Doping and Silicon
Reference: Chapter 4 Jaeger or Chapter 3 Ruska

- Recall dopants in silicon (column IV element)
- Column V extra electrons N type dopant $N_D$
- P Phosphorus, As Arsenic & Sb Antimony most common
- Column III holes (missing e’s) Acceptors P type dopant $N_A$
- B Boron, Al Aluminum most common
- For Diodes and transistors need to make P and N junctions
- Doping is inserting the impurities into the substrate

![Diagonal diagram of silicon lattice with dopant atoms](image1)

![Periodic table highlighting IIIA, IVA, and VA elements](image2)

![Cross-section diagram of an NMOS transistor](image3)

*Figure 11-4 Cross-section of a practical NMOS transistor.*
**Diffusion and Ion implantation**

- **N & P Dopants** determine the resistivity of material
- Very low levels for change 1 cm$^3$ Silicon has $5.5 \times 10^{22}$ atoms
- Significant resistivity changes at even $10^{10}$ dopant atoms/cc
- Typical doping begins at $10^{13}$ atoms/cc $N_A$ or $N_D$
- Note N lower resistivity than p: due to higher carrier mobility
- Near linear relationship below 0.2 ohm-cm ($\sim 10^{16}$ cm$^{-3}$)
- Above that high doping effects
- At $10^{19}$ get significant degeneracy effects
- There quantum effects become important
- Typical Si wafer substrate is about 1-10 ohm-cm or $10^{15}$-$10^{16}$ cm$^{-3}$

---

**Fig. 4.8** Room-temperature resistivity in $n$- and $p$-type silicon as a function of impurity concentration. (Note that these curves are valid for either donor or acceptor impurities but not for compensated material containing both types of impurities.) Copyright 1987 Addison-Wesley Publishing Company. Reprinted with permission from ref. [3].
Diffusion and Dopant Location

- Doping is adding impurities to Silicon
- Thermal diffusion process easiest
- Directly implanting (injecting) more expensive
- Dopant Atoms Substitutional – replaces Si:
- Called activated dopants – ie n and p carriers created
- Interstitial dopant: pushes out Si
- True Interstitial dopant atoms: not activated – no carriers

Fig. 4.1 Atomic diffusion in a two-dimensional lattice. (a) Substitutional diffusion, in which the impurity moves among vacancies in the lattice; (b) interstitial mechanism, in which the impurity atom replaces a silicon atom in the lattice, and the silicon atom is displaced to an interstitial site; (c) interstitial diffusion, in which impurity atoms do not replace atoms in the crystal lattice.
Diffusion under Concentration Gradient

- Dopant moves from heavy concentration area to lower concentration area
- Reason: simple statistics of motion: More dopant in heavy area
- Hence more heading in lower dopant direction
- Higher the temperature the faster dopants move
- Hence for doping done in a furnace

Figure 3.4 Diffusion under a concentration gradient: (a) two slices with different concentrations. (b) Focus on plane $P_1$ showing change in concentration. (c) Focus on slice 2 showing flux.
Diffusion Theory

Reference: Chapter 4 Jaeger or Chapter 3 Ruska

- Diffusion equations for the flux of dopants into the substrate
- Diffusion flow follows **Fick's First Law**

\[ J = -D \frac{\partial N(x,t)}{\partial x} = -D \nabla N(x,t) \]

Where:
- \( N \) = Impurity concentration: atoms/cc
- \( J \) = particle flux (atoms/cc/sec)
- \( D \) = diffusion coefficient (cm²/sec)

- **Continuity Equation**
- Now relate the flux to the changes in time and position of dopant
- Continuity Equation: **Fick's Second Law**

\[ \frac{\partial N}{\partial t} = - \frac{\partial J}{\partial x} = \frac{\partial}{\partial x} D \frac{\partial N(x,t)}{\partial x} = D \frac{\partial^2 N(x,t)}{\partial x^2} = D \nabla^2 N \]

Where: \( t \) = time

- This is the Diffusion differential equation in 1 dimension
- Assumption is that \( D \) is constant with \( x \)
Diffusion Solutions

