markov-perfect equilibria.

[3]Maskin and Tirole (2001) provide a definition and justification of this equilibrium concept.

period commitment and, in the case that it is not, which variable is chosen by private agents before they observe the government's policy choice. The two cases with simultaneous actions have different implications on the problem faced by the government. If consumption is determined residually, then the government faces a static problem since it cannot affect current capital accumulation. If savings are determined residually, then the problem of the government is dynamic. For all cases considered, the government is subjected to a balanced-budget constraint (i.e., there is no public debt), an assumption that is consistent with the literature cited above.

Under suitable assumptions—most notably, 100% (not tax-deductible) capital depreciation—I provide analytical solutions for the infinite-horizon equilibrium in each case. These equilibria are the limit of finite horizon equilibria, which is a refinement I maintain throughout the paper to address the concern that comparisons across tax regimes may not be valid if we are comparing different types of equilibria. The analysis yields two main results. First, higher tax rates are not necessarily associated with lower welfare, a point also raised previously by Klein, Krusell and Ríos-Rull (2008). Second, the environment with within-period government commitment implies the highest welfare, in contrast with the findings in Ortigueira (2006), under partial tax-deductible capital depreciation. These results motivate further exploration.

I then relax the assumption of full capital depreciation and rely on numerical analysis. I show that how the three tax regimes rank in terms of tax rates and welfare varies significantly with the depreciation rate. Thus, the analytical results are a special case in this respect. For depreciation rates close to those measured for developed economies, within-period commitment actually implies the lowest welfare, even though it does not feature the highest tax rate. I also show that differences across regimes are only significant for sufficiently large government expenditure. This is to be expected since all the differences come from how distortions are internalized by the government.

Next, I consider that case of tax-deductible capital depreciation (the standard assumption in the literature) and obtain the following results. First, the lowest tax rates correspond to the case with within-period commitment (government moves first). Second, the highest welfare is implied by the case with simultaneous actions with consumption determined residually (i.e., when the government faces a static problem); the steady state of this regime coincides with the long-run allocation of the Ramsey (full intertemporal commitment) problem. Third, the case with simultaneous actions and residual savings features high taxes and (very) low welfare due to the large dynamic inefficiencies that arise when capital depreciation is tax deductible.

As for the question of how the depreciation allowance affects welfare, I find the following. First, welfare is typically higher if capital depreciation is tax deductible.[4] When the government moves first, the welfare loss of making depreciation not tax deductible is of the order of 1% of steady state consumption. Second, differences across tax regimes appear more pronounced when depreciation is tax deductible.

The paper is organized as follows. Section 2 describes the environment and defines the Markov-perfect equilibrium for each of the cases considered. Section 3 derives some analytical results under the assumption of full not tax-deductible capital depreciation. Sections 4 and 5 perform the numerical analysis described above. Section 6 concludes.

[4]The exception being the case with simultaneous actions and residual savings, when it is subjected to large dynamic inefficiencies as described in the preceding paragraph. See section 5.1 for a detailed explanation.

2 The Basic Model

2.1 Environment

Consider an economy populated by a continuum of identical households. Time is discrete and indexed by $t = 0, \ldots, T$, where $T \leq \infty$. Households are endowed with one unit of time per period for work and leisure, and may accumulate capital, which depreciates at rate $\delta \in [0, 1]$ every period.

Lifetime utility of a representative household is given by

$$\sum_{t=0}^{T} \beta^t u(c_t, 1 - n_t, G_t),$$

where $\beta \in (0, 1)$ is the discount factor, c is consumption of an homogenous private good, n is labor and G is consumption of a public good. The instantaneous utility function u is continuously differentiable, strictly increasing and strictly concave in all its arguments.

Firms maximize profits and use capital and labor as inputs. They produce the private good using a constant returns to scale production function $F(K, N)$, where F is differentiable, strictly concave and strictly increasing in both arguments. Markets are competitive and thus, factor payments r and w equal the marginal product of capital and labor, respectively.

There is a benevolent government that supplies the public good, G financed with revenue from taxes τ on household income. Assume for now that capital depreciation is not tax deductible. The government period budget constraint is

$$G_t = \tau_t F(K_t, N_t).$$

Given factor prices r_t and w_t, and income taxes τ_t, the budget constraint of the household is

$$c_t + k_{t+1} = (1 - \tau_t)(r_t k_t + w_t n_t) + (1 - \delta) k_t.$$

Assume the government cannot commit to policies beyond the current period. Further assume that households and the government base their decisions solely on fundamentals.[5] Thus, let us analyze Markov-perfect equilibria, i.e., equilibria where aggregate household behavior and government policy depend only on the aggregate capital stock.

Household behavior and government policy will depend on the timing of decisions within a period. There are three cases to be considered. First, suppose the government has the ability to commit to a policy announcement within a period. Then, it will move first by announcing the current tax rate τ_t at the beginning of each period. Alternatively, suppose the government lacks commitment power even within a period. In this case, households and the government move simultaneously. However, since households do not observe the government's action until their decision is made, this setup spans two cases: one where households determine k_{t+1} and leave c_t as a residual, and one where households determine c_t and leave k_{t+1} as a residual.

[5]Below, in order to compare the different tax regimes, I will further refine the equilibrium concept to include only equilibria which are the limit of finite horizon equilibria. This requirement alone rules out the type of equilibria studied by Chari and Kehoe (1990) and Phelan and Stacchetti (2001), among others, which require an infinite horizon.

Let us consider household behavior first. To ease exposition, assume for now that the horizon is infinite. Suppose government policy follows stationary decision rules for taxes and expenditure, so that $\tau = \mathcal{T}(K)$ and $G = \mathcal{G}(K)$. Since households take government policy parametrically, the problem of the household is functionally equivalent in all cases, i.e., regardless of the timing of government actions. Thus, given $\mathcal{T}(K)$, $\mathcal{G}(K)$ and factor prices, the problem of the household is

$$v(k, K) = \max_{c,k',n} u(c, 1 - n, \mathcal{G}(K)) + \beta v(k', K')$$

subject to

$$c + k' = (1 - \mathcal{T}(K))(r(K)k + w(K)n) + (1 - \delta)k.$$

Given some stationary government policy $\mathcal{G}(K)$, a recursive competitive equilibrium is a list of functions $\mathcal{C}(K)$, $\mathcal{N}(K)$, $\mathcal{H}(K)$ and $\mathcal{T}(K)$ satisfying the following four conditions. First, we have the Euler equation from the household problem

$$-u_c\big(\mathcal{C}(K), 1 - \mathcal{N}(K), \mathcal{G}(K)\big) + \beta u_c\big(\mathcal{C}(\mathcal{H}(K)), 1 - \mathcal{N}(\mathcal{H}(K)), \mathcal{G}(\mathcal{H}(K))\big) \times \big\{1 - \delta + \big(1 - \mathcal{T}(\mathcal{H}(K))\big) F_K\big(\mathcal{H}(K), \mathcal{N}(\mathcal{H}(K))\big)\big\} = 0, \tag{1}$$

where u_c is the marginal utility of consumption and F_i is the derivative of $F(K, N)$ with respect to $i = \{K, N\}$. Second, the static first-order condition from the household problem

$$u_c\big(\mathcal{C}(K), 1 - \mathcal{N}(K), \mathcal{G}(K)\big)\big(1 - \mathcal{T}(K)\big) F_N\big(K, \mathcal{N}(K)\big) - u_\ell\big(\mathcal{C}(K), 1 - \mathcal{N}(K), \mathcal{G}(K)\big) = 0, \tag{2}$$

where u_ℓ is the marginal utility of leisure. Third, the aggregate resource constraint

$$F\big(K, \mathcal{N}(K)\big) + (1 - \delta)K - \mathcal{C}(K) - \mathcal{H}(K) - \mathcal{G}(K) = 0. \tag{3}$$

Fourth, the government budget constraint

$$\mathcal{T}(K) F(K, \mathcal{N}(K)) - \mathcal{G}(K) = 0. \tag{4}$$

Below, we will address how $\mathcal{G}(K)$ is determined, depending on the timing of government actions within a period and whether consumption or savings are determined residually.

2.2 Case 0: government moves first

Consider the case where the government can commit to policy announcements within a period. This environment corresponds to the one analyzed by Klein, Krusell and Ríos-Rull (2008).

