Markov-Perfect Capital and Labor Taxes

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May 28, 2009

Abstract

This paper analyzes the Markov-perfect equilibrium of an economy were a benevolent government that lacks the ability to commit to future policy choices, uses taxes on capital and labor income to finance the provision of a public good. The main finding is that the government taxes capital and subsidizes labor so that only the dynamic inefficiency of future capital taxes remains. If agents' preference for the public good is sufficiently high, then capital is confiscated. Setting bounds on taxes alleviates the dynamic inefficiency inherent in capital taxation, but some implementations carry a high welfare cost. Allowing for endogenous capital utilization makes the current capital tax distortionary and implies capital and labor tax rates that are relatively close to those measured for the U.S. economy.

Keywords: Time-consistency, Markov-perfect equilibrium, optimal taxation, capital tax.

JEL classification: E62, H21.

*Email: fmartin@sfu.ca. I thank José-Victor Ríos-Rull for support and advice, and David Andolfatto, Jesús Fernández-Villaverde, Alex Karaivanov, Dirk Krueger, Per Krusell and Iourii Manovskii for various comments and conversations. I also thank two anonymous referees for helpful comments. A version of this paper previously circulated under the title "Optimal Taxation without Commitment".

1 Introduction

A classic issue in public finance concerns the question of how a benevolent government should distribute the burden of taxation between capital and labor income. A substantial part of the literature has addressed this issue in the context of the Ramsey approach to optimal taxation. The distinctive feature of this framework is that governments are assumed to have access to a commitment technology, but are restricted to use linear taxes only (i.e., no lump-sum taxes). In this regard, as first noted by Judd (1985) and Chamley (1986), the government should in the long-run set capital income taxes to zero and finance its expenditure with labor income taxation.[1]

It is well understood that the Ramsey policy prescription is in general time-inconsistent.[2] The reason is that, in the absence of a commitment technology, the government—even a benevolent one—would be tempted to exploit the inelastic nature of installed capital and tax it heavily to finance its expenditure. To analyze the problem of optimal taxation without commitment, Chari and Kehoe (1990) introduce the concept of sustainable equilibrium and show that reputation mechanisms (basically, coordinated punishment strategies) may substitute for commitment if private agents are sufficiently patient. Phelan and Stacchetti (2001) build on Abreu, Pearce and Stacchetti (1990) and develop methods to find the entire set of sustainable equilibria.[3] A drawback of this approach is that equilibria sustained with collective threats require an infinite horizon and are typically not renegotiation-proof.

This paper analyzes the fundamental Markov-perfect equilibrium of the Ramsey taxation problem without commitment.[4] Specifically, in the context of the standard neoclassical growth model, the paper characterizes and solves the problem of a benevolent government that: (i) uses linear taxes on capital and labor income to finance the (endogenous) provision of a public good; (ii) cannot commit to policy choices beyond the current period; and (iii) bases its decisions solely on fundamentals (in this case, the beginning-of-period aggregate capital stock). The Markov-perfect equilibria considered here are those of finite horizon economies and the limit of these equilibria as the horizon tends to infinity. This allows us to rule-out potential reputation-like equilibria which can only be supported if the horizon is infinite, and are thus not as robust as the limit of finite horizon equilibria.

The main result of the paper is that government policy in an environment as described above features labor subsidies and positive capital taxes. The intuition for this result is as follows. Future capital taxes create a wedge in the trade-off between current and future private-good consumption, which the current government cannot eliminate because of lack of commitment. Instead, the government eliminates the static wedge between leisure and the consumption of the public good. However, since the capital tax tomorrow discourages labor today, it is not sufficient to set labor taxes to zero, i.e., labor needs to be subsidized. The wedge between leisure and public-good consumption can be eliminated since the government is financing the labor subsidy with capital taxes, which are viewed by the current government as non-distortionary. In this way, taxation in the

[1] This result remains robust in various settings, as for example Chari, Christiano and Kehoe (1994) who study taxation over the business cycle or Stockman (2001) that looks at a government that follows a balanced-budget rule. See also Jones, Manuelli and Rossi (1997) and Zhu (1992) for related work.

[2] The time-consistency problem in public policy was first stated by Kydland and Prescott (1977). The problem, in general, dates back to Strotz (1956).

[3] See also Benhabib and Rustichini (1997) and Fernández-Villaverde and Tsyvinski (2002).

[4] Note that the methods developed by Phelan and Stacchetti (2001) cannot be used to identify and interpret Markov-perfect equilibria.

current period is effectively lump-sum and only the dynamic inefficiency created by future capital taxes remains. If the distortions necessary to finance the public good are large enough, then capital taxes are confiscatory and the economy shuts down.

The equilibrium described above is derived analytically for a version of the neoclassical growth model where capital depreciates fully in every period. For an equilibrium with non-confiscatory capital taxes to exist, the parameter that controls the preference for the public good needs to be below some critical value, which is a decreasing function of the discount factor. Furthermore, this critical value approaches zero as the discount factor approaches one. Numerical simulations show that these findings carry over to more elaborated economies, say with partial capital depreciation. In particular, standard calibrations of the model imply that capital is confiscated in equilibrium.

A bound on tax rates is a simple institution that alleviates the dynamic inefficiency created by capital taxes. If labor subsidies are not allowed, then in equilibrium, the government behaves as if it only had access to capital income taxes. In this case, it will heavily tax capital income, but will not confiscate the capital stock. On the other hand, if capital income taxes are not to exceed some bound and are confiscatory in the absence of such bound, then capital income taxes are set equal to the maximum allowed rate for any level of capital. The welfare implications of tax bounds differ significantly across implementation. For example, not allowing for labor subsidies implies a welfare loss relative to setting a 50% upper bound on capital taxes. Furthermore, welfare is decreasing in the capital tax bound.

The behavior of the government in a Markov-perfect equilibrium, as described in the paragraphs above, is essentially driven by the fact that the standard taxation model features an inelastic supply of capital within each period. With commitment, this is only an issue in the initial period, since capital is viewed as elastic from the on. Without commitment, the inelasticity of capital makes current capital taxes non-distortionary in every period and thus plays a crucial role in the determination of policy. Thus, the environment is modified to allow for endogenous capital utilization, as in Greenwood, Hercowitz and Huffman (1988), which results in capital taxes being viewed as distortionary by the current government, a point first raised by Zhu (1995). A calibrated economy delivers capital and labor tax rates that are both positive, and close to those measured for the U.S. by Mendoza, Razin and Tesar (1994). In this case, the estimated welfare loss due to lack of commitment appears much smaller than the loss due to lack of non-distortionary tax instruments.

There are a few closely related papers that also look at fiscal policy without commitment focusing on Markov-perfect equilibria. Early work includes Cohen and Michel (1988) and Currie and Levine (1993). Klein and Ríos-Rull (2003) solve for the Markov-perfect equilibrium in a model with both capital and labor taxes. However, since they assume that the government can commit to the following period's capital tax, their analysis cannot be used to answer the questions raised in this paper. Furthermore, they solve the model with a linear quadratic approximation to the government's objective function and thus, do not derive interpretable conditions. More recently, Klein, Krusell and Ríos-Rull (2008) study a model of public expenditure and characterize and solve for the equilibrium of the dynamic game between successive governments. Their analysis restricts the government to only one tax instrument and thus, has no implications for how the burden of taxation should be distributed between capital and labor income. The single-tax assumption also prevents the government from using capital taxes to eliminate other distortions and hence, the existence of an interior equilibrium is not an issue. Other related papers are Ortigueira (2006) and Martin (2009) who analyze the effects of assuming different timings in the government's decision.

Both these papers assume a single tax instrument.

The paper is organized as follows. Section 2 develops a basic model of taxation, characterizes the solution and derives conditions under which an interior Markov-perfect equilibrium exist. Section 3 perturbs the basic model along some dimensions. First, the model is modified to allow for partial capital depreciation and the implications of standard calibrations is discussed. Second, the effect of bounds on tax rates is analyzed. Finally, capital utilization is made endogenous and the model is calibrated to the U.S. economy. Section 4 concludes.

2 A Simple Model of Taxation

Consider the standard, deterministic neoclassical growth model with a long-lived representative household and a benevolent government. Time is discrete and indexed by $t = 1, \ldots, T$, where $T \leq \infty$. Households value leisure and the consumption of a private and a public good. Let $\beta \in (0, 1)$ be the discount factor. Firms maximize profits and use capital and labor as inputs. They produce the private good using a constant returns to scale production function $F(K, L)$, where F is differentiable, strictly concave and strictly increasing in both arguments. Markets are competitive and thus, factor payments r and w equal the marginal product of capital and labor, respectively.

The government provides a public good G that it finances with proportional taxes on both capital and labor income (no lump-sum taxes). Let θ be the capital income tax and τ the labor income tax. The government is subject to a period by period budget constraint, i.e., there is no public debt. Suppose the government cannot commit to policies beyond the current period. Hence, at the beginning of each period it sets the tax policy for the period and then households choose current consumption and leisure. The current government takes the policies implemented by future governments as given.

The following assumptions will allow for an analytical solution to the problem of the government.

Assumption 1 *The production function is*

$$Y_t \equiv F(K_t, L_t) = K_t^{\gamma} L_t^{1-\gamma},$$

where $\gamma \in (0, 1)$.

Assumption 2 *The capital depreciation rate is* 100% *per period and not tax deductible. Thus, the aggregate resource constraint is*

$$C_t + G_t + K_{t+1} = Y_t \tag{1}$$

and the government budget constraint is

$$G_t = \theta_t r_t K_t + \tau_t w_t L_t. \tag{2}$$

Assumption 3 *The utility function of the representative household is*

$$U(c, 1-l, G) = \alpha_c \ln c + \alpha_\ell \ln(1-l) + \alpha_g \ln G,$$

where $\alpha_c = \alpha(1-\alpha_g)$ *and* $\alpha_\ell = (1-\alpha)(1-\alpha_g)$, *for* $\alpha \in (0, 1)$, $\alpha_g \in (0, 1)$, *so that* $\alpha_c + \alpha_\ell + \alpha_g = 1$.