• Solutions depend on Boundary Conditions
• Solutions in terms of Dt (Diffusion coef x time)
• Two typical cases depending on the source conditions

Constant Source Diffusion

• Constant source one common condition: ie unlimited dopant

\[ N(x,t) = N_0 \, \text{erfc}\left(\frac{x}{2\sqrt{Dt}}\right) \]

• Total impurity concentration

\[ Q = \int_{0}^{\infty} N(x,t) \, dx = 2N_0 \sqrt{\frac{Dt}{\pi}} \]

Limited Source Diffusion

• Total Dopant is fixed

\[ N(x,t) = \left[ \frac{Q}{\sqrt{\pi Dt}} \right] \exp\left( -\frac{x}{\sqrt{2Dt}} \right) \]
Constant Source Diffusion Solutions

- Constant source one common condition: ie unlimited dopant
- Surface concentration is fixed for all diffusion time

\[ N(x,t) = N_0 \, \text{erfc} \left( \frac{x}{2 \sqrt{Dt}} \right) \]

- Note this involves the Complementary Error Function
- Total impurity concentration

\[ Q = \int_0^\infty N(x,t) \, dx = 2N_0 \sqrt{\frac{Dt}{\pi}} \]

Fig. 4.2 A constant-source diffusion results in a complementary error function impurity distribution. The surface concentration \( N_0 \) remains constant and the diffusion moves deeper into the silicon wafer as the \( Dt \) product increases. \( Dt \) can change as a result of increasing diffusion time, increasing diffusion temperature, or a combination of both.
Useful Error Function erfc(x) Approximations

- Error function erf(x), Complementary Error Function erfc(x) are

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-s^2} ds
\]

\[
erfc(x) = 1 - \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-s^2} ds
\]

- erf(x) hard to find but easy to approximate with

\[
erf(x) = 1 - \left( a_1 t + a_2 t^2 + a_3 t^3 \right) e^{-x^2}
\]

\[
t = \frac{1}{1 + px} \quad \text{where} \quad p = 0.47047
\]

\[
a_1 = 0.3480242, \quad a_2 = -0.0958798, \quad a_3 = 0.7478556
\]

- See Abramowitz & Segun (Handbook of Mathematical Functions)

- Error on this is < 2.5x10^{-5} for all x

- We are using complementary error function

\[
erfc(x) = 1 - \text{erf}(x) \quad \text{erfc}(0) = 1 \quad \text{erfc}(\infty) = 0
\]

- Approximation has <2% error for x << 5.5

- For x > 5.5 use asymptotic approximation

\[
erfc(x) \to \frac{e^{-x^2}}{x\sqrt{\pi}} \left[ 1 - \frac{1}{2x^2} \right] \quad \text{as} \quad x \to \infty
\]

- Excel spreadsheet has erf and erfc built in.

but become inaccurate for x>5.4 – then use asymptotic equ.

- For x > 5.4 then ierfc(x) becomes

\[
ierfc(x) \to \frac{e^{-x^2}}{2x^2\sqrt{\pi}} \quad \text{as} \quad x \to \infty
\]
Limited Source Diffusion Solutions

- Where total dopant is fixed
- Surface dopant falls with time while dopant goes deeper

\[ N(x, t) = \left[ \frac{Q}{\sqrt{\pi Dt}} \right] \exp \left( - \left[ \frac{x}{\sqrt{2Dt}} \right]^2 \right) \]

- Often do constant source first (high concentration very shallow)
- Then drive in deeper using limited source

Fig. 4.3 A Gaussian distribution results from a limited-source diffusion. As the \(Dt\) product increases, the diffusion front moves more deeply into the wafer and the surface concentration decreases. The area under each of the three curves is the same.
Comparison of Normalized Gaussian & ERFC

- \( \text{erfc}(x) \) has much steeper curve than Gaussian
- Thus sharper boundary

\[ x = \frac{x}{2 \sqrt{Dt}} \]