Given the expectation that future governments will follow $\mathcal{G}^0(K)$, which in turn implements $\mathcal{C}^0(K)$, $\mathcal{N}^0(K)$, $\mathcal{H}^0(K)$ and $\mathcal{T}^0(K)$, the problem of the current government can be written as

$$\mathcal{V}(K) = \max_{K',N,G} u(F(K, N) + (1 - \delta)K - G - K', 1 - N, G) + \beta \mathcal{V}(K')$$

subject to

$$-u_c\big(F(K, N) + (1 - \delta)K - G - K', 1 - N, G\big) + \beta u_c\big(\mathcal{C}^0(K'), 1 - \mathcal{N}^0(K'), \mathcal{G}^0(K')\big) \times \Big\{1 - \delta + \Big(1 - \frac{\mathcal{G}^0(K')}{F(K', \mathcal{N}^0(K'))}\Big) F_K\big(K', \mathcal{N}^0(K')\big)\Big\} = 0 \tag{5}$$

and

$$u_c\big(F(K,N)-G-K',1-N,G\big)\Big(1-\frac{G}{F(K,N)}\Big)F_N(K,N)-u_\ell\big(F(K,N)-G-K',1-N,G\big)=0. \quad (6)$$

Definition 1 *When the government moves first, a Markov-perfect equilibrium is a set of functions* $\{\mathcal{C}^0,\mathcal{N}^0,\mathcal{H}^0,\mathcal{G}^0,\mathcal{T}^0,\mathcal{V}^0\}:\mathbb{R}_+\to\mathbb{R}^6$*, such that for all* $K>0$*:*

$$(i)\ \{\mathcal{H}^0(K),\mathcal{N}^0(K),\mathcal{G}^0(K)\}=\underset{K',N,G}{\operatorname{argmax}}\,u(F(K,N)+(1-\delta)K-G-K',1-N,G)+\beta\mathcal{V}(K')$$

subject to (5) *and* (6)*;*

$$(ii)\ \mathcal{V}^0(K)=u(\mathcal{C}^0(K),1-\mathcal{N}^0(K),\mathcal{G}^0(K))+\beta\mathcal{V}^0(\mathcal{H}^0(K));$$
$$(iii)\ \mathcal{C}^0(K)+\mathcal{H}^0(K)+\mathcal{G}^0(K)=F(K,\mathcal{N}^0(K))+(1-\delta)K;\ \textit{and}$$
$$(iv)\ \mathcal{G}^0(K)=\mathcal{T}^0(K)F(K,\mathcal{N}^0(K)).$$

If there exists an equilibrium with differentiable policy functions,[6] then it can be characterized using the first-order conditions of the government's problem. For clarity of exposition, let us switch to short-hand notation (i.e., ignoring the arguments of functions) and eliminate the "0" superscript (that functions are indexed by "0" is thus understood). Furthermore, let us write constraints (5) and (6) compactly as $\Phi(K,K',\mathcal{H}(K'),N,\mathcal{N}(K'),G,\mathcal{G}(K'))=0$ and $\Psi(K,K',N,G)=0$, respectively.[7] With associated Lagrange multipliers λ and μ on the two constraints, the first-order conditions are

$$\begin{aligned}-u_c+\beta\mathcal{V}'_K+\lambda(\Phi_{K'}+\Phi_{K''}\mathcal{H}'_K+\Phi_{N'}\mathcal{N}'_K+\Phi_{G'}\mathcal{G}'_K)+\mu\Psi_{K'}&=0\\ u_cF_N-u_\ell+\lambda\Phi_N+\mu\Psi_N&=0\\ -u_c+u_g+\lambda\Phi_G+\mu\Psi_G&=0.\end{aligned}$$

We can use the last two equations above to solve for the Lagrange multipliers. Thus,

$$\begin{aligned}\lambda&=-\frac{\Psi_N(u_c-u_g)+\Psi_G(u_cF_N-u_\ell)}{\Phi_N\Psi_G-\Phi_G\Psi_N}\\ \mu&=\frac{\Phi_N(u_c-u_g)+\Phi_G(u_cF_N-u_\ell)}{\Phi_N\Psi_G-\Phi_G\Psi_N}.\end{aligned}$$

The envelope condition implies

$$\mathcal{V}_K=u_c(F_K+1-\delta)+\lambda\Phi_K+\mu\Psi_K.$$

Let $\Sigma_{K'}\equiv\Phi_{K'}+\Phi_{K''}\mathcal{H}'_K+\Phi_{N'}\mathcal{N}'_K+\Phi_{G'}\mathcal{G}'_K$. We thus get (arranged by wedges)

$$\begin{aligned}-u_c+\beta u'_c(F'_K+1-\delta)&+\frac{(u_c-u_g)(\Phi_N\Psi_{K'}-\Psi_N\Sigma_{K'})+(u_cF_n-u_\ell)(\Phi_G\Psi_{K'}-\Psi_G\Sigma_{K'})}{\Phi_N\Psi_G-\Phi_G\Psi_N}\\ &+\frac{\beta(u'_c-u'_g)(\Phi'_N\Psi'_K-\Psi'_N\Phi'_K)+\beta(u'_cF'_n-u'_\ell)(\Phi'_G\Psi'_K-\Psi'_G\Phi'_K)}{\Phi'_N\Psi'_G-\Phi'_G\Psi'_N}=0.\end{aligned} \quad (7)$$

Since this equation contains the derivatives of policy functions, it is typically called a Generalized Euler Equation or GEE.

[6]In a section below, we will show that a differentiable equilibrium exists for $\delta=1$.

[7]Note that for the Euler equation we first substitute $\mathcal{C}(K')$ with $F(K',\mathcal{N}(K'))+(1-\delta)K'-\mathcal{G}(K')-\mathcal{H}(K')$.

2.3 Case 1: simultaneous actions with consumption as residual

Suppose now that the government cannot commit to an announcement within the period. Thus, government and households move simultaneously. Here, we will consider the case where the household decides k' and thus consumption is determined as a residual. In other words, if the current government decides to deviate from the expected tax rule, then households will not know their consumption until after they observe the tax rate.

Given the expectation that future governments will follow $\mathcal{G}^1(K)$, which in turn implements $\mathcal{C}^1(K)$, $\mathcal{N}^1(K)$, $\mathcal{H}^1(K)$ and $\mathcal{T}^1(K)$, and households' belief that the current government will behave as future governments, the problem of the current government can be written as

$$\mathcal{V}(K) = \max_G u(F(K, \mathcal{N}^1(K)) + (1-\delta)K - G - \mathcal{H}^1(K), 1 - \mathcal{N}^1(K), G) + \beta \mathcal{V}(\mathcal{H}^1(K)).$$

Notice that the current government takes current savings and labor as given, but can affect current consumption. Since savings are not affected by current government actions, the problem of the government is static.

Definition 2 *When households and the government move simultaneously, and consumption is determined residually at the end of the period, a Markov-perfect equilibrium is a set of functions* $\{\mathcal{C}^1, \mathcal{N}^1, \mathcal{H}^1, \mathcal{G}^1, \mathcal{T}^1, \mathcal{V}^1\} : \mathbb{R}_+ \to \mathbb{R}^6$*, such that for all* $K > 0$*:*

$$(i)\ \mathcal{G}^1(K) = \operatorname*{argmax}_G u(F(K, \mathcal{N}^1(K)) + (1-\delta)K - G - \mathcal{H}^1(K), 1 - \mathcal{N}^1(K), G) + \beta \mathcal{V}(\mathcal{H}^1(K));$$

$$(ii)\ -u_c\big(\mathcal{C}^1(K), 1-\mathcal{N}^1(K), \mathcal{G}^1(K)\big) + \beta u_c\big(\mathcal{C}^1(\mathcal{H}^1(K)), 1 - \mathcal{N}^1(\mathcal{H}^1(K)), \mathcal{G}^1(\mathcal{H}^1(K))\big) \times\big\{1 - \delta + \big(1 - \mathcal{T}^1(\mathcal{H}^1(K))\big) F_K\big(\mathcal{H}^1(K), \mathcal{N}^1(\mathcal{H}^1(K))\big)\big\} = 0;$$

$$(iii)\ u_c\big(\mathcal{C}^1(K), 1-\mathcal{N}^1(K), \mathcal{G}^1(K)\big)\big(1-\mathcal{T}^1(K)\big) F_N(K, \mathcal{N}^1(K)) - u_\ell\big(\mathcal{C}^1(K), 1-\mathcal{N}^1(K), \mathcal{G}^1(K)\big) = 0;$$

$$(iv)\ \mathcal{V}^1(K) = u(\mathcal{C}^1(K), 1 - \mathcal{N}^1(K), \mathcal{G}^1(K)) + \beta \mathcal{V}^1(\mathcal{H}^1(K));$$
$$(v)\ \mathcal{C}^1(K) + \mathcal{H}^1(K) + \mathcal{G}^1(K) = F(K, \mathcal{N}^1(K)) + (1-\delta)K;\ \textit{and}$$
$$(vi)\ \mathcal{G}^1(K) = \mathcal{T}^1(K) F(K, \mathcal{N}^1(K)).$$