2.1 The problem of the household

Households take factor prices, taxes and the provision of the public good as given. Thus, for some initial capital $k_1 > 0$ the problem of the household is to maximize

$$\sum_{t=1}^{T} \beta^{t-1} U(c_t, 1 - l_t, G_t)$$

subject to $c_t + k_{t+1} = (1 - \theta_t) r_t k_t + (1 - \tau_t) w_t l_t$, for all $t = 1, \ldots, T$.

Eliminating the arguments of functions, the first-order conditions at an interior solution are

$$-U_{c,t} + \beta U_{c,t+1}(1 - \theta_{t+1}) r_{t+1} = 0 \tag{3}$$

$$U_{c,t}(1 - \tau_t) w_t - U_{\ell,t} = 0, \tag{4}$$

where $U_{c,t}$ is the partial derivative of $U(c_t, 1 - l_t, G_t)$ with respect to c_t, $U_{\ell,t}$ and is the partial derivative of $U(c_t, 1 - l_t, G_t)$ with respect to $1 - l_t$.

2.2 2 periods

Assume for now that $T = 2$, i.e., the economy lasts only 2 periods. In a Markov-perfect equilibrium, households and the government base their decisions solely on fundamentals; in this case, the aggregate capital stock at the beginning of each period. In the first period, the government observes K_1 and chooses θ_1, τ_1 and G_1 that satisfy its budget constraint, taking as given: (i) that households maximize utility and markets are competitive, as described above; and (ii) the policy implemented by the government in period 2, which is a function of K_2. The reason the government in period 1 needs to take into account the policy it will follow in period 2, is that period-2 policy affects the decisions taken by the household in period 1, as shown by (3). In the second period, the government observes K_2 and sets θ_2, τ_2 and G_2, taking as given that households maximize utility. Lack of commitment implies that the government in period 2 will not internalize how its actions affected the decisions made in period 1.

We are going to solve the problem of the government by backward induction. In the second period, households consume all their remaining capital and hence by the resource constraint (1), aggregate consumption is simply $C_2 = Y_2 - G_2$. We can use the government budget constraint (2) and the static first-order condition of the household's problem (4) to write taxes as functions of K_2, L_2 and G_2:

$$\tau_2 = 1 - \frac{U_{\ell,2}}{U_{c,2} F_{L,2}} \tag{5}$$

$$\theta_2 = \frac{1}{\gamma} \frac{G_2}{Y_2} - \frac{1-\gamma}{\gamma} \Big(1 - \frac{U_{\ell,2}}{U_{c,2} F_{L,2}} \Big), \tag{6}$$

where we also replaced the wage rate by the marginal product of labor.

Hence, for any given $K_2 > 0$ the problem of the government in the second period can be simply written as

$$\max_{L_2,\ G_2} U(Y_2 - G_2, 1 - L_2, G_2).$$

The first-order conditions are

$$\begin{aligned} U_{c,2}F_{L,2} - U_{\ell,2} &= 0 \qquad (7)\\ U_{c,2} - U_{g,2} &= 0. \qquad (8)\end{aligned}$$

Equations (5) and (7) imply $\tau_2 = 0$ and thus, all the burden of taxation falls on capital.[5] This is so, since the government is facing a static problem and the labor income tax is distortionary, whereas the capital income tax is not (capital is already installed). Hence, the government achieves the first-best in the second period. This is the case even if K_2 is very small, since θ_2 can take any value—in particular it can be larger than one—as long as it satisfies the government budget constraint. We can also solve for the remaining variables:

$$\begin{aligned} \theta_2 &= \frac{\rho_2}{\gamma}\\ L_2 &= \frac{1-\gamma}{1-\gamma+\rho_2\frac{\alpha_\ell}{\alpha_g}}\\ C_2 &= (1-\rho_2)Y_2\\ G_2 &= \rho_2 Y_2,\end{aligned}$$

where $\rho_2 \equiv \frac{\alpha_g}{\alpha_c+\alpha_g}$. Note that τ_2, θ_2 and L_2 do not depend on K_2, but only on parameter values. On the other hand, C_2 and G_2 are functions of K_2. Given that $\rho_2 \in (0,1)$, allocations are interior for any K_2.

Let us consider the first period now. Use the Euler equation (3) to get $K_2 = \upsilon_1(Y_1 - G_1)$, where

$$\upsilon_1 \equiv \frac{\beta\gamma(1-\theta_2)}{\beta\gamma(1-\theta_2)+(1-\rho_2)} = 1 - \frac{\alpha_c}{\alpha_c(1+\beta\gamma)-\alpha_g\beta(1-\gamma)}.$$

Thus, K_2 is just a fixed fraction of $Y_1 - G_1$. Moreover, the problem of the agent has an interior solution with $K_2 > 0$ if and only if $\theta_2 < 1$ ($\upsilon_1 > 0$), i.e., $\alpha_g < \frac{\gamma\alpha}{1-\gamma(1-\alpha)}$. In other words, households save in the first period if and only if capital is not confiscated in the second period. Thus, in order to get an interior solution in the 2-period economy, we need to restrict the possible values of α_g.

Assumption 4 *For the 2-period economy, let* $\alpha_g \in \left(0, \frac{\gamma\alpha}{1-\gamma(1-\alpha)}\right)$.

If α_g is greater or equal than the upper bound in Assumption 4, then the incentives faced by the government in the second period are such that it will confiscate the entire capital stock. Thus, agents do not save in the first period and the economy shuts down.[6]

The resource constraint implies $C_1 = (1-\upsilon_1)(Y_1 - G_1)$. From (2) and (4) we get τ_1 and θ_1 as functions of K_1, L_1 and G_1. Hence, in the 2-period economy, given some initial $K_1 > 0$, the problem of the period-1 government can be written as

$$\max_{L_1,\ G_1} U(C_1, 1-L_1, G_1) + \beta\ U(C_2, 1-L_2, G_2)$$

[5] This result is due to Fisher (1980).

[6] Alternatively, we could have assumed a production function that features $F(0,L) > 0$ for $L > 0$. In such a case, the equilibrium would feature labor as the only factor of production in the second period.

where

$$\begin{aligned} C_1 &= (1-\upsilon_1)(Y_1 - G_1) \\ K_2 &= \upsilon_1(Y_1 - G_1) \\ C_2 &= (1-\rho_2)Y_2 \\ G_2 &= \rho_2 Y_2. \end{aligned}$$

The first-order conditions are

$$U_{c,1}(1-\upsilon_1)F_{L,1} - U_{\ell,1} + \beta\upsilon_1 F_{L,1}F_{K,2}\big(U_{c,2}(1-\rho_2) + U_{g,2}\rho_2\big) = 0 \tag{9}$$

$$-U_{c,1}(1-\upsilon_1) + U_{g,1} - \beta\upsilon_1 F_{K,2}\big(U_{c,2}(1-\rho_2) + U_{g,2}\rho_2\big) = 0. \tag{10}$$

Combining these two equations we get

$$U_{g,1}F_{L,1} = U_{\ell,1}.$$

Note that this is the same condition that a government with access to lump-sum taxes would face. To see this, consider that a government with lump-sum taxes sets $U_{c,t} = U_{g,t}$ for all t, which due to (4) also implies $U_{g,t}F_{L,t} = U_{\ell,t}$. The government in the second period behaves in a similar way—just combine (7) and (8) to get $U_{g,2}F_{L,2} = U_{\ell,2}$. The difference however, is that the government in the first period additionally faces an intertemporal distortion created by the capital tax in the second period, θ_2.

From the maximization in the second period we know $U_{c,2} = U_{g,2}$ and from the Euler equation of the household (3) we have $U_{c,1} = \beta U_{c,2}(1-\theta_2)F_{K,2}$. Plug these expressions into (10) to get

$$U_{c,1}\Omega_1 = U_{g,1},$$

where

$$\Omega_1 \equiv \frac{(\alpha_c + \alpha_g)\beta\gamma + \alpha_c}{(\alpha_c + \alpha_g)\beta\gamma + \alpha_c - \alpha_g\beta}.$$

It is straightforward to show that under *Assumption 4*, $\Omega_1 > 1$ (note that since $U_{c,2} = U_{g,2}$, $\Omega_2 = 1$). Thus, the margin between private and public goods consumption is distorted in the first period.

We can now find analytical solutions for all remaining variables

$$\begin{aligned} \tau_1 &= 1 - \Omega_1 \\ \theta_1 &= \frac{\rho_1 - (1-\gamma)(1-\Omega_1)}{\gamma} \\ L_1 &= \frac{1-\gamma}{1-\gamma+\rho_1\frac{\alpha_\ell}{\alpha_g}} \\ C_1 &= (1-\upsilon_1)(1-\rho_1)Y_1 \\ G_1 &= \rho_1 Y_1 \\ K_2 &= \upsilon_1(1-\rho_1)Y_1, \end{aligned}$$

where

$$\rho_1 \equiv \frac{\alpha_g}{(\alpha_c + \alpha_g)(1+\beta\gamma)}.$$

Labor and taxes are only functions of parameter values, whereas consumption of private and public goods as well as capital carried over to the second period are constant fractions of aggregate output, and thus functions of K_1. The main results from this section are summarized in the following proposition.

Proposition 1 *The Markov-perfect equilibrium in the 2-period economy features*

1. $\tau_1 < \tau_2 = 0$.
2. $\theta_1 > \theta_2$ *if and only if* $\rho_1 - (1-\gamma)(1-\Omega_1) > \rho_2$.
3. $L_1 < L_2$.
4. $U_{c,1} < U_{g,1}$ *and* $U_{c,2} = U_{g,2}$.

In the second period, the government finances its expenditure with capital income taxes alone. Since capital taxation is non-distortionary, the economy is at the first-best. In the first period, the government also taxes capital income heavily, but it subsidizes labor. Note that, even though the government cannot avoid the distortion generated by its use of taxes (i.e., $U_{c,1} < U_{g,1}$), it manages to achieve the first-best marginal trade-off between the provision of the public good and leisure (i.e., $U_{g,1}F_{L,1} = U_{\ell,1}$). Thus, only one margin is distorted. In terms of the relation of capital taxes across periods, a sufficient condition for $\theta_1 > \theta_2$ is $\gamma \leq 1/2$ (standard calibrations typically set this value around a third).

