ASSIGNMENT #1
PHYS 490 - Relativity and Gravitation

Due Wednesday, October 10

PROBLEMS:

1. Carroll 1.4

2. Useful formulas:
   a) Show that $\Gamma^{\beta}_{\alpha\beta} = \partial_{\alpha} \ln \sqrt{g}$, where $g = \det(g_{\mu\nu})$.
   b) If $V^\mu$ is a vector, show that
      $$V^\mu_{\ ;\mu} = \frac{1}{\sqrt{g}} \partial_{\mu} \left( \sqrt{g} V^\mu \right)$$
   c) If $F^{\mu\nu}$ is an antisymmetric tensor, show that
      $$F_{\mu\nu;\lambda} + F_{\nu\lambda;\mu} + F_{\lambda\mu;\nu} = F_{\mu\nu,\lambda} + F_{\lambda\mu,\nu} + F_{\lambda\nu,\mu}$$
      $$F^{\mu\nu}_{\ ;\nu} = \frac{1}{\sqrt{g}} \partial_{\nu} \left( \sqrt{g} F^{\mu\nu} \right)$$
      $$F^{\mu\nu}_{\ ;\mu\nu} = 0$$
      (These provide a way to write Maxwell's equations without explicit use of Christoffel symbols.)
   d) If $T^{\mu\nu}$ is a symmetric tensor, show that
      $$T^{\nu}_{\mu;\nu} = \frac{1}{\sqrt{g}} \partial_{\nu} \left( \sqrt{g} T^{\nu}_{\mu} \right) - \frac{1}{2} T^{\alpha\beta} \partial_{\mu} g_{\alpha\beta}$$

3. Carroll 3.3

4. Carroll 3.5

5. Carroll 3.8 (Hint: Calculate components of $R^{\alpha\beta}_{\mu\nu}$, it will make contractions easier. I will show you later how to do this kind of calculation using computer algebra software, but you have to do this once in your life by hand.)