We can further characterize government policy with the first-order condition of the government's problem. In this case, since the problem of the government is static and given our assumptions on primitives, there is no multiplicity of equilibria.[8] Switching to short-hand notation we get

$$u_c = u_g. \tag{8}$$

This is the same condition that a government with access to lump-sum taxes would implement. Thus, when consumption is determined as a residual, government policy eliminates the wedge

[8]That is, regardless of how future governments behave, the best the current government can do is to equate the marginal utilities of private and public consumption. Given the assumptions on u and F, there is a unique solution to this. Since all governments behave the same way, there is a unique Markov-perfect equilibrium.

between private and public good consumption. However, the economy is not at the first-best; we still have the wedges between consumption and leisure, and current and future consumption.

Note that under special assumptions on preferences, a government with full intertemporal commitment would implement $u_c = u_g$ in the long-run, but only if capital depreciation is tax deductible (see section 4 and the Appendix for further analysis).

2.4 Case 2: simultaneous actions with savings as residual

Now assume that households choose consumption and labor at the same time as the government chooses the tax rate. Thus, savings are left as a residual. This environment corresponds to the one analyzed by Ortigueira (2006), based on the earlier work by Turnovsky and Brock (1980) and Judd (1998).

Given the expectation that future governments will follow $\mathcal{G}^2(K)$, which in turn implements $\mathcal{C}^2(K)$, $\mathcal{N}^2(K)$, $\mathcal{H}^2(K)$ and $\mathcal{T}^2(K)$, and households' belief that the current government will behave as future governments, the problem of the current government can be written as

$$\mathcal{V}(K) = \max_G u(C^2(K), 1 - \mathcal{N}^2(K), G) + \beta \mathcal{V}(F(K, N^2(K) + (1-\delta)K - G - C^2(K)).$$

In this case, the problem of the government is dynamic, since policy affects capital accumulation.

Definition 3 *When households and the government move simultaneously, and savings are determined residually at the end of the period, a Markov-perfect equilibrium is a set of functions* $\{\mathcal{C}^2, \mathcal{N}^2, \mathcal{H}^2, \mathcal{G}^2, \mathcal{T}^2, \mathcal{V}^2\} : \mathbb{R}_+ \to \mathbb{R}^6$*, such that for all* $K > 0$*:*

$$(i)\ \mathcal{G}^2(K) = \operatorname*{argmax}_G u(C^2(K), 1 - \mathcal{N}^2(K), G) + \beta \mathcal{V}(F(K, N^2(K) - G - C^2(K));$$

$$\begin{aligned}(ii)\ \ & - u_c\big(\mathcal{C}^2(K), 1 - \mathcal{N}^2(K), \mathcal{G}^2(K)\big) + \beta u_c\big(\mathcal{C}^2(\mathcal{H}^2(K)), 1 - \mathcal{N}^2(\mathcal{H}^2(K)), \mathcal{G}^2(\mathcal{H}^2(K))\big) \\ & \qquad \times \big\{1 - \delta + \big(1 - \mathcal{T}^2(\mathcal{H}^2(K))\big) F_K\big(\mathcal{H}^2(K), \mathcal{N}^2(\mathcal{H}^2(K))\big)\big\} = 0;\end{aligned}$$

$$(iii)\ u_c\big(\mathcal{C}^2(K), 1 - \mathcal{N}^2(K), \mathcal{G}^2(K)\big)\big(1 - \mathcal{T}^2(K)\big) F_N(K, \mathcal{N}^2(K)) - u_\ell\big(\mathcal{C}^2(K), 1 - \mathcal{N}^2(K), \mathcal{G}^2(K)\big) = 0;$$

$$\begin{aligned}(iv)\ & \mathcal{V}^2(K) = u(C^2(K), 1 - \mathcal{N}^2(K), \mathcal{G}^2(K)) + \beta \mathcal{V}(\mathcal{H}^2(K)); \\ (v)\ & \mathcal{C}^2(K) + \mathcal{H}^2(K) + \mathcal{G}^2(K) = F(K, \mathcal{N}^2(K)) + (1-\delta)K;\ \textit{and} \\ (vi)\ & \mathcal{G}^2(K) = \mathcal{T}^2(K) F(K, \mathcal{N}^2(K)).\end{aligned}$$

Switching to short-hand notation, the first-order condition of the government's problem is

$$u_g - \beta \mathcal{V}'_K = 0. \tag{9}$$

The envelope condition implies

$$\mathcal{V}_K = u_c \mathcal{C}_K - u_\ell \mathcal{N}_K + \beta \mathcal{V}'_K (F_K + 1 - \delta + F_N \mathcal{N}_K - \mathcal{C}_K),$$

which after using $u_g = \beta \mathcal{V}'_K$ from (9) implies the following GEE (arranged by wedges)

$$-u_g + \beta u'_g(F'_k + 1 - \delta) + \beta(u'_c - u'_g)\mathcal{C}'_K + \beta(u'_g F'_N - u'_\ell)\mathcal{N}'_K = 0. \tag{10}$$

As opposed to (8), the equation above contains the derivatives of decision rules. This reflects the dynamic nature of the government's problem when savings are the residual. Compared to (7) from case 0 (government moves first), the GEE here features $\mathcal{C}'_K$. We can substitute this expression by totally differentiating (3) with respect to K. Thus, $\mathcal{C}_K = F_K + F_N\mathcal{N}_K - \mathcal{G}_K - \mathcal{H}_K$, and (10) can be written as

$$-u_g + \beta u'_c(F'_K + 1 - \delta) + \beta(u'_c - u'_g)(\mathcal{G}'_K + \mathcal{H}'_K) + \beta(u'_c F'_N - u'_\ell)\mathcal{N}'_K = 0. \tag{11}$$

This way of writing the GEE makes all three cases easier to compare, as they can all be solved in terms of $\mathcal{H}$, $\mathcal{N}$ and $\mathcal{G}$ only.

3 Full Capital Depreciation

Under suitable assumptions, we can find analytical solutions for all the cases considered. The algorithm is as follows: (i) assume $T < \infty$ and use the fact that $K_{T+1} = 0$ to solve the problem of the government using backwards induction; (ii) use the finite-horizon solution for the initial period and take the limit as the horizon goes to infinity to obtain the stationary policy functions. The algebraic details of the backward iteration are omitted here for brevity, but the procedure is described in Martin (2009*a*). The analytical equilibria found this way are the limit of finite horizon equilibria, which ensures that we are comparing equilibria of the same type.