2.3 Finite horizon, many periods

Assume the economy lasts $T \in (2, \infty)$ periods and let us continue solving the problem of the government backwards. After some work—described in Appendix A—one can show that, at an interior solution, the first-order conditions of the government's problem are

$$U_{g,T-j}\ F_{L,T-j} = U_{\ell,T-j} \tag{11}$$

$$U_{g,T-j} = U_{c,T-j}\ \Omega_{T-j}, \tag{12}$$

where

$$\Omega_{T-j} = \frac{(\alpha_c + \alpha_g)\ \lambda_j - \alpha_g}{(\alpha_c + \alpha_g)\ \lambda_j - \alpha_g\ \mu_j}$$

and

$$\lambda_j = \frac{1-(\beta\gamma)^{j+1}}{1-\beta\gamma}$$

$$\mu_j = \frac{1-\beta^{j+1}}{1-\beta}.$$

As in the 2-period economy, equation (11) also describes the behavior of a government with lump-sum taxes. This implies that the Markov government wants to implement the first-best G_t/L_t (and since K_t is given at t, this implies the first-best G_t/Y_t). The government is thus setting its taxes such that one of the margins (G_t/L_t) is not distorted. It cannot do the same with the other one (C_t/G_t), since it is using distortionary taxation. Again, the reason the government can successfully arrange taxes to accomplish this, is that the current capital income tax is non-distortive since capital is already installed. Hence, the burden of taxation falls on capital income and the labor income tax is set such that the public good-leisure distortion is eliminated.

Given

$$\begin{aligned}\rho_{T-j} &= \frac{\alpha_g}{(\alpha_c+\alpha_g)\ \lambda_j}\\ \upsilon_{T-j} &= 1-\frac{\alpha_c}{(\alpha_c+\alpha_g)\ \lambda_j-\alpha_g\ \mu_j}\end{aligned}$$

an interior Markov-perfect equilibrium in the T-period economy is characterized by

$$\begin{aligned}\tau_{T-j} &= 1-\Omega_{T-j}\\ \theta_{T-j} &= \frac{\rho_{T-j}-(1-\gamma)(1-\Omega_{T-j})}{\gamma}\\ L_{T-j} &= \frac{1-\gamma}{1-\gamma+\frac{\alpha_\ell}{\alpha_g}\ \rho_{T-j}}\\ C_{T-j} &= (1-\upsilon_{T-j})(1-\rho_{T-j})Y_{T-j}\\ G_{T-j} &= \rho_{T-j}Y_{T-j}\\ K_{T-j+1} &= \upsilon_{T-j}(1-\rho_{T-j})Y_{T-j}\end{aligned}$$

for $j=0,\ldots,T-1$ and given some initial $K_1>0$.

Given $K_1>0$, let a Markov-perfect equilibrium for the T-period economy be *interior* if it features positive consumption, labor, leisure and government expenditure in all periods. Essentially, the requirement is that households save some capital in all periods, except the last one. Below, we establish existence of a Markov-perfect equilibrium in a finite horizon economy and characterize its main properties.

Let

$$\omega_j \equiv \frac{\alpha}{\frac{\mu_j}{\gamma\lambda_j}-1+\alpha}.$$

Then, the following result follows.

Proposition 2 *If $\alpha_g\in(0,\omega_{T-2})$, then there exists an interior Markov-perfect equilibrium in the T-period economy. Furthermore, this equilibrium features*

$$\begin{aligned}\tau_T=0 \ \textit{and}\ \tau_{T-j}<0,\ j=1,\ldots,T-1;\\ \theta_{T-j}>0,\ j=0,\ldots,T-1;\\ U_{g,T-j}\ F_{L,T-j}=U_{\ell,T-j},\ j=0,\ldots,T-1;\\ U_{c,T}=U_{g,T}\ \textit{and}\ U_{c,T-j}<U_{g,T-j},\ j=1,\ldots,T-1.\end{aligned}$$

Proof. See Appendix B. ■

Thus, given α_g small enough, we get a Markov-perfect equilibrium in which the burden of taxation falls on capital income, while labor is subsidized. This policy implies a distortion in the private-public good consumption margin. On the other hand, the public good-leisure margin is at the first-best.

2.4 Limit of finite horizon equilibria

Now we will show that as the time-horizon expands, the equilibrium for the initial period, $T-j=1$, converges to a limit. To guarantee such convergence, we only need to keep track of ρ_{T-j}, υ_{T-j} and Ω_{T-j} along the iteration path. Since β and γ are both strictly between 0 and 1, all three variables converge to a finite number as j approaches infinity.

$$\begin{aligned}\rho &= \frac{(1-\beta\gamma)\alpha_g}{\alpha_c+\alpha_g} \\ \upsilon &= \frac{\beta((1-\beta)\gamma\alpha_c-(1-\gamma)\alpha_g)}{(1-\beta)\alpha_c-\beta(1-\gamma)\alpha_g} \\ \Omega &= \frac{(1-\beta)(\alpha_c+\beta\gamma\alpha_g)}{(1-\beta)\alpha_c-\beta(1-\gamma)\alpha_g}.\end{aligned}$$

The upper bound on α_g from *Proposition 2* also converges to a limit, call it ω, where

$$\omega = \frac{(1-\beta)\gamma\alpha}{(1-\beta)\gamma\alpha+1-\gamma}.$$

Note that $\omega \in (0,1)$, is decreasing in β and increasing in α and γ. Since ω_j is strictly decreasing in j (see *Lemma 1* in Appendix B), setting $a_g < \omega$, guarantees that an interior equilibrium exists in all periods.

If $a_g \in (0,\omega)$, the limit of the finite-horizon Markov-perfect equilibria features

$$\begin{aligned}L &= \frac{1-\gamma}{1-\gamma+(1-\beta\gamma)\frac{\alpha_\ell}{\alpha_c+\alpha_g}} \\ \tau &= -\frac{(1-\beta\gamma)\beta\alpha_g}{(1-\beta)\alpha_c-\beta(1-\gamma)\alpha_g} \\ \theta &= \frac{\alpha_c\alpha_g(1-\beta\gamma)^2}{\gamma\left(\alpha_c+\alpha_g\right)\left((1-\beta)\alpha_c-\beta(1-\gamma)\alpha_g\right)} \\ C &= (1-\upsilon)(1-\rho)K^\gamma L^{1-\gamma} \\ G &= \rho K^\gamma L^{1-\gamma} \\ K' &= \upsilon(1-\rho)K^\gamma L^{1-\gamma}.\end{aligned}$$

Labor is identical to the case of a government with lump-sum taxes, but because of distortionary taxation, the capital stock will be lower. Note that $G/Y=\rho$, which is also the first-best ratio. The upper bound on α_g determines an upper bound on G/Y. In particular, since ρ is strictly increasing in α_g we have that $\alpha_g < \omega$ implies $G/Y < \gamma(1-\beta)$.

The equation for K' implies that there exists a steady state with

$$K = (\upsilon(1-\rho))^{\frac{1}{1-\gamma}} L.$$

If the horizon is infinite, then the first-order conditions of the government's problem look identical to (11) and (12), but without the time-subscripts. Thus, the following results follows.

Proposition 3 *If $\alpha_g \in (0,\omega)$ then there exists an interior Markov-perfect equilibrium of the infinite horizon economy that is the limit of the Markov-perfect equilibrium of the finite horizon economy.*

How large is the set of economies with an interior Markov-perfect equilibrium? Clearly, as β goes to 1, ω goes to zero. In other words, the existence of an interior equilibrium is more difficult for higher discount factors. The intuition for this result is as follows. The discount factor weights the cost of future distortions. Thus, for any given future capital tax rate, a higher β implies lower incentives to work and save for the agent today. In turn, this implies the government needs to subsidize labor more and hence, tax current capital more. For β high enough, the government ends up confiscating the capital stock and so an interior equilibrium fails to exist.

Table 1 shows steady state statistics for $\alpha = 0.3,\ \beta = 0.5,\ \gamma = 0.36$ and selected values of $\alpha_g < \omega$ $(= 0.0778)$.

Table 1: Steady state statistics for $\alpha = 0.3,\ \beta = 0.5,\ \gamma = 0.36$.

	$\alpha_g = 0.010$	$\alpha_g = 0.050$	$\alpha_g = 0.075$
G/Y	0.027	0.122	0.174
C/G	32.538	6.624	4.692
L	0.257	0.282	0.298
θ	0.124	0.628	0.961
τ	-0.028	-0.162	-0.268

This section provided an example of economies that have a closed-form solution. For any given discount factor, a long-run Markov-perfect equilibrium exists only if the preference for the public good is low enough. In this case, governments tax capital and subsidize labor so that a lump-sum tax is effectively implemented and only an intertemporal distortion remains: that created by tomorrow's capital income tax, which in turn creates a wedge between the marginal utilities of current private and public good consumption. If the preference for the public good is too high, then the dynamic inefficiency created by capital income taxation is so large that governments end up confiscating all capital stock and the economy shuts down.

3 Sensitivity Analysis

In this section, we will analyze how general the results of the previous section are. First, we will show that the results carry over if we assume that only a fraction of capital depreciates. Then, we will analyze the effects of setting bounds to tax rates. Finally, we will show that making capital utilization endogenous makes capital taxes distortionary and allows to solve the equilibrium for an economy with standard calibration targets. Throughout this section we will maintain *Assumptions 1* and *3*, which specified the functional forms of the production and utility functions, respectively. For consistency, we will also keep the assumption that capital depreciation is not tax deductible.[7] Except for the case with endogenous capital utilization, all results are robust to making depreciation tax deductible.

[7]This assumption poses some inconsistencies with actual economies as tax codes typically do prescribe a depreciation allowance. Note however, that not all types of capital depreciation are allowed to be deducted from taxable income (e.g., inventories and land) and that the depreciation allowance refers to the accounting depreciation, which in general differs from the actual economic depreciation.

3.1 Partial capital depreciation

Here, we will modify *Assumption 2* to allow for partial capital depreciation.

