Diffraction
• Diffraction occurs when light waves is passed by an aperture/edge
• Huygen's Principal:
  each point on wavefront acts as source of another circular wave
• Consider light from point source at infinity or parallel beam (laser)
• Light diffracts around objects
• Consider a slit in a plate: light from edge diffracts
• Edge creates interference effects between the waves at each point
• Thus at edges asymmetric effects results in bending
• Diffractions sets the resolution limits for lens and optics systems

Figure 13-14  Straight-edge diffraction.
(d) Diffraction fringes from a straight line.
(From M. Cagnac, M. Francon, and J. C. Thirer, Atlas of Optical Phenomenon, Plate 32,
Berlin: Springer-Verlag, 1962.)

Figure 13-18  Fresnel shadow of a screw.
(From M. Cagnac, M. Francon, and J. C. Thirer, Atlas of Optical Phenomenon, Plate 36,
Berlin: Springer-Verlag, 1962.)

HUYGEN’S PRINCIPLE states that each point on a propagating wavefront is an emitter of secondary wavelets.
Fresnel and Fraunhofer Interference

- Image created depends on distance from slit light is seen
- Strongly seen in interference at the edge or small objects
- Two types: Fresnel and Fraunhofer Interference
- Both assume light at infinity nearly parallel light – laser beam

Fraunhofer Interference

- Diffracted light sensed at infinity (ie a focused image)
- Simpler equations

Fresnel interference

- Pattern created near the diffraction point
- Much more complex equations: involve near E-mag field
- Pattern very dependent on the distance & slit/object size

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![Image](image_url)

*Fig. 2.8.14. Transition from Fresnel to Fraunhofer diffraction. A portion of a wave, $W'$, passes through a slit of width, $d$. Intensity distributions across the wave are shown for planes $P_1$ (close to the slit), $P_2$ (just inside the Fresnel distance), and $P_3$ (beyond the Fresnel distance).*

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Fresnel Interference from laser

Fraunhofer Interference
Fresnel Number

- Fresnel number measures if the image is close or far field
- Consider a slit width b
- Measuring image at a distance R
- Fresnel number is
  \[ F > \frac{b^2}{R\lambda} \]
- \( F << 1 \) Fraunhofer interference (Far Field)
- \( F >> 1 \) Fresnel – complicated image function of distance R
Fraunhofer Interference

- Consider a single slit width \( b \) but infinite length
- Diffraction effects “seen at infinity” which means distances \( R \)

\[
R > \frac{b^2}{\lambda}
\]

- If focus slit with lens get the same as at infinity
- Most common effect
- Intensity follows the pattern of a synch function

\[
I(\beta) = I_0 \left[ \frac{\sin(\beta)}{\beta} \right]^2
\]

where

\[
\beta = \frac{\pi b \sin(\theta)}{\lambda}
\]

\( \theta = \) angular deviation of pattern from minimum

**FIGURE 15C**

Geometrical construction for investigating the intensity in the single-slit diffraction pattern.
Fraunhofer Interference Pattern

- Zeros are at
  \[ \beta = \pm N\pi \]
  where \( N \) is any integer

- Large \( d \) little pattern (a), small \( d \) pattern spreads out (c)

**FIGURE 15D**
Amplitude and intensity contours for Fraunhofer diffraction of a single slit, showing positions of maxima and minima.

**FIGURE 15B**
Photographs of the single-slit diffraction pattern.
Circular Fraunhofer Interference

- Interference changes for circular opening
- Called an Airy Disk
- Lenses act as circular apertures
- Most important for laser systems
- For single circular aperture diameter $D$
- Intensity follows the pattern

$$I(\beta) = I_0 \left[ \frac{2J_1(\beta)}{\beta} \right]^2$$

where

$$\beta = \frac{\pi D \sin(\theta)}{\lambda}$$

$J_1$ = Bessel function of first kind, order 1
$\theta$ = angular deviation of pattern from minimum