Normalized distance from surface, \( \tilde{x} \)

Fig. 4.4 A graph comparing the Gaussian and complementary error function (erfc) profiles. We will use this curve to evaluate the \( \text{erfc} \) and its inverse.
Diffusion Constants in Si

- For common dopants: Change with temperature
- Follows Arrhenius Formula ($E_A = \text{activation energy of diffusion}$)

$$D = D_0 \exp\left(\frac{-E_A}{KT}\right)$$

$E_A = \text{activation energy of diffusion}$

---

**Fig. 4.5** Diffusion constants in silicon for (a) substitutional diffusers (above) and (b) interstitial diffusers (next page)


**Diffusion Constants in Si**

- High diffusion coef $D$ for poisons: Cu, Au, Fe & Li

![Graph showing diffusion coefficients for various elements vs. temperature](image)

**Fig. 4.5 (continued)**

$$D = D_0 \exp \left( \frac{-E_A}{kT} \right)$$

**Table 4.1** Typical Diffusion Coefficient Values for a Number of Impurities.

<table>
<thead>
<tr>
<th>Element</th>
<th>$D_0$(cm$^2$/sec)</th>
<th>$E_A$(eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>10.5</td>
<td>3.69</td>
</tr>
<tr>
<td>Al</td>
<td>8.00</td>
<td>3.47</td>
</tr>
<tr>
<td>Ga</td>
<td>3.60</td>
<td>3.51</td>
</tr>
<tr>
<td>In</td>
<td>16.5</td>
<td>3.90</td>
</tr>
<tr>
<td>P</td>
<td>10.5</td>
<td>3.69</td>
</tr>
<tr>
<td>As</td>
<td>0.32</td>
<td>3.56</td>
</tr>
<tr>
<td>Sb</td>
<td>5.60</td>
<td>3.95</td>
</tr>
</tbody>
</table>
Formation of PN Junction

- For diodes and transistors want to create a PN junction (interface)
- When diffusion falls below background dopant
- May be substrate level (diode) or previous diffusion
- Carrier level becomes

\[ p-n = N_A - N_D \]

Fig. 4.7 Formation of a \textit{pn} junction by diffusion. (a) An example of a \textit{p}-type Gaussian diffusion into a uniformly doped \textit{n}-type wafer; (b) net impurity concentration in the wafer. The metallurgical junction occurs at the point \( x = x_j \), where the net concentration is zero. The material is converted to \textit{p}-type to the left of \( x_j \) and remains \textit{n}-type to the right of \( x_j \).
Limits to Diffusion: Solid Solubility

- Sets upper limit to diffusion
- Silicon participates out the dopant at higher levels
- Limit is set the solid solubility of particular dopant in Si
- Complicated function of Temperature at diffusion

Figure 3-6 Solid solubilities of various impurities in silicon. (Reference 7. Reprinted with permission from the Bell System Technical Journal. Copyright 1960, AT&T.)
Common Process: Predeposition & Drive in

- Use diffusion to create thin layer of highly doped material
- Then drive in dopant from this layer as limited source at surface

Fig. 4.9 Calculated boron impurity profiles for Example 4.2. (a) Following the predeposition step at 900 °C for 15 min; (b) following a subsequent 5-hr drive-in step at 1100 °C. The final junction depth is 2.77 μm with a surface concentration of $1.1 \times 10^{18}$/cm$^3$. The initial profile approximates an impulse.
Dopant And Masks

- Commonly use patterned layer (oxide mostly) as mask
- Hence grow oxide, pattern with resist, etch oxide, strip for mask
- Then diffuse dopant at high temp (too high for resist)
- Dopant diffuses under mask

Figure 3.8 Pattern of diffusion near a masking window. (a) View of the window, showing one- and multidimensional diffusion zones. (b) Calculated two-dimensional diffusion profiles near a window. (Figure
Dopant Diffusion Under Mask

- Under mask diffusion depends on type: Constant or limited source

![Graph](image)

