Transfer Characteristics

- Often define circuits by their "Transfer Characteristics"
- Apply an input voltage to one side of a circuit
- Output voltage measured across some part of the circuit
- Transfer characteristics: Plots the output against input
- Thus that state what the output will be for any input
Op Amp Integrator

- Recall resistor followed by a capacitor RC integrator
- If the RC time constant is long relative to period
- The resistor dominates the voltage drop and
- The voltage across the capacitor becomes the integral
- Consider an inverting op amp circuit
- But replace $R_f$ with a capacitor $C_f$
- Since summing point SP = a virtual ground.

$$V_{sp} = 0 \quad I_{sp} = 0$$

- As with the regular inverting op amp

$$I_{in} = I_s = I_i = \frac{V_{in}}{R_s}$$

- For the following capacitor then the current is

$$I_f = C_f \frac{dV_f}{dt}$$
Op Amp Integrator Cont'd

- Since there can be no current through the op amp
  \[ I_s = I_f \]
  \[ I_f = C_f \frac{dV_f}{dt} = \frac{V_{in}}{R_s} = I_s \]

- Thus the voltage across the output capacitor is
  \[ V_f = \frac{1}{R_s C_f} \int V_{in} dt \]

- Since
  \[ V_{out} = -V_f \]

- Thus the op amp output voltage is
  \[ V_{out} = -\frac{1}{R_s C_f} \int V_{in} dt \]

- Where \( \tau = R_s C_f \) = time constant of RC circuit
- However the op amp supplies the current
- And the summing point is a ground
- Thus RC need not be longer than the input period.
Op Amp Integrator Single Pulse Input

- Consider an op amp integrator circuit for a single square pulse
- 4 V for 10 ms duration
- What is the response?
- Assuming C is initially uncharged then
  \[ \tau = R_s C_f = 5000 \times 10^{-6} = 5 \text{ msec} \]

- During the pulse; t < 10 msec
  \[ V_{out} = -\frac{1}{R_s C_f} \int V_{in} dt = -\frac{1}{0.005} \int_0^t 4 \, dt = -800t \, V \]

- After the pulse t = 10 msec for all times
- Because only period when input current flows is important
  \[ V_{out} = [-800t]_0^{0.01} = -8 \, V \]

- Op amp will maintain this
Op Amp Integrator For a Single Pulse

- Result: slope to a constant value of 8 V
- Falling edge of pulse does not matter
- Only the period of voltage input
Op Amp Integrator for a Stream of Pulses

- For a stream of pulses period $T$
- When period $T < RC$ get a triangle wave output
- Negative voltage gives positive rising edge
- Slope of out wave is
  \[ V_{out} = -\frac{1}{R_s C_f} \int V_{in} dt = -\frac{V_{in}}{R_s C_f} \]

- Input of positive voltage starts decreasing voltage portion
- Called a sawtooth wave or triangular wave output
Op Amp Capacitive Differentiator

- Can change the op amp circuit to a Differentiator
- Exchange the resistor and capacitor
- Have the capacitor on the input, resistor as feedback
- Want RC time constant short relative to period of any signal
- For the feedback side

\[ I_f = \frac{V_f}{R_f} \]

- Recall that for a capacitor

\[ I_{in}(t) = C_s \frac{dV_{in}}{dt} \]

- Since the summing point SP is a ground this equation is exact
Op Amp Capacitive Differentiator Output

• Again for inverting op amp circuits

\[ I_s = I_f \quad V_f = -V_{out} \]

• Thus the output becomes

\[ V_{out} = -R_f I_{in} = -R_f C_s \frac{dV_{in}}{dt} \]

• Where \( \tau = R_f C_s \) is the time constant of the RC circuit.
• Note the response time of op amp limits the operation
• Even if RC is very small
Op Amp Capacitive Differentiator & Stream of Pulses

• Thus for a string of pulses (square wave)
• Get a sudden change called an impulse
• Direction opposite to that of falling/rising edge
• Followed by an exponential decay
• Decay is as capacitor charges/discharges
• Decay time set by the RC time constant
• Other waveforms integrated eg sin wave gives cos wave
Second Order Systems (EC 8)

- Second Order circuits involve two energy storage systems
- Create second order Differential Equations
- Transfer of energy from one storage to another and back again
- In circuits Resistors, Inductors and Capacitors
- Called RLC circuits
- L stores energy in Magnetic field from the current
- C stores energy in Electric field from stored charge
- As L discharges energy from B field it is stored in C
- As C discharges charge it is stored in L
- Resistor is always loosing energy
- Eg. series Voltage source, Resistor, Inductor and Capacitor
- Also parallel RLC (equations different)
Damped Spring DE (EC 8)

- Math often uses the damped spring with weight for 2\textsuperscript{nd} order DE

\[
m \frac{d^2y}{dt^2} + c \frac{dy}{dt} + ky = 0
\]

Where  
\begin{itemize}
  \item m=mass of weight
  \item c=damping constant
  \item k=spring constant
  \item y=vertical displacement
\end{itemize}

- Energy is stored in momentum of weight
- Energy also in position of spring
- Energy lost in damping pot
Solution of Second Order Systems

- General solution to Second Order circuits
- Proceeds similar to First Order Circuits

1. Use Kirchhoff's laws for circuit equation

2. Manipulate to get I or V in terms of derivatives in time

3. Generate the "Differential Equation form"
   - also called "Homogeneous equation form"

4. Solve the Differential Equation:
   - Solution substitution method: assume a solution
   - For step change assume exponential type solution.
   - Second order equations generally have two solutions
   - Response is combination of both solutions

5. Use initial or final conditions for constants of integration
   - Conditions may include derivatives at those times
Solution of Series RLC Second Order Systems

- Consider a series RLC with voltage source suddenly applied
- For series RLC used KVL
- Note for parallel will use KCL

(1) Using KVL to write the equations:

\[ V_0 = L \frac{di}{dt} + iR + \frac{1}{C} \int_0^t idt \]

(2) Want full differential equation
- Differentiating with respect to time

\[ 0 = L \frac{d^2i}{dt^2} + \frac{di}{dt} R + \frac{1}{C} i \]

(3) This is the differential equation of second order
- Second order equations involve 2nd order derivatives

![RLC Circuit Diagram](image)
Comparison of RLC and Damped Spring DE (EC 8)

- Looking at the damped spring with weight 2nd order DE

\[ m \frac{d^2y}{dt^2} + c \frac{dy}{dt} + ky = 0 \]

Where \( m = \) mass of weight  
\( c = \) damping constant  
\( k = \) spring constant  
\( y = \) vertical displacement

- Energy is stored in momentum of weight and spring

- For the RLC series the DE is

\[ L \frac{d^2i}{dt^2} + R \frac{di}{dt} + \frac{1}{C} i = 0 \]

- Current \( i \) is related to the displacement \( y \)
- \( L \) is equivalent to the momentum energy stored \( m \)
- \( 1/C \) is equivalent to the spring constant \( k \)
- \( R \) is equivalent to the damping loss \( c \)