- Shrinks rings as wavelength decreases
- White light creates multicolour rings
Bessel Functions

- Bessel functions: from solutions to cylindrical coordinate problems
- Comes from solutions to the DE

\[ x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + \left( x^2 - n^2 \right)y = 0 \]

- Typical applications: Electromagnetic waves in cylindrical cavities
- Heat flow in cylinder (e.g., laser heating of material)
- Bessel functions of first kind order \( n \) are \( J_n(x) \)
- Bessel function \( J_1(x) \) are given by the expansion

\[ J_1(x) = \frac{x}{2} - \frac{x^3}{2^2 \cdot 4} + \frac{x^5}{2^2 \cdot 4^2 \cdot 6} \ldots \]

- Look like a decaying sin/cos wave
- Thus for Airy disk at the centre

\[
I(\beta \to 0) \to I_0 \left[ \frac{2J_1(\beta)}{\beta} \right]^2 \approx I_0 \left[ \frac{\beta}{2} \cdot \frac{2}{\beta} \right]^2 \to I_0
\]
Comparison of Circular and Slit Fraunhofer Interference

- Slit produces smaller width pattern
- Circular Airy disks is wider than slit of same size/wavelength
- Nulls occur further out
- Peaks smaller after nulls than for slit
- Higher % energy in central peaks

\[
y_n = \left(\frac{2J_1(x)}{x}\right)^2, \text{ where } J_1(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{x^{n+1}}{n+1}
\]

Note: \( J_1(x) \) is the Bessel function of the first kind of order unity.

\[
x = \frac{\pi}{\lambda} D \sin \theta
\]

\( \lambda \) = Wavelength

\( D \) = Aperture diameter

\( \theta \) = Angular radius from pattern maximum

\[
y_n = \left(\frac{\sin x}{x}\right)^2, \text{ where } x = \frac{\pi}{\lambda} w \sin \theta
\]

\( \lambda \) = Wavelength

\( w \) = Slit width

\( \theta \) = Angular deviation direction of pattern maximum

FRAUNHOFER DIFFRACTION PATTERN of a singlet slit superimposed on the Fraunhofer Diffraction Pattern of a circular aperture.
Minimum Spotsize of Focused Laser Beam

- For beam much smaller than lens limited by waist size of input beam
- Hence if waist in at the focus then

\[ w_0 = \frac{\lambda f}{\pi w} \]

- NOTE: lens aberration may modify this by a factor
- eg He-Ne laser is focused through a lens with
  \( f = 7 \text{ mm} \quad \lambda = 632.8 \text{ nm} \quad w_{\text{out}} = 0.4 \text{ mm} \)
- What is the minimum spot produced?
- Assume input waist is at focus

\[ w_0 = \frac{\lambda f}{\pi w} = \frac{6.328 \times 10^{-7} (0.007)}{\pi \times 4 \times 10^{-4}} = 3.5 \times 10^{-6} = 3.5 \mu m \]

- For singlet lens multiply this by 1.333 for 4.7 micron

CONCENTRATION OF LASER BEAM by a laser line focusing singlet. Size of the focal waist has been greatly exaggerated for illustrative purposes.
**Diffraction Limited spot**

- If laser beam fills the lens then diffraction limited
- Opening of width D
- Minimum spot is to point of first zero in diffraction pattern

\[ I = \frac{b \sin(\theta)}{\lambda} = \frac{bd_{\text{min}}}{\lambda 2f} \]

\[ d_{\text{min}} = \frac{2f\lambda}{D} \]

- Since circular effectively Airy diffraction
  add a factor of 1.22

\[ d_{\text{min}} = \frac{2.44 f\lambda}{D} \]

---

*Fig. 2.15. Diagram illustrating the diffraction limited spot size.*
Resolution of Spots