Assumption 5 *The capital depreciation rate is $\delta \in (0,1)$ and not tax deductible. This implies the aggregate resource constraint is*

$$C + G + K' = F(K,L) + (1-\delta)K$$

and the government budget constraint is

$$G = \theta r K + \tau w L.$$

Assuming government policy allows for an interior solution, household behavior in a competitive equilibrium is characterized by

$$-U_c + \beta U_c' \big(1 - \delta + (1-\theta')F_K'\big) = 0 \quad (13)$$

$$U_c(1-\tau)F_L - U_\ell = 0. \quad (14)$$

In a Markov-perfect equilibrium, government policy will depend only on capital. Let $\theta = \Theta(K)$ and $\tau = \mathcal{T}(K)$ be the equilibrium tax functions. Households understand that in equilibrium, government policy follows functions Θ and $\mathcal{T}$, and thus, (13) and (14) yield stationary decision rules for capital tomorrow and labor today, that depend only on the capital stock today. Call them $\mathcal{H}(K)$ and $\mathcal{L}(K)$, respectively.

Let us write (13) and (14) in a more compact form

$$\eta(K, K', \mathcal{H}(K'), L, \mathcal{L}(K'), \theta, \Theta(K'), \tau, \mathcal{T}(K')) = 0 \quad (15)$$

$$\varepsilon(K, K', L, \theta, \tau) = 0. \quad (16)$$

Equations (15) and (16) characterize household behavior for the current period, for any arbitrary policy of the current government, given that future governments follow Θ and $\mathcal{T}$ which implement $\mathcal{H}$ and $\mathcal{L}$.

To simplify exposition, define the following aggregate functions

$$\mathcal{G}(K, L, \theta, \tau) \equiv \big(\gamma\theta + (1-\gamma)\tau\big)F(K,L)$$

$$\mathcal{C}(K, K', L, \theta, \tau) \equiv \big(1 - \gamma\theta - (1-\gamma)\tau\big)F(K,L) + (1-\delta)K - K',$$

which follow from the government budget and aggregate resource constraints, respectively.[8]

Given the perception that future governments will follow some tax policy $\Theta(K)$ and $\mathcal{T}(K)$, which in turn induces future household behavior given by $\mathcal{H}(K)$ and $\mathcal{L}(K)$, the problem of the current government can be written recursively as

$$V(K) = \max_{K', L, \theta, \tau} U\big(\mathcal{C}(K, K', L, \theta, \tau), 1 - L, \mathcal{G}(K, L, \theta, \tau)\big) + \beta\, V(K')$$

subject to (15) and (16).

[8] Note that we are using the functional form of the production function—*Assumption 1*—to simplify these expressions.

Definition 1 *Under Assumption 5, an interior Markov-perfect equilibrium is a set of functions $\mathcal{H}$, $\mathcal{L}$, Θ, $\mathcal{T}$ and V, such that for all $K \in (0, \bar{K}]$, where $K^* = \mathcal{H}(K^*) < \bar{K}$:*
(i) $\mathcal{H}(K) > 0$, $\mathcal{L}(K) \in (0,1)$, $\mathcal{C}(K, \mathcal{H}(K), \mathcal{H}(K), \Theta(K), \mathcal{T}(K)) > 0$ *and* $\mathcal{G}(K, \mathcal{L}(K), \Theta(K), \mathcal{T}(K)) > 0$*;*

$$(ii)\ \{\mathcal{H}(K), \mathcal{L}(K), \Theta(K), \mathcal{T}(K)\} = \underset{K', L, \theta, \tau}{\operatorname{argmax}}\ U\big(\mathcal{C}(K, K', L, \theta, \tau), 1 - L, \mathcal{G}(K, L, \theta, \tau)\big) + \beta\ V(K')$$

subject to $\eta(K, K', \mathcal{H}(K'), L, \mathcal{L}(K'), \theta, \Theta(K'), \tau, \mathcal{T}(K')) = 0$ *and* $\varepsilon(K, K', L, \theta, \tau) = 0$*; and*

$$(iii)\ V(K) = U\big(\mathcal{C}(K, \mathcal{H}(K), \mathcal{H}(K), \Theta(K), \mathcal{T}(K)), 1 - \mathcal{L}(K), \mathcal{G}(K, \mathcal{L}(K), \Theta(K), \mathcal{T}(K))\big) + \beta\, V(\mathcal{H}(K)).$$

Krusell and Smith (2003) show that this class of dynamic policy games may feature both smooth and non-differentiable Markov-perfect equilibria. The infinite-horizon equilibrium analyzed in the previous section was the limit of the finite-horizon equilibria and featured differentiable policy functions.[9] Since we are exploring extensions to the model of the previous section, let us analyze the differentiable Markov-perfect equilibrium, which in this case also corresponds to the limit of finite horizon equilibria.

To characterize a differentiable Markov-perfect equilibrium, we can use the first-order conditions from the problem of the government. After some work—see Appendix C for derivation—we get

$$U_g F_L - U_\ell = 0. \tag{17}$$

As in the basic model of the previous section, (17) also describes the behavior of a government with access to lump-sum taxes. Such a government would set $U_c = U_g$; this, together with the static first-order condition of the private sector (which with lump-sum taxes would be just $U_c F_L - U_\ell = 0$) implies (17). Hence, the Markov government sets the first-best ratio ratio between the marginal utilities of leisure and public good consumption, i.e., eliminates the leisure—public good wedge. Equations (14) and (17) imply $U_c(1 - \tau) = U_g$. As shown in the previous section, distortionary taxation implies $U_c < U_g$. Hence, in equilibrium we have $\tau < 0$, i.e., labor income taxes are negative and the burden of taxation falls on capital. Note that this result does not rely on policy functions being differentiable or the limit of finite horizon policies. The intuition for this result is as follows. Given that current capital taxes are non-distortive, the current government uses labor taxes solely to eliminate the leisure—public good wedge. Whether current labor taxes are positive, zero or negative depends on future capital taxes (since they affect the incentives to work today). Regardless, labor taxes are not used to finance expenditure, i.e., the burden falls on capital taxes. Thus, since in equilibrium future capital taxes are always positive, the current government needs to provide agents with incentives to work hard enough to eliminate the leisure-public good wedge, i.e., labor is subsidized.

The other equation characterizing government behavior—see Appendix C for derivation—is

$$\frac{\tau\ \xi}{U_{cc}} + \frac{(1 - \mathcal{T}')(1 - \delta + F'_K)}{1 - \delta + (1 - \Theta')F'_K} - 1 = 0, \tag{18}$$

[9] See Krusell, Martin and Ríos-Rull (2006) for an environment with non-differentiable finite-horizon equilibria.

where U_{cc} is the partial derivative of U_c with respect to consumption (note that by *Assumption 3* cross-derivatives are zero), and ξ is the total derivative of (15) with respect to K', i.e.,

$$\xi = U_{cc} + \beta U_c'\big((1-\Theta')(F_{KK}' + F_{KL}'\mathcal{L}_K') - F_K'\Theta_K'\big) + \beta U_{cc}'\big(1-\delta+(1-\Theta')F_K'\big)\times \\ \Big\{\big(1-\gamma\Theta'-(1-\gamma)\mathcal{T}'\big)(F_K' + F_L'\mathcal{L}_K') + 1 - \delta - \mathcal{H}_K' - \big(\gamma\Theta_K' + (1-\gamma)\mathcal{T}_K'\big)F(K',\mathcal{L}')\Big\}.$$

Since the expression for ξ contains the derivatives of the decision rules, (18) is typically referred to as a Generalized Euler Equation or GEE.

Definition 2 *Under Assumptions 1, 3 and 5, an interior differentiable Markov-perfect equilibrium is a set of smooth functions $\mathcal{H}$, $\mathcal{L}$, Θ and $\mathcal{T}$, that for all $K \in (0, \bar{K}]$ satisfy equations* (15)*,* (16)*,* (17) *and* (18) *with equality.*

The Euler equation of the agent, (13), clearly indicates that any interior solution has tomorrow's capital income tax bounded above. In particular, an interior solution requires $1-\delta+(1-\Theta')F_K' > 0$, i.e.,

$$\Theta(K) < \bar{\Theta}(K) \equiv 1 + \frac{1-\delta}{F_K}. \tag{19}$$

Note that the endogenous upper bound for $\Theta(K)$ is equivalent to a tax rate that confiscates all capital, both income and stock.[10] If the government confiscates all capital tomorrow, then the household today will not carry over any capital and thus (13) is no longer held with equality. Also note that $\bar{\Theta}(K)$ is increasing in K and approaches 1 as K approaches 0, i.e., the upper bound is stricter for lower capital stocks.

In steady state, (13) implies $1 = \beta(1-\delta+(1-\theta)F_K)$. Thus, the steady state capital income tax needs to be lower than 100% for an interior steady state to exist. The intuition for this results is straightforward: if agents are taxed all their capital income, then they will gradually deplete their capital holdings.

Why are the usual Inada conditions not sufficient to guarantee an interior solution? As in the previous section, the reason is that the endogenous bound on capital income taxes applies to next period's taxes. Governments can set capital income taxes as high as they want and do not internalize the bound since it only affects the problem of previous governments. Hence, even though the utility function satisfies Inada conditions, it is possible to have an equilibrium in which the solution is not interior. In that case, all capital is confiscated today and the economy shuts down.

Let us now proceed with the computation of the interior differentiable Markov-perfect equilibrium. There are two basic approaches: local and global. The local approach follows the perturbation method proposed by Klein, Krusell and Ríos-Rull (2008) to solve for the steady state. This approach gets around the problem that, due to the presence of the derivatives of policy functions in (18), there are more unknowns than equations in steady state. The basic idea of the perturbation method is to assume that equilibrium functions are polynomials of some degree n and then use the system of equations given by the the four first-order conditions—(15) to (18)—and their first to nth derivatives with respect to K (which are all equal to zero), all evaluated at the steady state. In the computations below, I consider cubic polynomials.

The global method solves for the policy functions applying the following algorithm.

[10] For the case with tax-deductible capital depreciation, the upper bound on capital taxes is $1 + \frac{1}{F_K - \delta}$.

1. Define a grid over K and guess the decision rules: $\mathcal{H}^0(K)$, $\mathcal{L}^0(K)$, $\Theta^0(K)$ and $\mathcal{T}^0(K)$.