Assumption 1 $u(c, 1-n, G) = \alpha_c \ln c + \alpha_\ell \ln(1-n) + \alpha_g \ln G,\ \alpha_c, \alpha_\ell, \alpha_g > 0.$

Assumption 2 $F(K, N) = K^\gamma N^{1-\gamma},\ \gamma \in (0,1).$

Assumption 3 $\delta = 1.$

Proposition 1 *Suppose Assumptions 1, 2 and 3 hold. Then, for all $K > 0$ and $i = \{0, 1, 2\}$, the equilibrium policy functions are:*

$$\begin{aligned}
\mathcal{C}^i(K) &= (1-\beta\gamma)(1-\rho^i)K^\gamma N^{1-\gamma} \\
\mathcal{N}^i(K) &= N \\
\mathcal{H}^i(K) &= \beta\gamma(1-\rho^i)K^\gamma N^{1-\gamma} \\
\mathcal{G}^i(K) &= \rho^i K^\gamma N^{1-\gamma} \\
\mathcal{T}^i(K) &= \rho^i,
\end{aligned}$$

where

$$N \equiv \frac{1-\gamma}{1-\gamma+\frac{\alpha_\ell(1-\beta\gamma)}{\alpha_c}}$$

and

$$\rho^0 \equiv \frac{\alpha_g(1-\beta\gamma)}{\alpha_c+\alpha_g}$$
$$\rho^1 \equiv \frac{\alpha_g(1-\beta\gamma)}{\alpha_c+\alpha_g(1-\beta\gamma)}$$
$$\rho^2 \equiv \frac{\alpha_g(1-\beta\gamma)}{\alpha_c+\alpha_g(2-\beta\gamma)}.$$

Proof. It is straightforward to verify that these functions satisfy the equilibrium conditions for the respective cases, i.e., the two first-order conditions from the agent's problem, the GEE from the government's problem, the resource constraint and the government budget constraint. ■

Note that both labor and taxes are independent of the level of capital, as is typical in environments with full capital depreciation. Perhaps more interesting is the fact that labor is the same in all three cases. In particular, it implies that all the qualitative results in this section would also hold in an environment with inelastic labor supply.

In an interior equilibrium, we have $\mathcal{C}(K)>0$, $\mathcal{H}(K)>0$, $\mathcal{N}(K)\in(0,1)$ and $\mathcal{G}(K)>0$ for all $K>0$. It is easy to show that $\rho^i\in(0,1)$ for $i=\{0,1,2\}$ and thus, the Markov-perfect equilibrium is interior in all three cases.

The following statements compare the cases considered.

Proposition 2 *Under Assumptions 1, 2 and 3, for all* $K>0$*:*

$$(i)\ \mathcal{T}^2(K)<\mathcal{T}^0(K)<\mathcal{T}^1(K);$$
$$(ii)\ \frac{\mathcal{C}^2(K)}{\mathcal{G}^2(K)}>\frac{\mathcal{C}^0(K)}{\mathcal{G}^0(K)}>\frac{\mathcal{C}^1(K)}{\mathcal{G}^1(K)};$$
$$(iii)\ \frac{K}{\mathcal{Y}^2(K)}>\frac{K}{\mathcal{Y}^0(K)}>\frac{K}{\mathcal{Y}^1(K)},$$

where $\mathcal{Y}^i(K)\equiv F(K,\mathcal{N}^i(K))$.

Proof. Follows from $\rho^2<\rho^0<\rho^1$. ■

The results for the above proposition hold for any admissible parameter value. The most salient feature is that the case "government moves first" is always in between the two simultaneous move environments.

To compare welfare across environments, we need to calculate the value functions first. We get

$$\mathcal{V}^i(K)=\frac{\Omega(K)+\omega^i}{(1-\beta)(1-\beta\gamma)},$$

where

$$\begin{aligned}\Omega(K)\equiv{}&\alpha_c(1-\beta\gamma)\ln(1-\beta\gamma)+(\alpha_c+\alpha_g)\beta\gamma\ln(\beta\gamma)+\alpha_\ell(1-\beta\gamma)\ln(1-N)\\&+(\alpha_c+\alpha_g)(1-\gamma)\ln N+(\alpha_c+\alpha_g)(1-\beta)\gamma\ln K\end{aligned}$$

and

$$\omega^i \equiv (\alpha_c + \alpha_g \beta\gamma) \ln(1-\rho^i) + \alpha_g(1-\beta\gamma) \ln \rho^i.$$

Thus, for any $K > 0$, the value functions differ only by the constant term ω^i. Next, we verify the properties of the value functions.

Proposition 3 *Under Assumptions 1, 2 and 3, $\mathcal{V}^i(K)$ is C^∞, strictly increasing and strictly concave for all $K > 0$.*

Proof. C^∞ follows from the properties of the ln function. Taking the derivative of $\mathcal{V}^i(K)$ with respect to K, we get $\mathcal{V}^i_K = \frac{(\alpha_c+\alpha_g)\gamma}{(1-\beta\gamma)K} > 0$ and $\mathcal{V}^i_{KK} = -\frac{(\alpha_c+\alpha_g)\gamma}{(1-\beta\gamma)K^2} < 0$ for all $K > 0$. ■

The following proposition compares welfare across environments.

Proposition 4 *Under Assumptions 1, 2 and 3, for all $K > 0$:*
(i) $\mathcal{V}^0(K) > \mathcal{V}^1(K)$;
(ii) $\mathcal{V}^0(K) > \mathcal{V}^2(K)$; and
(iii) The sign of $\mathcal{V}^1(K) - \mathcal{V}^2(K)$ is equal to the sign of $(\alpha_c+\alpha_g) \ln \frac{\alpha_c+\alpha_g(2-\beta\gamma)}{\alpha_c+\alpha_g(1-\beta\gamma)} - (\alpha_c+\alpha_g\beta\gamma) \ln \frac{\alpha_c+\alpha_g}{\alpha_c}$, which may be positive, zero or negative.

Proof. First note that to get the sign of $\mathcal{V}^i(K) - \mathcal{V}^j(K)$ it is sufficient to get the sign of $\omega^i - \omega^j$.

(i) After some rearranging, $\omega^0 > \omega^1$ implies

$$(\alpha_c + \alpha_g\beta\gamma) \ln\left(\frac{\alpha_c + \alpha_g\beta\gamma}{\alpha_c}\right) - (\alpha_c + \alpha_g) \ln\left(\frac{\alpha_c + \alpha_g}{\alpha_c + \alpha_g(1-\beta\gamma)}\right) > 0.$$

This inequality can be written as $f(x_1) - f(x_2) > 0$, where $f(x) \equiv x \ln \frac{x}{x-\alpha_g\beta\gamma}$, $x_1 \equiv \alpha_c + \alpha_g\beta\gamma$ and $x_2 = \alpha_c + \alpha_g$. The inequality holds since $f(x)$ is decreasing and $x_1 < x_2$.

(ii) $\omega^0 > \omega^2$ implies

$$(\alpha_c + \alpha_g) \ln\left(\frac{\alpha_c + \alpha_g(2-\beta\gamma)}{\alpha_c + \alpha_g}\right) - (\alpha_c + \alpha_g\beta\gamma) \ln\left(\frac{\alpha_c + \alpha_g}{\alpha_c + \alpha_g\beta\gamma}\right) > 0.$$

This inequality can be written as $g(x_2) - g(x_1) > 0$, where $g(x) \equiv x \ln \frac{x+\alpha_g(1-\beta\gamma)}{x}$ and , x_1 and x_2 are as defined above. The inequality holds since $g(x)$ is increasing.

(iii) This condition follows from rearranging $\omega^1 - \omega^2 > 0$. ■

Thus, even though the case 0 (government moves first) is in between the two simultaneous action environments in terms of policy (see *Proposition 2*), it is always better in terms of welfare. We can rely on a simple simulation to find out how the two cases with simultaneous actions rank in terms of welfare for arbitrary parameters.

Result 1 *Suppose Assumptions 1, 2 and 3 hold, and wlog assume $\alpha_c, \alpha_g \in (0, 1)$. Simulating the model one hundred million times for random values of α_c, α_g and $\beta\gamma$, drawn from a uniform distribution for each case, yields $\mathcal{V}^1(K) > \mathcal{V}^2(K)$ in 39% of the simulations.*

Overall, there are two important results to consider. First, higher welfare is not necessarily associated with lower tax rates. In particular, case 0 (government moves first) features the highest welfare even though its tax rate is in between the other two cases. Case 2 (simultaneous actions with savings determined as residual) has the lowest tax rate, but will sometimes feature the lowest welfare. Second, for the case with full, not tax deductible depreciation, more commitment within the period is better. This last result is in sharp contrast with the numerical findings reported by Ortigueira (2006) for the case with partial, tax-deductible capital depreciation.[9] Thus, in the following sections, we will analyze the effects of allowing for partial depreciation and of making depreciation tax deductible. Furthermore, some comparative statics are performed in order to analyze how the ranking of tax rates and welfare varies with fundamentals.