**Fig. 4.10** Normalized two-dimensional Gaussian and complementary error function diffusions near the edge of a window in the barrier layer. Copyright 1965 by International Business Machines Corporation; reprinted with permission from ref. [4].
Common Dopant Sources

- Often have solid, liquid and gaseous sources
- Different materials for each source type

<table>
<thead>
<tr>
<th>Type</th>
<th>Element</th>
<th>Compound Name</th>
<th>Formula</th>
<th>State</th>
<th>Diffusion Reactions*</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Antimony</td>
<td>Antimony Trioxide</td>
<td>Sb₂O₃</td>
<td>Solid</td>
<td>2AsH₃ + 3O₂ → As₂O₃ + 3H₂O</td>
</tr>
<tr>
<td></td>
<td>Arsenic</td>
<td>Arsenic Trioxide</td>
<td>As₂O₃</td>
<td>Solid</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Arsine</td>
<td>AsH₃</td>
<td>Gas</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Phosphorus</td>
<td>Phosphorus Oxychloride</td>
<td>POCl₃</td>
<td>Liquid</td>
<td>4POCl₃ + 3O₂ → 2P₂O₅ + 6Cl₂</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Phosphorus Pentoxide</td>
<td>P₂O₅</td>
<td>Solid</td>
<td>2PH₃ + 4O₂ → P₂O₅ + 3H₂O</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Phosphine</td>
<td>PH₃</td>
<td>Gas</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>Boron</td>
<td>Boron Tribromide</td>
<td>BBr₃</td>
<td>Liquid</td>
<td>4BBr₃ + 3O₂ → 2B₂O₃ + 6Br₂</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Boron Trioxide</td>
<td>B₂O₃</td>
<td>Solid</td>
<td>B₂H₆ + 3O₂ → B₂O₃ + 3H₂O</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Diborane</td>
<td>B₂H₆</td>
<td>Gas</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Boron Trichloride</td>
<td>BCl₃</td>
<td>Gas</td>
<td>BCl₃ + 3H₂ → 2B + 6HCl</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Boron Nitride</td>
<td>BN</td>
<td>Solid</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gold</td>
<td>Gold</td>
<td>Au</td>
<td>Solid</td>
<td>(Evap.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Element</th>
<th>Formula</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron</td>
<td>Fe</td>
<td>Undesirable impurities</td>
</tr>
<tr>
<td>Copper</td>
<td>Cu</td>
<td>from contamination</td>
</tr>
<tr>
<td>Lithium</td>
<td>Li</td>
<td></td>
</tr>
<tr>
<td>Zinc</td>
<td>Zn</td>
<td></td>
</tr>
<tr>
<td>Manganese</td>
<td>Mn</td>
<td></td>
</tr>
<tr>
<td>Nickel</td>
<td>Ni</td>
<td></td>
</tr>
<tr>
<td>Sodium</td>
<td>Na</td>
<td></td>
</tr>
</tbody>
</table>

*Note: Only selected diffusion reactions are listed

Figure 11.18  Deposition source table.
Furnace Seceptor Sources

- Boron Nitride wafer seseptors, between wafers
- Grow layer of Boron oxide on surface (soft)
- In furnace oxide releases Boron to wafers
- Boron dopant on surface of wafers
- Note wafers front faces seceptor
- Easy to do but disks change over time

Fig. 13  Stacking pattern for solid disc type dopants in a diffusion furnace. Courtesy of Standard Oil Engineered Materials.
Gas Dopant Sources

- Dopant containing gas flows over wafer
- Usually have a carrier gas (nitrogen)
- Dangerous gas product output

Fig. 4.22  Open-furnace-tube diffusion systems. (a) Solid source in a platinum source boat in the rear of diffusion tube; (b) liquid-source system with carrier gas passing through a bubbler; (c) diffusion system using gaseous impurity sources.
Bubbler Dopant Source

- Use gas or liquid dopant in bubbler to furnace

Figure 3-9 Vaporizer or “bubbler” for use with liquid sources.
Safety and Dopant Sources

- Common sources very deadly
- Measure exposure limit for 8 hours in parts per million (ppm)