- Resolution tells how good is the fine detail on an image
- Simple case: what is the min separation of 2 small, close, spots
- When spots fully separated then can resolve
- In telescopes how close can 2 stars be seen as separate objects
- In astronomy this is called the angular separation
- Bigger telescope mirror – greater angular separation
- However on earth atmosphere density changes limits resolution
- Actually effective atm telescope diameter only 20 cm
- Later can see how to compensate for atmosphere
- Hence space based telescopes much better (Hubble)
Rayleigh’s Criteria Images

- What determines when we can separate two spots in an image
- When spots overlap enough cannot separate
- Different systems determine how much overlap allowed

$$d_{\text{min}} = \frac{1.22 f \lambda}{D}$$

- This is Rayleigh’s Criteria most common
- Where two separate peaks can just be separated
**Depth of Focus**

- Spot is in focus if waist expands less than 5%
- Using waist formula

\[ w(z) = w_0 \left[ 1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2 \right]^{1/2} \]

- Solve this waist formula for 5% change then

\[ \Delta z = \pm \frac{0.32\pi w_0^2}{\lambda} \]

- eg previous He-Ne laser is focused through a lens with
  \( f = 7 \text{ mm} \quad \lambda = 632.8 \text{ nm} \quad w_0 = 3.5 \text{ micron} \)
  what is the depth of focus of spot produced?

\[ \Delta z = \pm \frac{0.32\pi (3.5 \times 10^{-6})^2}{6.328 \times 10^{-7}} = 1.97 \times 10^{-5} \text{ m} = 19.7 \text{ \mu m} \]

**CONCENTRATION OF LASER BEAM** by a laser line focusing singlet. Size of the focal waist has been greatly exaggerated for illustrative purposes.
Diffraction Limit and Gaussian Optics

- Both give slightly different answer because looking at different beam parts
- eg He-Ne laser is focused through a lens with $f = 50 \text{ mm}$, $\lambda = 632.8 \text{ nm}$, $D = 10 \text{ mm}$
- What is the minimum spot produced?
- If want first minimum of Airy disk then
  
  $$d_{\min} = \frac{2.44 \, f \lambda}{D} = \frac{2.44(0.05)(6.328 \times 10^{-7})}{0.01} = 7.7 \times 10^{-6} \text{ m} = 7.7 \mu \text{m}$$

- If assume input waist is at focus using range formulas
- Let spot be typical 1/3 lens so waist $w_\text{out} = 3.3 \text{ mm}$
  
  $$w'' = \frac{\lambda \, f}{\pi \, w_0} = \frac{6.328 \times 10^{-7}(0.05)}{\pi \times 0.0033} = 3.05 \times 10^{-6} = 3.05 \mu \text{m}$$

- Difference comes from 1st minimum diameter of Airy disk verses 1/e² radius
Young’s Double Slit Experiment

- Thomas Young 1801 experiment to prove light is a wave nature
- Consider 2 slits width b
- Separated by space a (centre to centre of slits)
- Now the pattern created by one slit creates interference with other
- At time considered the proof that light behaved as a wave

\[ \Sigma_a \]

Wavefronts

Slotted plane

Interference fringes

(c)

Figure 10.20 (a, b) Interference fringes formed when a monochromatic coherent plane wave strikes a thin opaque plane with two neighbouring narrow slits in it (the Young Experiment).
(c) The common source \( S_0 \) ensures coherence
Double Slit Interference

- Get the single slit pattern forming envelope
- Interference of two slits modulating that.

\[
I(\beta) = 4I_0 \left[ \frac{\sin(\beta)}{\beta} \right]^2 \cos(\alpha) \quad \beta = \frac{\pi b \sin(\theta)}{\lambda} \quad \alpha = \frac{\pi a \sin(\theta)}{\lambda}
\]

\(\theta\) = angular deviation of pattern from minimum

- For zeros: \(\beta = \pm N\pi\)
- Principal Maximums occur at

\[
sin(\theta) = \frac{m\lambda}{a}
\]

where \(m = \) any integer, order of the diffraction

---

**Figure 10.13** (a) Double-slit geometry. Point \(P\) on \(\sigma\) is essentially infinitely far away. (b) A double-slit pattern (a = 3b).
Young’s Double Slit & Single Slit