4 Partial Capital Depreciation

4.1 Benchmark calibration

In this section, we will relax the assumption of full capital depreciation and rely on numerical methods to analyze the three environments. For now, we maintain the assumption that depreciation is not tax deductible to make the results comparable with those in the previous section. In section 5 we will consider the case where the whole period depreciation is tax deductible (as is Ortigueira, 2006 and Klein, Krusell and Ríos-Rull, 2008). Theoretically, both cases have been considered in the Ramsey taxation literature; in the real world, tax codes fall somewhere in between.[10]

We maintain *Assumptions 1* and *2*, which specify the functional forms for the utility and production functions. The benchmark parametrization is borrowed from Klein, Krusell and Ríos-Rull (2008) that calibrate their economy with no commitment and only labor income taxes to match some statistics of the post-war U.S. economy. The parametrization would have been similar if instead we had calibrated the model to match case 0 to the U.S. economy.[11] The period length is set to a year. The parameter values are presented in Table 1.

Table 1: Benchmark parameters

Parameter	α_c	α_ℓ	α_g	β	δ	γ
Value	0.261	0.609	0.130	0.960	0.080	0.360

Source: Klein, Krusell and Ríos-Rull (2008).

[9]Ortigueira also compares welfare between cases 0 and 2 in a two-period period economy with inelastic labor supply and full not tax-deductible capital depreciation. For his benchmark parameters, he also finds that case 0 implies higher welfare than case 2 but attributes this to the short time-horizon. Proposition 4 shows that this result is more general and not due to the time-horizon.

[10]Not all types of capital depreciation are allowed to be deducted from taxable income, e.g., inventories and land. In Canada and the U.S., the tax code divides assets into categories and specifies the depreciation schedule for each. Thus, the tax allowance is based on a definition which does not necessarily correspond to the actual economic depreciation.

[11]See Table 2. Typical targets in the literature are: capital-output ratio around 2.5, government expenditure to GDP of 20% and hours worked about one quarter of discretionary time. In addition, we have a capital depreciation rate of 8% annual and a share of capital income in total income around one-third, which pin-down the values of δ and γ, respectively.

In this class of problems, the Markov-perfect equilibrium that is the limit of finite horizon equilibria, is typically differentiable.[12] Thus, the numerical methods used in this paper look for an approximation to a differentiable equilibrium of each case considered. The model is solved using a variety of global methods to verify the results and look out for multiplicity of equilibria. Throughout the paper, I have not found more than one differentiable equilibrium in each environment considered.

The first—and preferred—algorithm is a variant of a standard projection method. First, define a grid over K. Second, make initial guesses for the value of the functions $\mathcal{H}$, $\mathcal{N}$ and $\mathcal{G}$ at each gridpoint. The number of unknowns is thus three times the number of gridpoints. Third, solve a system of equations, which includes the two first-order conditions of the agent's problem and the GEE from the government's problem, evaluated at all the gridpoints. Thus, the number of equations is three times the number of gridpoints. Since some of the equations include variables evaluated at K', we need to interpolate the values of policy functions between gridpoints; for this, I use cubic splines. Hence, for each update in the guess, the splines are also updated.

Typically, I use either 50 or 100 gridpoints, depending on which yields higher accuracy (this also applies to all the methods used). To evaluate the approximation, I evaluate the resulting functions at 1,000 gridpoints within the grid end-points and take the sum of squared residuals of the equations used. As a reference, the sum of squared residuals for the GEE of case 0 at benchmark is 10^{-11}. The other two cases feature even higher accuracy.

The second method involves value function iteration. First, define a grid and guess some policy and value functions. Second, solve the maximization problem for each K in the grid; replace the guess with the solution and continue iterating until convergence is achieved. Accuracy of the solution is evaluated as in the projection method. The case "government moves first" involves a constrained maximization problem which solves for all three variables simultaneously. The cases with simultaneous actions involve an unconstrained maximization problem, where after each iteration a new solution to $\mathcal{G}$ is found. Next, we need to solve for functions $\mathcal{H}$ and $\mathcal{N}$ which are consistent with the updated $\mathcal{G}$, using the first-order conditions of the agent's problem. Then, we update $\mathcal{H}$ and $\mathcal{N}$, solve again for $\mathcal{G}$ and continue the process until convergence is achieved.

The third method is used to verify that the solution is indeed the limit of the finite horizon equilibria. I start by solving the static problem of the last period. Then, I use backwards induction to solve the model. In each iteration I use the first-order conditions of the agent's problem and the GEE. The iteration stops when the policy functions for two consecutive periods are sufficiently close.

4.2 Analysis of equilibria at benchmark

Table 2 shows the steady state statistics for the benchmark parameters, assuming that depreciation is not tax deductible. As a reference, the table includes the statistics for the first-best (i.e., with lump-sum taxes) and the Ramsey (full intertemporal commitment) long-run allocation[13]. Except for labor, all statistics appear significantly different for the three Markov taxation cases. In particular, the size of the government varies quite dramatically. Relative to the first-best, all three Markov

[12]For an exception, see Krusell, Martin and Ríos-Rull (2006).

[13]See Appendix for a formulation and characterization of the Ramsey problem. Klein, Krusell and Ríos-Rull (2008) provide a thorough comparison between Ramsey and Markov-perfect policies, so I omit a similar analysis here.

cases exhibit too little capital and labor, which is typical of models with linear taxation. Less typical is the fact that both cases with simultaneous actions feature higher government expenditure over GDP than the first-best.

Table 2: Steady state statistics when depreciation is not tax deductible

	First-best	Ramsey	Case 0	Case 1	Case 2
K/Y	2.959	2.208	2.345	2.144	2.196
G/Y	0.254	0.254	0.207	0.275	0.258
C/G	2.008	2.244	2.917	2.008	2.196
K	1.906	0.911	1.001	0.870	0.903
N	0.350	0.264	0.264	0.264	0.264
τ	–	0.254	0.207	0.275	0.258

Case 0: government moves first; case 1: simultaneous actions with consumption as residual; case 2: simultaneous actions with savings as residual.

Figure 1 shows tax rates and welfare measures for the three Markov taxation cases. Welfare is measured as the permanent fraction of private good consumption an agent would be willing to give up in order to be indifferent between staying at the first-best equilibrium and switching to a particular Markov taxation case. Specifically, define the welfare gain associated with each case as

$$\zeta^i(K) \equiv \exp\left\{\frac{(1-\beta)(\mathcal{V}^i(K) - \mathcal{V}^*(K))}{\alpha_c}\right\} - 1,$$

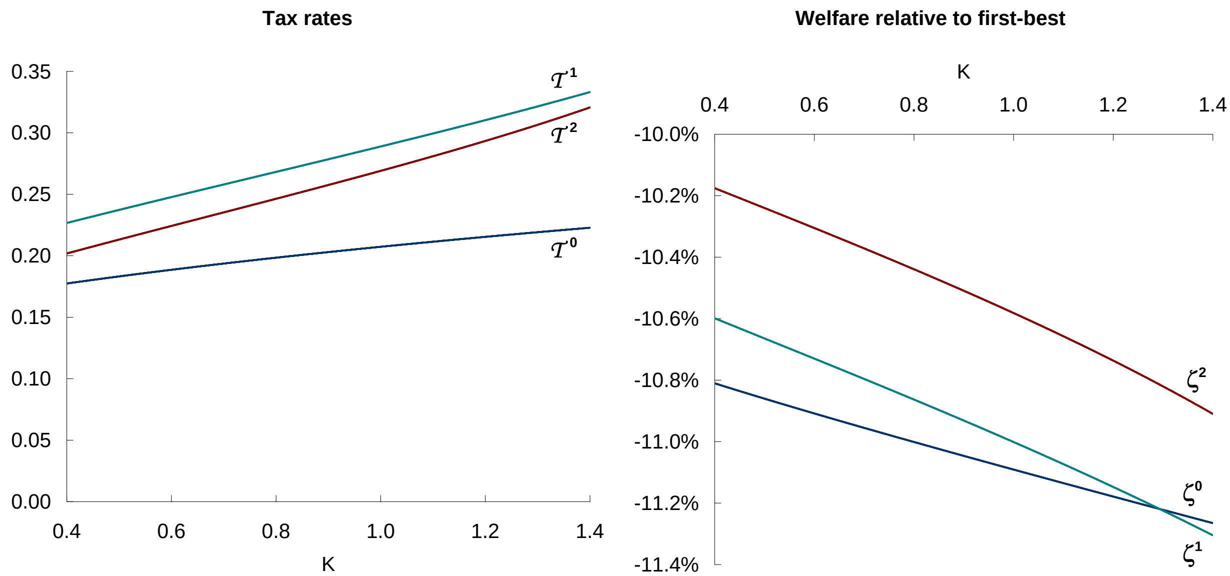
where $\mathcal{V}^*$ is the value function in the first-best equilibrium. A negative value for $\zeta^i(K)$ indicates a welfare loss relative to the first-best. The lower the number, the larger the loss.