<table>
<thead>
<tr>
<th>Source</th>
<th>8-hour exposure level (ppm)</th>
<th>Life-threatening exposure</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diborane (B₂H₆)</td>
<td>0.10</td>
<td>160 ppm for 15 min</td>
<td>Colorless, sickly sweet, extremely toxic, flammable.</td>
</tr>
<tr>
<td>Phosphine (PH₃)</td>
<td>0.30</td>
<td>400 ppm for 30 min</td>
<td>Colorless, decaying fish odor, extremely toxic, flammable. A few minutes’ exposure to 2000 ppm can be lethal.</td>
</tr>
<tr>
<td>Arsine (AsH₃)</td>
<td>0.05</td>
<td>6–15 ppm for 30 min</td>
<td>Colorless, garlic odor, extremely toxic. A few minutes’ exposure to 500 ppm can be lethal.</td>
</tr>
<tr>
<td>Silane (SiH₄)</td>
<td>0.50</td>
<td>Unknown</td>
<td>Repulsive odor, burns in air, explosive, poorly understood.</td>
</tr>
<tr>
<td>Dichlorosilane (SiH₂Cl₂)</td>
<td>5.00</td>
<td>...</td>
<td>Colorless, flammable, toxic. Irritating odor provides adequate warning for voluntary withdrawal from contaminated areas.</td>
</tr>
</tbody>
</table>

*Data from the 1979 American Conference of Governmental Hygienists (ACGIH).*
Uniformity of Dopant Distribution

- Variation with Vapour source Dopants
- Doping level varies with gas flow
- Note variation with flow direction

Figure 3-14 Doping patterns of phosphine under different flow conditions. Substrates were loaded parallel to the flow; arrows show flow direction. (Reference 22. Reprinted with permission of Solid State Technology, published by Technical Publishing, a company of Dun & Bradstreet.)
Spin-on Glass Dopants

- Glasses with dopant dissolved in solvent
- Spin on like photoresist
- Viscosity and spin speed control thickness
- Usually diluted with ethanol
- Types available:
  - As (arsenosilica)
  - B (Borosilica)
  - P (phosphorosilica)
  - Sb (antimoysilica)
- After spin on bake: 250 °C, 15 min.
- Baking densifies film, removes water
- Diffusion proceeds as with constant source diffusion

<table>
<thead>
<tr>
<th>Dopant</th>
<th>Gaseous Source</th>
<th>Liquid Source</th>
<th>Solid Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>As</td>
<td>AsH₃, AsF₃</td>
<td>arsenosilica&lt;sup&gt;S&lt;/sup&gt;</td>
<td>AlAsO₄&lt;sup&gt;d&lt;/sup&gt;</td>
</tr>
<tr>
<td>P</td>
<td>PH₃, PF₃</td>
<td>POC₁&lt;sub&gt;3&lt;/sub&gt;, phosphosilica&lt;sup&gt;S&lt;/sup&gt;</td>
<td>NH₄H₂PO₄&lt;sup&gt;d&lt;/sup&gt;, (NH₄)₂H₂PO₄&lt;sup&gt;d&lt;/sup&gt;</td>
</tr>
<tr>
<td>B</td>
<td>B&lt;sub&gt;2&lt;/sub&gt;H₆, BF₃, BCl₃</td>
<td>BB₃&lt;sub&gt;3&lt;/sub&gt;, (CH₃O)₃B borosilica&lt;sup&gt;S&lt;/sup&gt;</td>
<td>BN&lt;sup&gt;d&lt;/sup&gt;</td>
</tr>
<tr>
<td>Sb</td>
<td>SbH&lt;sub&gt;3&lt;/sub&gt;</td>
<td>Sb₂Cl₅, antimonysilica&lt;sup&gt;S&lt;/sup&gt;</td>
<td>Sb₂O₃, Sb₂O₄</td>
</tr>
</tbody>
</table>

<sup>d</sup> disc source   <sup>s</sup> spin on source  <sup>I</sup> ion implantation source only
Sheet Resistance Definition