- If take single slit
- Then add second slit see the one pattern on top of other
- If slits wide see both
- Slits narrow – nearly even pattern due to wide null in small slit

Figure 10.14 Single- and double-slit Fraunhofer patterns. (a) Photographs taken with monochromatic light. The faint cross-hatching arises entirely in the printing process. (Photos courtesy M. Cappel, M. Frascn, and J. C. Thierm: Atris optischer Erscheinungen). Berlin-Heidelberg-New York: Springer, 1962.) (b) When the slit spacing equals $b$, the two slits coalesce into one (of width $2b$) and the single-slit pattern appears—that's the first curve closest to you. The farthest curve corresponds to the two slits separated by $a = 10b$. Notice that the two-slit patterns all have their first diffraction minimum at a distance from the central maximum of $Z_0$. Note how the curves gradually match Fig. 10.13b as the slit width $b$ gets smaller in comparison to the separation $a$. (Reprinted from “Graphical Representations of Fraunhofer Interference and Diffraction” Am. J. Phys., 62, 6, (1994), with permission of A. B. Bartlett, University of Colorado and B. Moshly, Northeast Missouri State University and the American Association of Physics Teachers.)
Young’s Double Slit & Quantum Effects

- Classic description of Young’s double slit is a wave effect
- But what about light being particles: photons
- Taylor (1909) reduced light so one photon at time at slit
- Still get interference
- Davisson and Germer in 1927 proved particles acted as waves
- Reason is the wave – particle duality first proposed by de Brogile
- The wave of a particle detects both slits
- Claus Jönsson in 1962 showed electrons do same thing
- Because e’s have a quantum wave function
- Even if single particle going through one.
- Creates the dual slit pattern due to quantum probability field
- Wave function field of photon extends far enough to see other slit
- Alternative: the many worlds theory of quantum
- See impact of all possible probabilistic events from “all universes
Diffraction Gratings

- Diffraction gratings: extends the 2 slits to a repeated pattern used by many systems eg spectrometers, acousto-optic deflectors
- Recall the Interference from a single width b

\[ I(\beta) = I_0 \left[ \frac{\sin(\beta)}{\beta} \right]^2 \]

\[ \beta = \frac{\pi b \sin(\theta)}{\lambda} \]

\( \theta = \) angular deviation of pattern from minimum
- Now consider n slits width b spaced distance d apart
- Extending the 2 slits get the diffraction pattern from each slit
- But the diffraction patterns interfere
- The more slits, the more the interference patterns add up
- If have several slits then waves from each interfere
- More slits narrower beams

FIGURE 17A
Fraunhofer diffraction patterns for gratings containing different numbers of slits.
Diffraction Gratings Formulas

- Similar to the 2 slit the intensity becomes for n slits

\[
I(\beta) = I_0 \left[ \frac{\sin^2(\beta)}{\beta^2} \right] \left[ \frac{\sin^2(n\gamma)}{\sin^2(\gamma)} \right]
\]

\[
\gamma = \frac{\pi d \sin(\theta)}{\lambda}
\]

- Principal Maximums occur at

\[
\sin(\theta) = \frac{m\lambda}{d}
\]

where \( m = \) any integer, order of the diffraction

- The maxima vary within the single slit \( \beta \) function
- Result is that large \( n \) with small \( d \) gives narrow lines
- Peaks of different \( \lambda \) at different angles
- Single \( \lambda \) get peaks: white different peaks for each colour

**FIGURE 17C**
Fraunhofer diffraction by a grating of six very narrow slits and details of the intensity pattern.
Spectrometry

- Want to see the spectral distribution of the light
- Used for detecting presence of elements based on spectrum
- Two ways – prism & diffraction gratings
- Mostly use diffraction gratings
- Generally greater spread than prisms
- Black body continuous spectrum
- Hot glowing gas (plasma) get emission spectrum of material
- Pass black body light through cold gas get absorption spectrum