Figure 1 shows that how the three cases compare to each other differs from the analytical results derived in the previous section for the case with full capital depreciation. From the approximated equilibrium functions, we get $\mathcal{T}^0(K) < \mathcal{T}^2(K) < \mathcal{T}^1(K)$ for all K in the range considered, i.e., $K \in [0.4, 1.4]$. Compared to the analytical results, case 1 still exhibits the highest tax rate, but the order of cases 0 and 2 is reversed. If we take a look at the welfare functions (right panel of Figure 1), we see that $\mathcal{V}^2(K)$ is the highest for all K, and we get $\mathcal{V}^1(K) > \mathcal{V}^0(K)$, except for very high K (about 30% above the steady state level for case 0). Recall that for the case with full depreciation, case 0 (government moves first) features unequivocally higher welfare than either case with simultaneous actions.

4.3 Comparative statics

Figure 2 compares taxes and welfare for different depreciation rates, evaluated at $K = 0.1$. The reason I use such a low value for capital is that the steady state for high depreciation rates features very low capital. As a reference, steady state capital when the government moves first is 0.038 for $\delta = 1$, 0.107 for $\delta = 0.5$ and 1.001 for $\delta = 0.08$. The left panel of Figure 2 shows $\mathcal{T}^i(K = 0.1)$ for $i = \{0, 1, 2\}$ and $\delta \in [0.03, 1]$. Some of the result on taxes from *Proposition 2* seem to hold. In particular, $\mathcal{T}^1$ is the highest for any δ. On the other hand, $\mathcal{T}^2 < \mathcal{T}^0$ holds only for high enough

Figure 1: Taxes and welfare when depreciation is not tax deductible



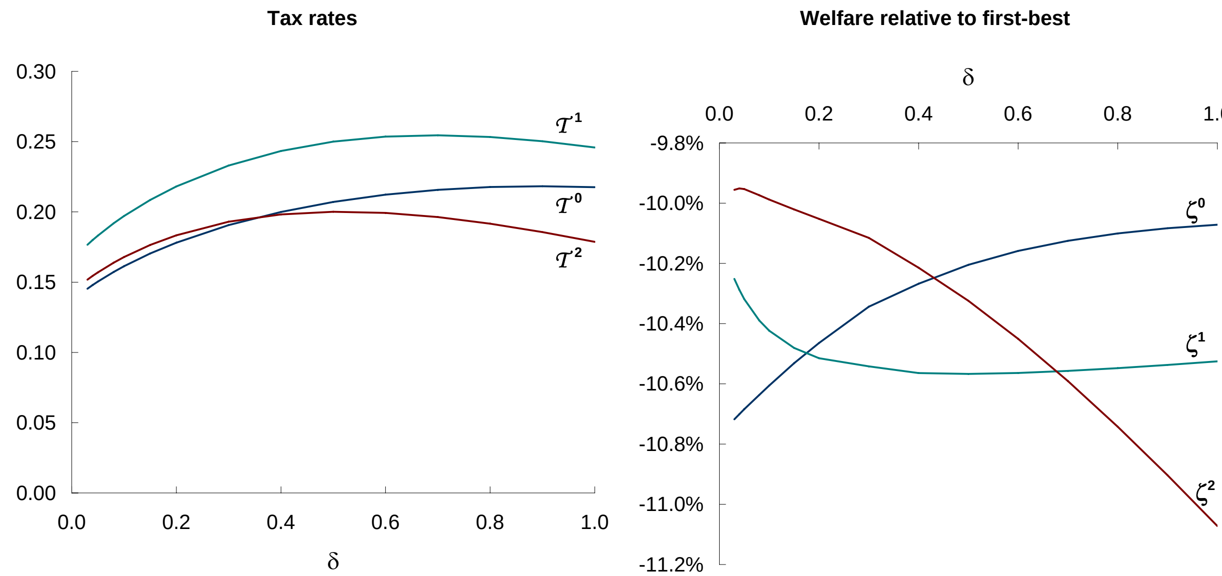
δ. Welfare appears less stylized. The right panel of Figure 2 shows that $\mathcal{V}^0$ is the highest for high depreciation rates—as established by *Proposition 4*—, but slides to lowest for low depreciation rates. Welfare in case 2 behaves in the opposite way, i.e., is the lowest for high depreciation rates and the highest for low depreciation rates. Case 1 remains in the middle for low and high depreciation rates, but is the lowest for intermediate values of δ. The main message of this paragraph is that how the different cases compare to each other may vary dramatically, depending on depreciation rates. Thus, different economies may be suitable for different tax structures.

Another parameter of interest is α_g. This determines the size of the government in the economy, which varies significantly across countries. It is clear that as α_g goes to zero, differences between cases disappear as they all converge to the first-best. The questions are then: how fast do these differences disappear and is the ordering of cases affected by the preference for the public good? To answer these questions, let us take the benchmark parametrization and evaluate the effects of changing α_g. Figure 3 summarizes this exercise. Taxes and welfare are evaluated at $K = 0.9$, which is a rough average of the steady states for the 3 cases considered at benchmark (see Table 2).

The left panel of Figure 3 shows that differences in tax rates are significant across cases, except for very low values of α_g. Furthermore, the ordering of cases appears constant, with case 1 (simultaneous actions with consumption as residual) featuring the highest tax rates and case 0 (government moves first) the lowest. Interestingly, the tax rates for the two simultaneous-action cases appear to grow with α_g at a much faster rate than for the case of government-moves-first.

The right panel of Figure 3 shows the effects of α_g on welfare. Cases 0 and 1 are very close—the difference between welfare functions $\zeta^i(K)$ is at most 0.1 percentage points for the range considered—, with case 1 featuring the higher welfare. Case 2 features higher welfare than the other two cases, although the difference vanishes quite rapidly as we reduce α_g. For example, the distance between welfare functions for cases 0 and 2 is -0.5 percentage points at $\alpha_g = 0.13$ and

Figure 2: Effects of δ evaluated at $K = 0.1$ when depreciation is not tax deductible



-0.1 percentage points at $\alpha_g = 0.06$.

In sum, the timing of taxation matters only when the preference for the public good is (sufficiently) high. Differences drop rapidly with α_g. However, the ordering of the cases in terms of tax rates and welfare seems invariant to the value of α_g.