- First measure of doped region is the change in resistivity
- Sheet resistance used for thin films or layers
- Measure resistance in Ohms per square
- Typically put in a test (unprocessed) wafer at that doping process
- Use these monitor wafers for sheet resistance during processing

\[ R = \rho \frac{L}{A} \]
\[ \rho = \frac{1}{\sigma} \]
\[ \sigma = \sigma_n + \mu \rho \]

Fig. 4.13 Resistance of a block of material having uniform resistivity. A uniform current distribution is entering the material perpendicular to the end of the block. The ratio of resistivity to thickness is called the sheet resistance of the material.

\[ R = (\rho/t)(L/W) = R_s(L/W) \]  \hspace{1cm} (4.14)

where \( R_s = (\rho/t) \) is called the sheet resistance of the layer of material.
Test Structures for Sheet Resistance

- Always create test structures to monitor process
- Typically place at edge of chip or special patterns in wafer
- Measure resistance sheet resistance Ohms/sq.
- Linear test structures

\[ R_s = \frac{\bar{\rho}}{x_j} = \left[ \int_0^{x_j} \sigma(x) \, dx \right]^{-1} \]

Fig. 4.14  Top and side views of two diffused resistors of different physical size having equal values of resistance. Each resistor has a ratio \( L/W \) equal to 7 squares. Each end of the resistor contributes approximately 0.65 additional squares.
Estimating Resistance

- Often state size of structure in terms of squares
- Thus for metal contact to diffusion pads get

![Diagram of resistor configurations with dimensions labeled.](image)

Corner = 0.56 squares

![Diagram of a corner configuration with dimensions labeled.](image)

0.65 squares

![Diagram of a straight configuration with dimensions labeled.](image)

0.14 squares

![Diagram of a corner configuration with dimensions labeled.](image)

0.35 squares

Fig. 4.15 Effective square contributions of various resistor end and corner configurations.
Surface Dopant Density vs Junction Depth

- Relationship between junction depth, Background $N_B$ and surface dopant concentration $N_0$
- Different charts for Constant and Limited source, n & p type
4 Point Probe Sheet Resistance Measurement

- Test structures often not measureable during processing
- Instead use 4 point probe stations
- Use 2 current sources, separate from V measurement
- Thus do not get resistive loss in measurement
- Use on test wafer

**Fig. 4.17** Four-point probe with probe spacing $s$ used for direct measurement of bulk wafer resistivity and the sheet resistance of thin diffused layers. A known current is forced through the outer probes, and the voltage developed is measured across the inner probes. (See eqs. (4.16) through (4.18).)
Common Resistance Test Structure: Van der Pauw

- 4 point probe type test structure for post fabrication tests
- Need to add metallization contacts first
- Measures sheet resistance

4 point structures on lab wafers – two for p and n dopants

Fig. 4.19  A simple van der Pauw test structure used to measure the sheet resistance of a diffused layer. Sheet resistance is calculated using eq. (4.20).
**Angle Lapping: Stain Measurement of Junction thickness**

- For all doping need to determine dopant depth/profile
- For diodes/transistors junction depth important process
- Typically put in a test (unprocessed) wafer at that doping process
- Lap (grind away) test wafer at shallow angle (< 2°)
- After lapping stain the wafer to identify dopant

**Staining N type Junction**

- Place drop of copper sulphate (CuSO₄) junction
- Illuminate junction with intense light (UV best) causes junction to forward bias
- Voltage causes Cu⁺⁺ to plate on n side
**Interference Technique for Grove**

- Angle lap & stain wafers
- Place Glass slide over wafer
- Illuminate with single wavelength light laser or sodium vapour light
- Get optical interference creating lines at half wavelength
- Junction depth by counting lines

\[ x_j = d \tan(\theta) = N \frac{\lambda}{2} \]

*Fig. 4.12* Junction depth measurement by the angle-lap and stain method. Interference fringe lines are used to measure the distance \( d \), which is related to the junction depth using eq. (4.12).
Cylinder Grove of Junctions