2. For every K in the grid solve for K', L, θ and τ given that $\mathcal{H}^0$, $\mathcal{L}^0$, Θ^0, $\mathcal{T}^0$ are followed from tomorrow on, using the system of equations given by (15), (16), (17) and (18). Call the solution $\{\mathcal{H}^1(K), \mathcal{L}^1(K), \Theta^1(K), \mathcal{T}^1(K)\}$.

3. Check convergence of all decision rules. If the convergence error is not below the desired tolerance, set $\mathcal{H}^0 = \mathcal{H}^1$, $\mathcal{L}^0 = \mathcal{L}^1$, $\Theta^0 = \Theta^1$, $\mathcal{T}^0 = \mathcal{T}^1$ and go to step 2.

This algorithm presumes the use of some interpolation method to evaluate functions in between grid points and calculate the derivatives of policy functions. To this effect, I use cubic splines.

The benchmark parametrization is borrowed from Klein, Krusell and Ríos-Rull (2008) that calibrate their economy with no commitment and labor income taxes to match some statistics of the post-war U.S. economy. The period length is set to a year. The parameter values are presented in Table 2.

Table 2: Benchmark parameters

Parameter	α	α_g	β	δ	γ
Value	0.30	0.13	0.96	0.08	0.36

Source: Klein, Krusell and Ríos-Rull (2008).

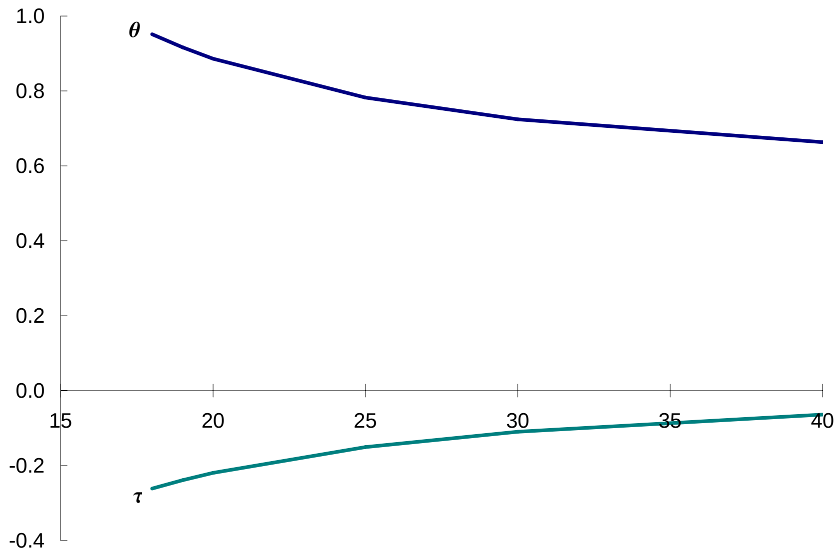
Under the benchmark parametrization, neither numerical method manages to find a solution. The analytical results of the previous section suggest that α_g may be too large. Furthermore, given that β is very close to 1, the level of public expenditure that can be supported in equilibrium may also be quite low. To better understand why an interior equilibrium fails to exist for the benchmark calibration, consider increasing the period length.[11] Let N be the length of a period, measured in years. Thus, the benchmark calibration features $N = 1$. For consistency, let $\beta(N) = \beta^N$ and $\delta(N) = 1 - (1 - \delta)^N$, where β and δ are equal to their benchmark values. Thus, for any given period length of N years, $\beta(N)$ and $\delta(N)$ specify the values used for the discount factor and the depreciation rate.

Figure 1 shows steady state capital and labor taxes as a function of period length, N, assuming $\alpha_g = 0.075$, which targets G/Y in the U.S. (about 19%) for $N = 25$. Steady state statistics were verified using both computational methods described above. As we can see, decreasing the period length increases steady state capital taxes. At $N = 18$, the capital income tax rate is 95% in steady state. The numerical algorithms fail to find a solution for $N < 18$. As the graph suggests, if the period length were short enough, steady state capital taxes would be above 100%, which as argued above, implies an interior steady state cannot exist. All these results are consistent with the analytical findings of the previous section.

Consider another illustrative example: $N = 10$ and $\alpha_g = 0.01$. This parametrization implies a steady state capital income tax rate of 20% and G/Y of 2.5%. Increasing α_g to 0.04 implies a steady state capital tax of 90% and G/Y of 9.1%. Alternatively, lowering N to 6 years implies a

[11] Alternatively, we could consider lowering just β. The results are similar in either case.

Figure 1: Steady state taxes as a function of period length (in years) for $\alpha_g = 0.075$



steady state capital tax of 82% and G/Y of 2.2%. If we further increase α_g or decrease N, a steady state fails to exist.

We should also verify that the lack of an interior Markov-perfect equilibrium for standard calibration targets is not related to the assumed utility function. Given that the existence problem basically stems from the Euler equation of the agent, we could analyze the effects of changing the curvature of the utility function in consumption.[12] Let $U(C, 1-L, G) = \alpha_c \frac{C^{1-\sigma}-1}{1-\sigma} + \alpha_\ell \ln(1-L) + \alpha_g \ln G$. Table 3 shows steady state statistics for different values of σ, given $N = 25$, $\alpha_g = 0.075$ and all other parameters at benchmark. As we can see, the steady state capital tax rate is increasing in σ, which makes the existence problem even worse. The reason for this is that a lower elasticity of intertemporal substitution in consumption makes future distortions more painful today, which decreases the agents' incentives to work today and thus, increases both current labor subsidies and capital taxes.

3.2 Bounds on tax rates

In this section we will analyze the effects of setting bounds on tax rates. Tax bounds are a simple institution that may alleviate the dynamic inefficiencies inherent in the problem of the government. A sufficiently high lower bound on labor taxes or a sufficiently low upper bound on capital taxes should allow for the existence of a Markov-perfect equilibrium with non-confiscatory capital taxes, in cases where an interior solution fails to exist. Note that binding tax bounds are subject to the usual time-inconsistency criticism. However, the amount of commitment required to implement bounds on taxes is much smaller than the one corresponding to the Ramsey problem. In other words, an institutional reform that imposes bounds on taxes is more plausible than one that fixes the sequence of all future taxes. Thus, it is worth analyzing what types of bounds could approximate

[12]Similar results obtain if we consider non-separable utility functions.

Table 3: Steady state statistics for $U(C, 1-L, G) = \alpha_c \frac{C^{1-\sigma}-1}{1-\sigma} + \alpha_\ell \ln(1-L) + \alpha_g \ln G$

σ	1	2	4
K/Y	0.030	0.025	0.019
G/Y	0.185	0.188	0.167
C/G	4.257	4.202	4.899
L	0.286	0.283	0.308
θ	0.782	0.815	0.863
τ	−0.151	−0.165	−0.225

Assumptions: $N = 25$, $\alpha_g = 0.075$. All other parameters at benchmark.

the welfare levels of the Ramsey solution.

Let us consider two distinct cases: (1) labor income taxes cannot be lower than zero, i.e., no subsidies on labor are allowed; and (2) capital income taxes cannot exceed some upper bound. As we will see, both cases have very different implications. The model is solved globally using the following algorithm.

1. Define a grid over K and guess the decision rules: $\mathcal{H}^0(K)$, $\mathcal{L}^0(K)$, $\Theta^0(K)$, $\mathcal{T}^0(K)$ and associated value function $V^0(K)$.

2. For every K in the grid solve:

$$\max_{K',L,\theta,\tau} U\big(\mathcal{C}(K,K',L,\theta,\tau), 1-L, \mathcal{G}(K,L,\theta,\tau)\big) + \beta V^0(K')$$

   subject to

$$\begin{aligned} \eta(K,K',\mathcal{H}^0(K'),L,\mathcal{L}^0(K'),\theta,\Theta^0(K'),\tau,\mathcal{T}^0(K')) &= 0 \\ \varepsilon(K,K',L,\theta,\tau) &= 0 \end{aligned}$$

   and the corresponding constraint on θ or τ. Call the solution $\{\mathcal{H}^1(K), \mathcal{L}^1(K), \Theta^1(K), \mathcal{T}^1(K)\}$ and let $V^1(K)$ be the updated value function.

3. Check convergence of all decision rules and the value function. If the convergence error is not below the desired tolerance, set $\mathcal{H}^0 = \mathcal{H}^1$, $\mathcal{L}^0 = \mathcal{L}^1$, $\Theta^0 = \Theta^1$, $\mathcal{T}^0 = \mathcal{T}^1$, $V^0 = V^1$ and go to step 2.

Using value function iteration instead of the first-order conditions of the government's problem allows us to use numerical maximization subroutines that allow for bounds on control variables. As in the previous section, functions are interpolated using cubic splines.

Table 4 shows the steady state statics for selected cases, under the benchmark parametrization. The first column shows the steady state allocation under commitment and is presented as reference.[13] Figure 2 shows equilibrium functions $\mathcal{H}$, Θ and $\mathcal{T}$ for the cases considered. The figures are informative in that they show that the bounds on tax rates bind for all K.

[13]The derivation of the Ramsey solution is standard and thus, omitted.