5 Tax Deductible Capital Depreciation

5.1 Analysis of equilibria at benchmark

Suppose now that capital depreciation is tax deductible. All equilibrium equations remain the same, except for the Euler equation from the household problem (1), the government budget constraint (4) and the GEE for the case when the government moves first (7). In equilibrium, the Euler equation is now

$$-u_c\big(\mathcal{C}(K), 1 - \mathcal{N}(K), \mathcal{G}(K)\big) + \beta u_c\big(\mathcal{C}(\mathcal{H}(K)), 1 - \mathcal{N}(\mathcal{H}(K)), \mathcal{G}(\mathcal{H}(K))\big) \\ \times\big\{1 + \big(1 - \mathcal{T}(\mathcal{H}(K))\big)\big(F_K\big(\mathcal{H}(K), \mathcal{N}(\mathcal{H}(K)\big) - \delta\big)\big\} = 0$$

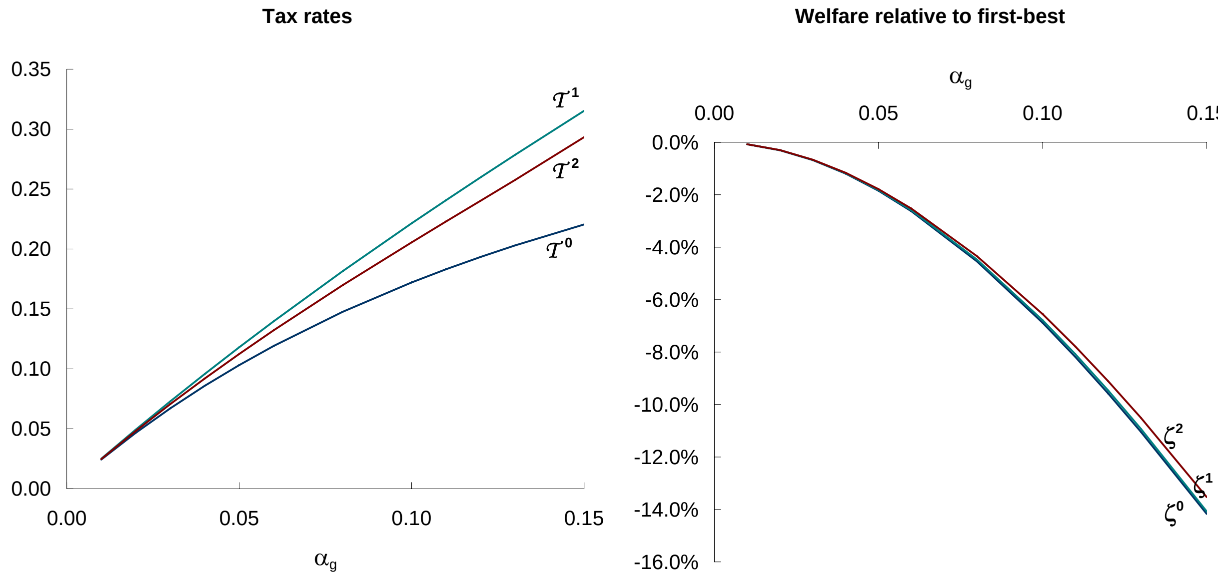
and the government budget constraint is

$$\mathcal{T}(K)\big(F(K, \mathcal{N}(K)) - \delta K\big) - \mathcal{G}(K) = 0.$$

The GEE for case 0 looks functionally the same, but the terms $\Phi_{K'}$, $\Phi_{N'}$ and $\Phi_{G'}$—i.e., the partial derivatives of the Euler equation—are different since the Euler equation is different.

Table 3 shows the steady state statistics at benchmark parameters when depreciation is tax deductible. Compared to the environment of the previous section, there are some similarities and

Figure 3: Effects of α_g evaluated at $K = 0.9$ when depreciation is not tax deductible



differences. First, steady state tax rates increase dramatically since the tax base is now smaller due to the depreciation allowance. This also implies lower labor and higher capital-output ratio. In terms of the size of the government, the equilibria for cases 0 and 1 show similar G/Y and C/G steady state statistics, when compared to the case without depreciation allowance. On the other hand, case 2 is strikingly different: for example, G/Y is about 38% if depreciation is deductible, compared to 26% for the case when depreciation is not tax-deductible (more on this below). Also note that, as mentioned in section 2.3, the Ramsey long-run allocation coincides with the steady state for case 1 (see Appendix for formal result).

Table 3: Steady state statistics when depreciation is tax deductible

	First-best	Ramsey	Case 0	Case 1	Case 2
K/Y	2.959	2.528	2.649	2.528	2.274
G/Y	0.254	0.265	0.201	0.265	0.383
C/G	2.008	2.008	2.930	2.008	1.137
K	1.906	1.090	1.183	1.090	0.907
N	0.350	0.256	0.258	0.256	0.251
τ	–	0.332	0.254	0.332	0.468

Case 0: government moves first; case 1: simultaneous actions with consumption as residual; case 2: simultaneous actions with savings as residual.

Figure 4 shows tax rates (left panel) and measures of welfare (right panel) as a function of capital for the three Markov taxation cases. At first glance, it is clear that there is much more curvature in the tax functions, when compared to the case with no depreciation allowance. The

Figure 4: Taxes and welfare when depreciation is tax deductible

reason is that, regardless of the case, the tax base becomes decreasing in capital for a high enough K. To see this, note that the change in the tax base as capital increases is equal to $r + w\mathcal{N}_K - \delta$. Given that the interest rate and labor are decreasing in capital, this expression turns negative for a sufficiently high K (as r approaches δ). As the tax base starts decreasing with capital, the tax rate has to start increasing much faster to keep up with the higher demand for the public good. This effect is exacerbated in both cases with simultaneous actions, since the government does not internalize the dynamic inefficiency faced by agents—the Euler equation is not a constraint in the government's problem. The end result is that an equilibrium may not exist for sufficiently high levels of capital. As we can see in Figure 4, case 2 was approximated for a smaller range of capital values[14] than the other two cases.

Comparing the tax functions for all three cases, we see a change in the ordering relative to the previous section. Case 0 (government moves first) still features the lowest tax rates at the benchmark parametrization. When depreciation is tax deductible, however, case 2 features the highest tax rate. This switch between cases 1 and 2 is related to issues described in the previous paragraph. Basically, the whole $\mathcal{T}^2$ function shifts up due to dynamic inefficiencies growing as the tax base shrinks. This effect is seen more dramatically when measuring welfare. The right panel of Figure 4 shows now that case 2 features the lowest welfare (it was the highest with non-deductible depreciation) and the difference with the other cases is quite significant. For the range considered, case 1 features higher welfare than case 0.

Quantitatively, welfare differences across cases appear more significant when depreciation is tax deductible. Take again $K = 0.9$; $\zeta^0 - \zeta^1$ is -0.11 percentage points when depreciation is not tax deductible and -1.02 percentage points when depreciation is tax deductible, i.e., an order of magnitude higher. A casual inspection of Figures 1 and 4 also reveals that the difference between

[14] The steady state capital stock in case 2, 0.907, is below the selected upper bound for capital, 0.92.

the best and worse case is much larger when depreciation is tax deductible.

Thus, how the cases compare appears more extreme when there is a tax allowance for capital depreciation. In spite of this, the highest welfare is to be found in the cases with deductible depreciation. For all K in the range considered, both cases 0 and 1 when depreciation is tax deductible feature higher welfare than the highest case with non-deductible depreciation (case 2). Case 1 with tax deductible depreciation features the highest welfare overall.

Suppose we are within a particular timing for taxation. What is the welfare gain of making depreciation tax deductible? To answer this, simply calculate welfare measures comparing the case with and without deductible depreciation, and evaluate at the steady state with non-deductible depreciation. When the government moves first (case 0), the welfare gain of making depreciation tax deductible is 1.0% of steady state consumption when depreciation is not tax deductible. When household and government move simultaneously and consumption is determined as a residual (case 1), this gain is equal to 2.1%. On the other hand, when savings are determined ex-post (case 2), there is a loss equivalent to 7.0% of steady state consumption.

5.2 Comparative statics

When capital depreciation is not tax deductible, which Markov taxation case features the highest welfare varies significantly with δ. This is no longer the case with tax deductible depreciation.[15] If we measure welfare relative to the first-best, the order of the different cases does not vary with δ. That is, case 1 always features the highest welfare and case 2 the lowest. The welfare loss from being in case 2 is particularly high for low depreciation rates and is mitigated for high ones.

Figure 5 shows the effects on taxes and welfare of varying α_g, again evaluated at $K = 0.9$. As for the case when depreciation is not tax deductible, the order of tax rates across cases does not vary with α_g. The tax rate when the government moves first (case 0) is still the lowest, with the difference disappearing for low values of α_g. For example, the difference in tax rates between cases 0 and 1 is only 1.6 percentage points at $\alpha_g = 0.06$. More significantly, tax rates for both cases with simultaneous action appear to converge as we decrease α_g. At benchmark values ($\alpha_g = 0.13$), the difference in tax rate at $K = 0.9$ is about 16 percentage points; at $\alpha_g = 0.06$ this difference drops to only 0.1 percentage points.