- To get shallow angle use a rotating cylinder
- Grove & stain, then measure linear distance
- Depth calculated as below

\[ x_j = \sqrt{(R^2 - b^2)} - \sqrt{(R^2 - a^2)} \]  \hspace{1cm} (4.10)

\[ (x + x_j) - x = \sqrt{R^2 - b^2} - \sqrt{R^2 - a^2} = R \left( \sqrt{1 - \left( \frac{b}{R} \right)^2} - \sqrt{1 - \left( \frac{a}{R} \right)^2} \right) \]

\[ \approx R \left\{ \left( 1 - \frac{1}{2} \frac{b^2}{R^2} \right) - \left( 1 - \frac{1}{2} \frac{a^2}{R^2} \right) \right\} \quad \text{if} \quad R \gg a, R \gg b \]

\[ x_j \approx \frac{a^2 - b^2}{2R} = \frac{(a+b)(a-b)}{2R} \]

**Fig. 4.11** Junction-depth measurement by the groove-and-stain technique. The distances \( a \) and \( b \) are measured through a microscope, and the junction depth is calculated using eq. (4.11).
### Advanced Techniques for Dopant Measurement

**TABLE 4**
Commonly used diffusion profile measurement techniques

<table>
<thead>
<tr>
<th>Profile techniques</th>
<th>Characteristics</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitance-Voltage</td>
<td>Carrier concentration at the edge of the depletion layer of a pn junction. Maximum total dopants $2 \times 10^{12}$ atoms/cm$^2$.</td>
<td>33</td>
</tr>
<tr>
<td>Differential conductance</td>
<td>Resistivity and Hall effect mobility of net electrically active species. Requires thin-layer removal, concentration range from $10^{20}$ to $10^{18}$ atoms/cm$^3$.</td>
<td>34</td>
</tr>
<tr>
<td>Spreading resistance</td>
<td>Resistance on angle-beveled sample. Good for comparison with known profiles and quick semi quantitative evaluation. $x_j \geq 1\mu$m.</td>
<td>35</td>
</tr>
<tr>
<td>SIMS</td>
<td>High sensitivity on many elements; for B and As detection limit is $5 \times 10^{15}$cm$^{-3}$. Capable of measuring total dopant profiles in 1000Å range. Needs standards.</td>
<td>36</td>
</tr>
<tr>
<td>Radioactive tracer analysis</td>
<td>Total concentration. Lower limit is $10^{15}$cm$^{-3}$. Limited to radioactive elements with suitable half-life times: P, As, Sb, Na, Cu, Au, etc.</td>
<td>37</td>
</tr>
<tr>
<td>Rutherford backscattering</td>
<td>Applicable only for elements heavier than Si.</td>
<td>38</td>
</tr>
<tr>
<td>Nuclear reaction</td>
<td>Measures total boron through $^{10}$B(n,$^4$He)$^7$Li, or $^{11}$B(p,$^\alpha$). Needs Van de Graaff generator.</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
</tr>
</tbody>
</table>
Spreading Resistance

- Make angle groove
- Now use 4 point probe across width of groove
- Good for junctions greater than 1 microns
- Gives junction profiles

Fig. 20 Commercial spreading resistance measuring system. Courtesy of Solid State Measurements, Inc.
Secondary Ion Mass Spectrometry (SIMS)

- Bombard surface in vacuum with ions (1-20 KeV)
- Nocks atoms off surface (sputtering)
- Sputtered atoms collect in Mass Spectrometer
- Count the number of atoms with specific charge/mass ratio
  Si different than dopants
- Can sputter down depth of sample measuring ratios
- Get a depth versus dopant profile
- Can map the dopants vs position
- Expensive: about $500/$1000 per profile

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**Fig. 18** (a) Schematic diagram of a secondary ion mass spectrometer. (b) Typical SIMS depth profile for boron into boron-doped silicon.
Scanning Ion Microscopy (SMIS)

- Get 2D map of dopant profile
- Expensive: about $1000 per profile
- Great for complex 2D structures