Figure 2: Equilibrium policies when taxes are bounded

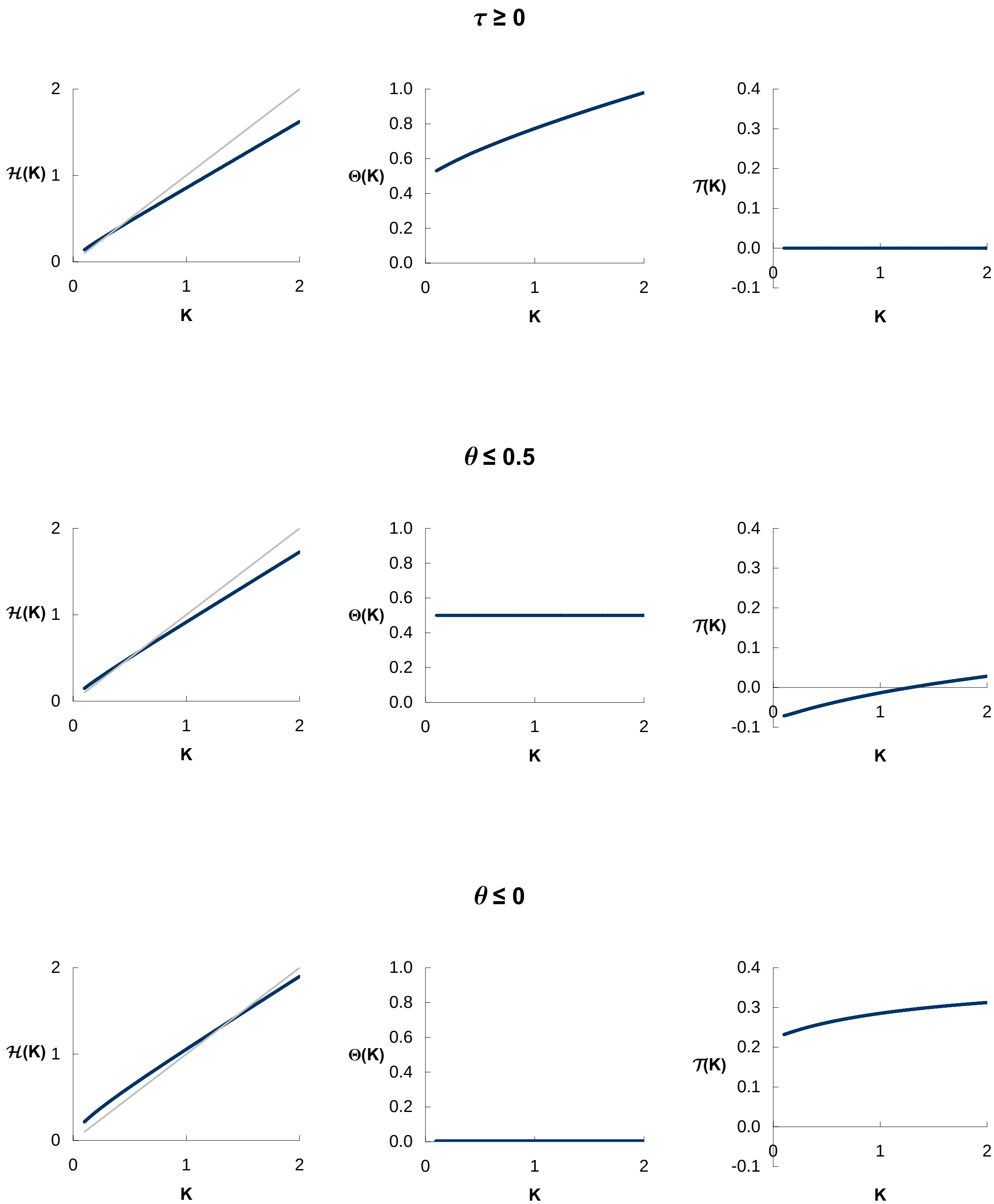


Table 4: Steady state statistics when tax rates are bounded—benchmark parameters

	Ramsey	$\tau \geq 0$	$\theta \leq 0.5$	$\theta \leq 0$
K/Y	2.959	1.150	1.479	2.959
G/Y	0.254	0.220	0.154	0.190
C/G	2.008	3.125	4.735	3.010
L	0.245	0.285	0.282	0.252
θ	0.000	0.612	0.500	0.000
τ	0.397	0.000	−0.041	0.297

Consider first the case with no subsidies on labor. Since we argued that labor is subsidized in an interior equilibrium, this constraint is binding and thus, the government behaves as if it had only access to capital income taxes. Next, consider the case where the capital tax rate cannot exceed some upper bound. From the analysis in the previous sections, it is clear that if an economy does not have an equilibrium with an interior solution, then any upper bound on capital income taxes—lower than the endogenous upper bound $\bar{\Theta}(K)$—will constitute an equilibrium. As argued above, if we want a Markov-perfect equilibrium with an interior steady state, we need to impose a bound below 100% on capital taxes. Consider then $\theta \leq 0.5$. The computations show that indeed, $\Theta(K) = 0.5$ for all K. Finally, suppose the government were not allowed to tax capital income at all, i.e., $\theta \leq 0$. In this case, it behaves in equilibrium as if it only had access to labor taxes.

Since we solve the model globally, it is straightforward to calculate the welfare implications of the different alternative bounds. For any given level of capital, the value function is decreasing in the upper bound on capital. Let us then measure the welfare loss relative to the steady state Ramsey allocation. Specifically, solve for ζ such that

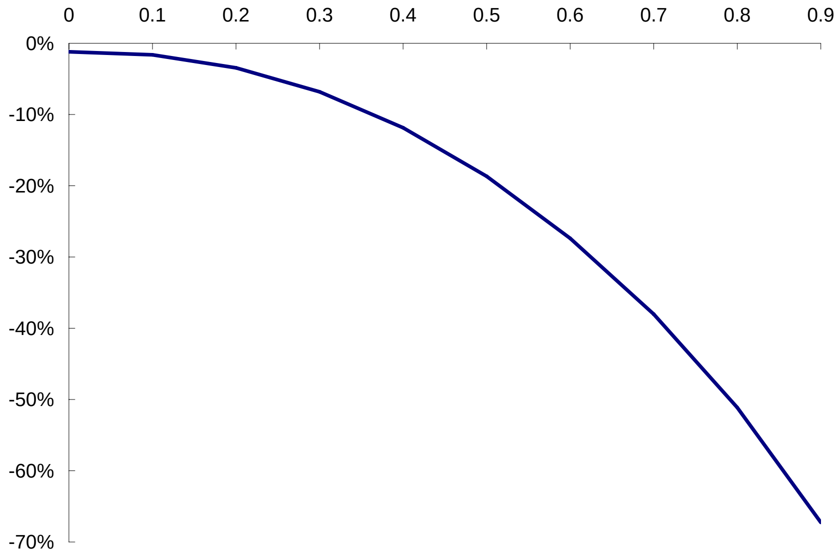
$$U(C^R(1+\zeta), 1-L^R, G^R) = (1-\beta)V(K^R),$$

where a superscript "R" refers to the variable being evaluated at the Ramsey steady state and V is the value function for the corresponding case with no commitment. This method allows for consistent comparisons across regimes, as all cases are evaluated at the same level of capital. As it turns out, the welfare loss due to lack of commitment varies widely, depending on the type of bound implemented. Not allowing the government to subsidize labor implies a welfare loss equivalent to 21% of steady state consumption under commitment. Figure 3 shows ζ as a function of the upper bound on θ. As we can see, the welfare loss relative to the Ramsey allocation is decreasing in the capital tax bound. Most notably, for the case $\theta \leq 0.5$, the welfare loss—roughly 19%—is smaller than for the case of no labor subsidies. Welfare losses decrease rapidly with more stricter bounds: for $\theta \leq 0.2$ and $\theta \leq 0$ welfare losses are 3.4% and 1.2%, respectively.

The literature on sustainable equilibria[14] relies on the identification of the worst-possible equilibrium to sustain the best equilibrium. If we restrict ourselves to competitive equilibria, then it is clear that if parameters are such that an interior Markov-perfect equilibrium does not exist, then the (corner solution) Markov-perfect equilibrium is the worst equilibrium. If follows then that any bounds on taxes we impose will affect the value of the worst equilibrium and thus the set of

[14]See Chari and Kehoe (1990), Phelan and Stacchetti (2001) and Fernández-Villaverde and Tsyvinski (2002).

Figure 3: ζ as a function of upper bound on θ



equilibria that can be sustained. As the analysis above shows, our choice of bounds affects the equilibrium significantly, not only in terms of allocations, but also in terms of welfare.

3.3 Endogenous capital utilization

The main results from the previous sections are basically driven by the fact that the standard model features an inelastic supply of capital within each period. If the government could commit, this would be a minor issue, since capital would only be viewed as inelastic in the initial period. In contrast, in the environments analyzed in this paper, the government views capital as inelastically supplied in every period and thus views current capital taxes as lump-sum.

Following Greenwood, Hercowitz and Huffman (1988), Zhu (1995) and Greenwood, Hercowitz and Krusell (1998) we now let households choose the utilization rate of capital, u. Given some stock of capital, this rate determines the flow of capital services uK and represents the intensity of capital use. Capital utilization affects its depreciation. Thus, let the depreciation rate be an increasing, convex function of utilization. For a given u, aggregate production is now $F(uK, L)$. Note that factor payments are still equal to the corresponding marginal products. We need to replace *Assumption 5* with the following.

Assumption 6 *The capital depreciation rate is* $\delta(u) = \frac{\chi_0}{\chi_1} u^{\chi_1}$*, with* $\chi_0 > 0$ *and* $\chi_1 > 1$*. Capital depreciation is not tax deductible. This implies the aggregate resource constraint is*

$$C + G + K' = F(uK, L) + (1 - \delta(u))K$$

and the government budget constraint is

$$G = \theta ruK + \tau wL.$$

Under the above assumption, the budget constraint of the household is

$$c + k' = (1-\theta)ruK + (1-\tau)wl + (1-\delta(u))k.$$

In a competitive equilibrium, the first-order conditions of the household's problem imply

$$-U_c + \beta U_c'\big((1-\theta')F_K'u' + 1 - \delta(u')\big) = 0 \tag{20}$$

$$U_c(1-\tau)F_L - U_\ell = 0 \tag{21}$$

$$(1-\theta)F_K - \delta_u = 0. \tag{22}$$

Clearly from (22), the current capital tax is now distortionary.[15]

In an interior equilibrium, the above set of equations is satisfied in every period. If we update (22) one period and plug it into (20) we get

$$-U_c + \beta U_c'\big(\delta_u'u' + 1 - \delta(u')\big) = 0. \tag{23}$$

Note that if the government had access to lump-sum taxes, the above expression would also apply. Indeed, (23) shows the first-best trade-off between consumption today and tomorrow. This implies that capital taxes—like labor taxes—create only a static wedge.

Writing the problem of the government involves the same steps an in previous sections, which we omit here for brevity. Still, (23) indicates that we can solve for u in steady state, even without looking at the problem of the government. In steady state, (23) reduces to $-1+\beta(\delta_u u+1-\delta(u)) = 0$ and thus

$$u^* = \Big(\frac{(1-\beta)\chi_1}{\beta\chi_0(\chi_1-1)}\Big)^{\frac{1}{\chi_1}} \tag{24}$$

$$\delta(u^*) = \frac{1-\beta}{\beta(\chi_1-1)}. \tag{25}$$

As argued above, u^* is also the rate of capital utilization that an economy with lump-sum taxes would feature in steady state.