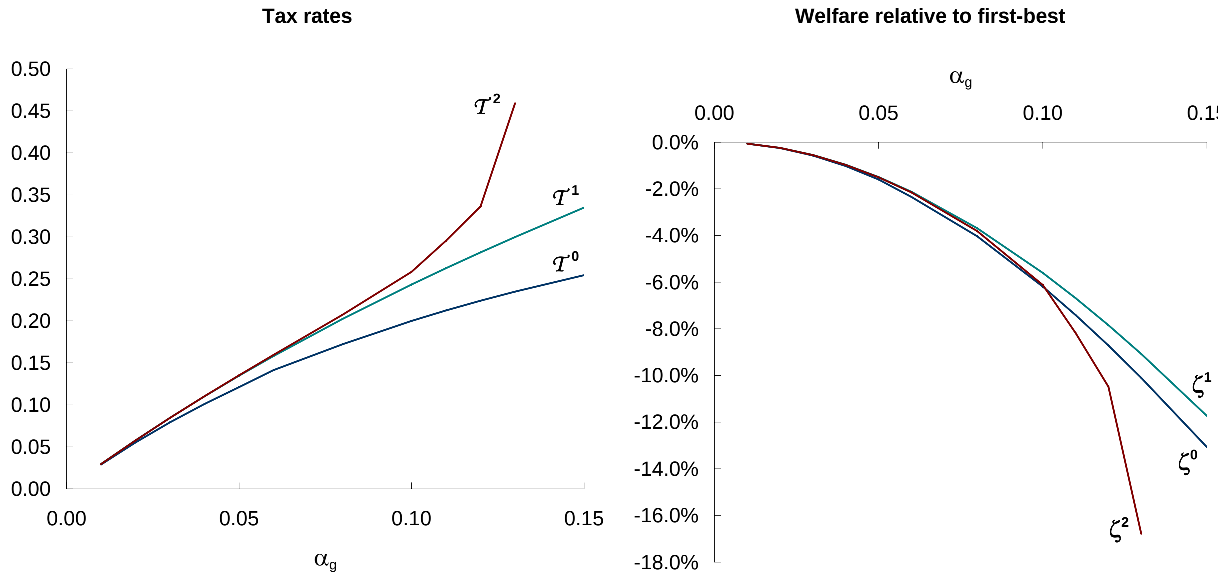
The right panel of Figure 5 shows the effects of α_g on welfare. First, let us compare cases 0 and 1. As for the case when depreciation is not tax deductible, case 1 features higher welfare than case 0 for all values of α_g considered.[16] However, here the difference is significant, except for very low α_g. Around benchmark parameters, the distance of the welfare measures is of the order of 1 percentage point.

Let us now compare cases 1 and 2, i.e., the two cases with simultaneous actions. For the case with no depreciation allowance, we saw that case 2 features the higher welfare for all α_g considered. For the case with tax deductible depreciation, we have already seen that at benchmark parameters, case 2 features the lowest welfare. However, as we lower α_g, welfare for case 2 approaches welfare for case 1. In this sense, the distance between welfare functions is about 7.7 percentage points at

[15] A graph is omitted for brevity.

[16] If we increase α_g beyond the range reported in Figure 5, the difference in welfare between cases 0 and 1 appears to widen.

Figure 5: Effects of α_g evaluated at $K = 0.9$ when depreciation is tax deductible



benchmark and drops to 0.03 percentage points at $\alpha_g = 0.06$. As with tax rates, cases 1 and 2 are quite different for high values of α_g, but virtually indistinguishable for lower values of α_g.

6 Concluding Remarks

As mentioned in the introduction, the case "government moves first" closely resembles the way fiscal policy is set up in modern developed economies. The cases with simultaneous actions could be thought of as economies where the fiscal authority has the capacity to substantially revise any previous announcements, e.g., due to a weak legislative branch. As for the depreciation allowance, the model does not capture all the details and nuances of actual tax codes. However, this allowance is typically seizable and thus, should be included in a model of an actual economy.

Within the context of the environments studied in this paper, there appears to be little room for improvement in the tax codes of developed economies. Consider case 0 with tax deductible capital depreciation as the benchmark. Making depreciation not tax deductible would actually result in a welfare loss. Removing within period commitment by implementing case 1 (simultaneous actions with consumption as residual) would bring only about 1% gain in terms of steady state consumption. In addition, since the Ramsey long-run allocation coincides with the case 1 steady state (when depreciation is tax deductible), we also obtain a small welfare loss due to lack of intertemporal commitment.[17]

[17] Small welfare losses due to lack of intertemporal commitment have been obtained in several different environments: see Martin (2009*a*) for an economy with capital and labor taxes and endogenous capital utilization; see Martin (2009*b*) for an economy with money and public debt, but no capital; see Azzimonti, Sarte and Soares (2009) for an economy with productive government expenditure.

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A The Ramsey problem

Suppose the government can commit at time zero to all future policy decisions, but is subjected to a balanced budget constraint, as in Stockman (2001) and Klein, Krusell and Ríos-Rull (2008). The problem of the government can be written as follows:

$$\max_{\{K_{t+1},N_t,G_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \Big\{ u\big(F(K_t,N_t)+(1-\delta)K_t-G_t, 1-N_t, G_t\big) + \lambda_t \Phi(K_t,K_{t+1},K_{t+2},N_t,N_{t+1},G_t,G_{t+1}) + \mu_t \Psi(K_t,K_{t+1},N_t,G_t) \Big\}$$

given K_0 and a standard transversality condition. Switching to short-hand notation, the first-order conditions are

$$\begin{aligned} -u_{c,t}+\beta u_{c,t+1}(1-\delta+F_{K,t+1})+\beta^{-1}\lambda_{t-1}\Phi_{K'',t-1}+\lambda_t\Phi_{K',t}+\beta\lambda_{t+1}\Phi_{K,t+1}+\mu_t\Psi_{K',t}+\beta\mu_{t+1}\Psi_{K,t+1} &= 0 \\ u_{c,t}F_{N,t}-u_{\ell,t}+\beta^{-1}\lambda_{t-1}\Phi_{N',t-1}+\lambda_t\Phi_{N,t}+\mu_t\Psi_{N,t} &= 0 \\ -u_{c,t}+u_{g,t}+\beta^{-1}\lambda_{t-1}\Phi_{G',t-1}+\lambda_t\Phi_{G,t}+\mu_t\Psi_{G,t} &= 0. \end{aligned}$$

In the long-run, the allocation $\{K^*,N^*,G^*,\lambda^*,\mu^*\}$ solves

$$\Phi(K^*,K^*,K^*,N^*,N^*,G^*,G^*) = 0 \qquad (12)$$

$$\Psi(K^*,K^*,N^*,G^*) = 0 \qquad (13)$$

$$u_c^*\big(-1+\beta(1-\delta+F_K^*)\big)+\lambda^*(\beta^{-1}\Phi_{K''}^*+\Phi_{K'}^*+\beta\Phi_K^*)+\mu^*(\Psi_{K'}^*+\beta\Psi_K^*) = 0 \qquad (14)$$

$$u_c^*F_N^*-u_\ell^*+\lambda^*(\beta^{-1}\Phi_{N'}^*+\Phi_N^*)+\mu^*\Psi_N^* = 0 \qquad (15)$$

$$-u_c^*+u_g^*+\lambda^*(\beta^{-1}\Phi_{G'}^*+\Phi_G^*)+\mu^*\Psi_G^* = 0. \qquad (16)$$

The following result relates the steady state of Markov case 1 with the Ramsey long-run allocation (Klein, Krusell and Ríos-Rull, 2008 make a similar observation).

Lemma 1 *Under Assumptions 1 and 2, if capital depreciation is tax deductible then the long-run Ramsey allocation coincides with the Markov case 1 steady state.*

Proof. Suppose Assumptions 1 and 2 hold and capital depreciation is tax deductible. Then, we can verify that $\beta^{-1}\Phi_{G'}^*+\Phi_G^*=\Psi_G^*=0$, so that (16) simplifies to $u_c^*=u_g^*$, which is identical to (8). Thus, $\{K^*,N^*,G^*\}$ is also the steady state of case 1. ■

Note that the above result does not hold if depreciation is not tax deductible; in this case, we get $\beta^{-1}\Phi_{G'}^*+\Phi_G^*<0$, $\Psi_G^*>0$ and generically, $u_c^*<u_g^*$.