We are going to calibrate the economy to match some post-war U.S. facts. The period is a year. The discount factor β is set to 0.96 to match a net return on capital of about 4% annual. Depreciation is 8% annual and the utilization rate is about 80%. Thus, using (25) we set χ_1 equal to 1.521 and using (24) set χ_0 equal to 0.171. The capital income share γ is taken directly from the data and set to 0.36. Government expenditure over GDP is about 19% and hours worked are around a quarter of total available time. We set α and α_g to match these targets. These choices are in line with the standards in the macroeconomic literature. Table 5 summarizes the parameter choice.

The model is solved using a suitable variant of the global method described above. Table 6 shows the steady state statistics of the economy. The first column (Ramsey) corresponds to an economy where the government can commit to future policy choices and is displayed for reference. The second column (Markov) corresponds to the economy without commitment described in this section.

[15] If depreciation were entirely tax deductible, then the first-order condition with respect to utilization implies $F_K = \delta_u$. In this case, the current capital tax does not affect any margin and is thus, non-distortionary. Note however, that to make capital taxes distortionary we only need depreciation not to be *entirely* tax deductible.

Table 5: Parameters for economy with endogenous capital utilization

Parameter	α	α_g	β	γ	χ_0	χ_1
Value	0.285	0.119	0.96	0.36	0.171	1.521

Table 6: Steady state statistics for economy with endogenous capital utilization

	Ramsey	Markov
K/Y	2.959	2.406
G/Y	0.245	0.190
C/G	2.110	3.254
L	0.233	0.250
θ	0.000	0.187
τ	0.383	0.191
u	0.800	0.800
δ	0.080	0.080

The standard result from the commitment literature still applies in this setting: capital taxes are set to zero in the long-run. In contrast, the optimal capital income tax when there is no commitment is around 19% and finances about a third of government expenditure. The capital tax rate is quite low when compared to the 42% calculated by Mendoza, Razin and Tesar (1994) for the period 1965-1988. The reason for the large discrepancy is that here the tax base does not account for capital depreciation. The equivalent capital tax rate that allows for depreciation being entirely tax deductible[16] is about 40%, very close to that estimated by the authors. The labor tax rate is also 19%, a bit below the 25% calculated by Mendoza, Razin and Tesar (1994).

We can also measure the welfare loss due to lack of commitment. As in the previous section, we can ask how much consumption agents would be willing to sacrifice to be indifferent between staying at the long-run Ramsey allocation and switching to the Markov policy. For the parameters considered, the welfare loss is equivalent to roughly 3% of long-run Ramsey period consumption. Consider instead the welfare loss of the Markov policy relative to the first-best. In this case, the welfare loss is equivalent to 13% of first-best long-run period consumption. Thus, in this environment, the welfare loss due distortionary instruments is much larger than the welfare loss due to lack of commitment.

For a closely related environment, Zhu (1995) demonstrates that if the government is allowed to issue debt with a sufficiently rich maturity and payoff structure, then the optimal Ramsey policy can be made time-consistent.[17] Two comments are in order, one normative and the other positive. First, as shown above, even if the government is not allowed to issue debt, welfare under the time-consistent policy is close to that under the commitment. A much larger inefficiency is due to the lack of lump-sum taxes. Second, as shown by Marcet and Scott (2007), the response of debt

[16] The equivalent tax rate $\hat{\theta}$ is defined as the one that raises the same revenue, i.e., $\hat{\theta}(F_K u - \delta(u)) = \theta F_K u$.

[17] One important difference is that Zhu includes costly investor's effort and investment tax credits. The Ramsey policy in his environment features zero effective taxes on the return to investment.

to aggregate shocks under incomplete markets is much more consistent with the data than under complete markets. This result casts some doubts on the empirical plausibility of government access to a wide set of policy instruments.

4 Concluding Remarks

In the absence of commitment, a benevolent government that follows a balanced-budget rule and bases its policies on fundamentals, taxes capital (heavily) and subsidizes labor. In this way, the government effectively implements a lump-sump tax and only the dynamic inefficiency created by future capital taxes remains. If the preference for the public good is high enough, then the government confiscates all capital or taxes it at the maximum allowed rate. With endogenous capital utilization the current capital tax creates a static wedge today that alters the government's incentives. For a calibrated economy, the implied optimal tax rates are actually very close to those in the data. The welfare loss due to lack of commitment appears much smaller than the loss due to lack of non-distortionary instruments.

A crucial assumption throughout the paper is the restriction to a balanced-budget rule. However, the idea of the paper is to provide an understanding of the basic incentives faced by governments in the conduct of fiscal policy and thus, the absence of public debt is a convenient simplification. It is easy to see what would happen for the case when current capital taxes are non-distortionary. The government would tax capital heavily so as to build up the necessary savings to finance government expenditure from tomorrow onwards, without the need to rely on distortionary taxation.[18] With endogenous capital utilization, we have that capital and labor taxes each create static distortions only. Public debt would allow the government to move these distortions across time, but the result that both capital and labor taxes are used to finance the burden of taxation would likely remain.

The restriction to Markov-perfect equilibria does not rule out the possibility that reputation considerations play a role in the determination of fiscal policy. In this sense, a compatible approach is to assume that the government's true type is private information, as in Barro and Gordon (1983) and more recently Phelan (2006). The current paper contributes in this direction by characterizing the fundamental equilibrium under full information, a necessary first step to understand the game with private information.

[18]See also Azzimonti, Sarte and Soares (2007).

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Appendix

A Derivation of (11), (12) and related variables.

Section 2.2 has the solution for the last two periods, $T-1$ and T. By continuing to iterate backwards, we can verify that consumption is a fixed fraction of output (like shown for the cases of C_T and C_{T-1}). Thus, if C_{T-j+1} is a constant fraction of Y_{T-j+1}, then we can use the dynamic Euler equation of the household (3) to get the choice of capital in the previous period. Following the same notation, let

$$C_{T-j+1} = (1-\upsilon_{T-j+1})(1-\rho_{T-j+1})Y_{T-j+1},$$

where

$$\rho_{T-j+1} \equiv \frac{G_{T-j+1}}{Y_{T-j+1}}.$$

Then

$$K_{T-j+1} = \upsilon_{T-j}(Y_{T-j} - G_{T-j}),$$

where

$$\upsilon_{T-j} \equiv \frac{\beta\gamma(1-\theta_{T-j+1})}{\beta\gamma(1-\theta_{T-j+1}) + (1-\upsilon_{T-j+1})(1-\rho_{T-j+1})}$$

and so by the aggregate resource constraint

$$C_{T-j} = (1-\upsilon_{T-j})(Y_{T-j} - G_{T-j}).$$

In turn, given K_{T-j}, L_{T-j} and G_{T-j}, the static first-order condition of the household (4) determines the labor income tax

$$\tau_{T-j} = 1 - \frac{U_{\ell,T-j}}{U_{c,T-j}\ F_{L,T-j}}$$

and the government budget constraint gives the capital income tax

$$\theta_{T-j} = \frac{1}{\gamma}\Big(\frac{G_{T-j}}{Y_{T-j}} - (1-\gamma)\tau_{T-j}\Big).$$

Hence, for any $j = 0, \ldots, T-1$ and given $K_1 > 0$ and $V^{T+1}(K) = 0$, the problem of the government can be written as

$$V^{T-j}(K_{T-j}) = \max_{L_{T-j},\ G_{T-j}} U(C_{T-j}, 1-L_{T-j}, G_{T-j}) + \beta\ V^{T-j+1}(K_{T-j+1})$$

where

$$\begin{aligned} C_{T-j} &= (1-\upsilon_{T-j})(Y_{T-j} - G_{T-j}) \\ K_{T-j+1} &= \upsilon_{T-j}(Y_{T-j} - G_{T-j}) \\ V^{T-j+1}(K_{T-j+1}) &= \sum_{i=T-j+1}^{T} U(C_i, 1-L_i, G_i). \end{aligned}$$

Note that $\{C_i, L_i, G_i\}_{i=T-j+1}^{T}$ is determined by the choice of K_{T-j+1}, given that we already solved the problem from $T-j+1$ to T.

The first-order conditions are

$$\begin{aligned} U_{c,T-j}\ (1-\upsilon_{T-j})\ F_{L,T-j} - U_{\ell,T-j} + \beta\ V_{k,T-j+1}^{T-j+1}\ \upsilon_{T-j}\ F_{L,T-j} &= 0 \\ -U_{c,T-j}\ (1-\upsilon_{T-j}) + U_{g,T-j} - \beta\ V_{k,T-j+1}^{T-j+1}\ \upsilon_{T-j} &= 0. \end{aligned}$$

Define

$$\Omega_{T-j} \equiv 1 - \upsilon_{T-j}\ \left(1 - \frac{\Omega_{T-j+1}}{1-\theta_{T-j+1}}\right).$$

After some work, similar to that of the section 2.2, the first-order conditions can be written as (11) and (12). We can exploit the recursive structure of the problem to find analytical solutions for the coefficients ρ_{T-j}, υ_{T-j} and Ω_{T-j}. First, define the following

$$\begin{aligned} \lambda_j &= \frac{1-(\beta\gamma)^{j+1}}{1-\beta\gamma} \\ \mu_j &= \frac{1-\beta^{j+1}}{1-\beta}. \end{aligned}$$

Note that $\lambda_0 = \mu_0 = 1$ and $\lambda_j > 1$, $\mu_j > 1$ for $j \geq 1$.

After some work we get

$$\begin{aligned} \rho_{T-j} &= \frac{\alpha_g}{(\alpha_c+\alpha_g)\ \lambda_j} \\ \upsilon_{T-j} &= 1 - \frac{\alpha_c}{(\alpha_c+\alpha_g)\ \lambda_j - \alpha_g\ \mu_j} \\ \Omega_{T-j} &= \frac{(\alpha_c+\alpha_g)\ \lambda_j - \alpha_g}{(\alpha_c+\alpha_g)\ \lambda_j - \alpha_g\ \mu_j}. \end{aligned}$$

B Proof of Proposition 2

Following the intuition from the 2-period economy, we need to ensure that the capital income tax remains bounded away from 1 to obtain an interior equilibrium. Clearly, if $\theta_{T-j+1} \geq 1$ then $K_{T-j+1} = 0$ and positive allocations cannot be supported from the on. Thus, in any period $T-j$ we need $\theta_{T-j+1} < 1$. Note that $\theta_{T-j+1} < 1$ is equivalent to $\alpha_g < \omega_{j-1}$. Thus, if ω_j is decreasing, we only need to verify that $\alpha_g < \omega_{T-2}$ (i.e., $\theta_2 < 1$) to ensure that $\theta_{T-j+1} < 1$, $\forall j = 1, \ldots, T-1$. Note that θ_1 can take any value and thus, we do not need to check $\alpha_g < \omega_{T-1}$.

Lemma 1 *ω_j is strictly decreasing in j.*

Proof. This is equivalent to $\frac{\mu_j}{\gamma\lambda_j}$ being strictly increasing in j. Applying the definitions of λ_j and μ_j, we get

$$\frac{1-\beta\gamma}{(1-\beta)\gamma}\ \frac{1-\beta^{j+1}}{1-(\beta\gamma)^{j+1}},$$

which is strictly increasing in j given that $\beta\gamma < 1$. ■

Thus, it is sufficient to restrict the value of α_g as in the statement of the proposition, i.e., $\alpha_g \in (0, \omega_{T-2})$. We now need to verify that this restriction indeed implies an interior equilibrium. Given that we already know from section 2.2 that $\rho_T \in (0,1)$ and $\upsilon_T = 0$, all we need to show is that $\rho_{T-j} \in (0,1)$ and $\upsilon_{T-j} \in (0,1)$ for $j = 1, \ldots, T-1$.

The following result follows directly from the definition of ρ_{T-j}.

Lemma 2 $\rho_{T-j} \in (0,1)$ *for all* $j = 0, \ldots, T-1$.

From the definitions of λ_j and μ_j we get $\frac{u_j - \lambda_j}{\lambda_j - 1} = \frac{\mu_{j-1}}{\gamma \lambda_{j-1}} - 1$. Thus, for $j = 1, \ldots, T-1$, $\alpha_g \in (0, \omega_{T-2})$ is equivalent to

$$\alpha_g < \frac{\alpha}{\alpha + \frac{\mu_j - \lambda_j}{\lambda_j - 1}},$$

which implies $(\alpha_c + \alpha_g)\lambda_j - \alpha_g \mu_j > \alpha_c$. It is now straightforward to show that $\alpha_g \in (0, \omega_{T-2})$ also implies the following results.

Lemma 3 $\upsilon_T = 0$ *and* $\upsilon_{T-j} \in (0,1)$ *for* $j = 1, \ldots, T-1$.

Lemma 4 $\Omega_T = 1$ *and* $\Omega_{T-j} > 1$ *for* $j = 1, \ldots, T-1$.

Lemmas 2 and *3* imply C_{T-j}, L_{T-j} and G_{T-j} are all positive for $j = 0, \ldots, T-1$. Also, by *Lemma 2* and $\alpha, \alpha_g > 0$, $L_{T-j} < 1$, so leisure is positive. Thus, the equilibrium is interior.

The features of the equilibrium as stated in the proposition follow directly from equations (11) and (12), and *Lemmas 2* and *4*. ■

C Derivation of (17) and (18).

With Lagrange multipliers λ and μ, the first-order conditions are (switching to short-hand notation[19])

$$\begin{aligned} U_c\, \mathcal{C}_{K'} + \beta V'_K + \lambda \xi + \mu \varepsilon_{K'} &= 0 \quad (26) \\ U_c\, \mathcal{C}_L - U_\ell + U_g\, \mathcal{G}_L + \lambda \eta_L + \mu \varepsilon_L &= 0 \quad (27) \\ U_c\, \mathcal{C}_\theta + U_g\, \mathcal{G}_\theta + \lambda \eta_\theta + \mu \varepsilon_\theta &= 0 \quad (28) \\ U_c\, \mathcal{C}_\tau + U_g\, \mathcal{G}_\tau + \lambda \eta_\tau + \mu \varepsilon_\tau &= 0, \quad (29) \end{aligned}$$

where $\xi \equiv \eta_{K'} + \eta_{K''} \mathcal{H}'_K + \eta_{L'} \mathcal{L}'_K + \eta_{\theta'} \Theta'_K + \eta_{\tau'} \mathcal{T}'_K$.

Totally differentiating $V(K)$ and applying the envelope condition gives

$$V_K = U_c\, \mathcal{C}_K + U_g\, \mathcal{G}_K + \lambda \eta_K + \mu \varepsilon_K. \quad (30)$$

Taking the partial derivatives of η and ε with respect to θ and τ we get: $\eta_\theta = -U_{cc}\, \mathcal{C}_\theta$, $\eta_\tau = -U_{cc}\, \mathcal{C}_\tau$, $\varepsilon_\theta = U_{cc}\, \mathcal{C}_\theta (1-\tau) F_L$ and $\varepsilon_\tau = U_{cc}\, \mathcal{C}_\tau (1-\tau) F_L - U_c F_L$, where U_{cc} is the partial

[19] Primes denote one period ahead (or two if there are two primes); functions with subscripts indicate the partial derivative of the function with respect to the subscript. For example, $\mathcal{C}'_{K'}$ is the partial derivative of $C(K, K', L, \theta, \tau)$ with respect to its second argument, K', evaluated tomorrow. That is, $\mathcal{C}'_{K'} = \frac{\partial C(K', K'', L', \theta', \tau')}{\partial K''}$.

derivative of U_c with respect to consumption (note that by *Assumption 3* cross-derivatives are zero). Further using $\mathcal{C}_\theta = -\mathcal{G}_\theta$ and $\mathcal{C}_\tau = -\mathcal{G}_\tau$, (28) and (29) can be rewritten as

$$\begin{aligned} -(U_g - U_c) - \lambda\, U_{cc} + \mu\, U_{cc}\, (1-\tau)F_L &= 0 \\ -(U_g - U_c) - \lambda\, U_{cc} + \mu\, U_{cc}\, (1-\tau)F_L &= \frac{\mu U_c F_L}{\mathcal{C}_\tau}, \end{aligned}$$

which implies the right-hand-side of the second equation is equal to zero. Using $\mathcal{C}_\tau = -(1-\gamma)F(K,L)$ we get $-\mu U_c/K = 0$. Given the assumptions on U, this implies that if $K > 0$ then $\mu = 0$. Using this result and updating (30) one period we get a system of three equations

$$U_c\, \mathcal{C}_{K'} + \lambda\xi + \beta\big(U_c'\, \mathcal{C}_K' + U_g'\, \mathcal{G}_K' + \lambda'\eta_K'\big) = 0 \tag{31}$$

$$U_c\, \mathcal{C}_L - U_\ell + U_g\, \mathcal{G}_L + \lambda\eta_L = 0 \tag{32}$$

$$U_g - U_c + \lambda\, U_{cc} = 0. \tag{33}$$

Equation (33) gives the value for the remaining Lagrange multiplier,

$$\lambda = -\frac{U_g - U_c}{U_{cc}}.$$

Moreover, taking the partial derivative of η with respect to L gives $\eta_L = -U_{cc}\, \mathcal{C}_L$. Hence, (32) reduces to $U_g(\mathcal{G}_L + \mathcal{C}_L) - U_\ell = 0$. Since $\mathcal{C}_L = F_L - \mathcal{G}_L$, we get (17).

Now let us turn to equation (31) to get the GEE. Use $\mathcal{C}_K = 1 - \delta + F_K - \mathcal{G}_K$, $\mathcal{C}_{K'} = -1$ and $\eta_K = -U_{cc}\, \mathcal{C}_K$ to re-write (31) as follows

$$-U_c - \frac{(U_g - U_c)\xi}{U_{cc}} + \beta\, U_g'\, (F_K' - \delta + 1) = 0. \tag{34}$$

This is the GEE. We can further simplify this expression. Equations (14) and (17) imply $U_c - U_g = \tau U_c$, which updated one period and re-arranged implies $U_g' = (1 - \mathcal{T}')U_c'$. Using (13) we get

$$\beta U_g' = \frac{(1-\mathcal{T}')U_c}{1 - \delta + (1-\Theta')F_K'}.$$

Replace these expressions in (34) to get (18).

Finally, we derive the expression for ξ. We get

$$\begin{aligned} \xi = \;& U_{cc} + \beta U_c'\big((1-\Theta')(F_{KK}' + F_{KL}'\mathcal{L}_K') - F_K'\Theta_K'\big) \\ & \beta U_{cc}'\big(1 - \delta + (1-\Theta')F_K'\big)\big(\mathcal{C}_K' - \mathcal{H}_K' - \mathcal{C}_L'\mathcal{L}_K' + \mathcal{C}_\theta'\Theta_K' + \mathcal{C}_\tau'\mathcal{T}_K'\big). \end{aligned}$$

Using $\mathcal{C}_K = \big(1 - \gamma\theta - (1-\gamma)\tau\big)F_K + 1 - \delta$, $\mathcal{C}_L = \big(1 - \gamma\theta - (1-\gamma)\tau\big)F_L$, $\mathcal{C}_\theta = -\gamma F(K,L)$ and $\mathcal{C}_\tau$ as derived above, we get

$$\begin{aligned} \xi = \;& U_{cc} + \beta U_c'\big((1-\Theta')(F_{KK}' + F_{KL}'\mathcal{L}_K') - F_K'\Theta_K'\big) + \beta U_{cc}'\big(1 - \delta + (1-\Theta')F_K'\big)\times \\ & \Big\{\big(1 - \gamma\Theta' - (1-\gamma)\mathcal{T}'\big)(F_K' + F_L'\mathcal{L}_K') + 1 - \delta - \mathcal{H}_K' - \big(\gamma\Theta_K' + (1-\gamma)\mathcal{T}_K'\big)F(K', \mathcal{L}')\Big\}. \end{aligned}$$