# Econ 802

## First Midterm Exam

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All questions have equal weight. If something is unclear, please ask. You may want to work first on the questions where you feel most confident.

1. A firm has two inputs $(x_1, x_2) \geq 0$ and one output $y \geq 0$. For any $n = 0, 1, 2 \ldots$, if <u>both</u> inputs are $\geq n$ but <u>either</u> input is $< n+1$ then the maximum output is $y = n$.

(a) Draw a graph showing the isoquant set $Q(n)$ and the input requirement set $V(n)$ for some integer $n \geq 1$. Explain your answer in words. You can assume free disposal of inputs and outputs.

(b) Under what conditions, if any, would the profit maximization problem for this firm have a solution? Justify your answer.

(c) Assuming output y is an integer, do you think the cost minimization problem for this firm would normally have a solution? Why or why not?

2. Let the production function be $y = \ln(x_1 + 1) + \ln(x_2 + 1)$ where $x_1 \geq 0$ and $x_2 \geq 0$.

(a) Consider profit maximization for a perfectly competitive firm with output price $p > 0$ and input prices $w_1 > 0$, $w_2 > 0$. Does this problem always have a solution? If so, is the solution always unique? Explain your reasoning.

(b) Using Kuhn-Tucker multipliers, derive conditions under which $x_1 > 0$ would hold, and conditions under which $x_1 = 0$ would hold. Then do the same for $x_2$.

(c) Suppose the first order conditions yield a solution $x_1^* > 0$ and $x_2^* > 0$ (i.e. the Kuhn Tucker multipliers are irrelevant). Would the necessary second order condition hold at $x^*$? What about the sufficient SOC? Justify your answer.

3. Assume all of the functions described below are well defined.

(a) Prove that the <u>unconditional</u> input demand functions $x(p, w)$ are homogeneous of degree zero in $(p, w)$.

(b) Prove that the <u>conditional</u> input demand functions $x(w, y)$ are homogeneous of degree zero in w when the output y is held constant.

(c) Prove that the profit function $\pi(p, w)$ is homogenous of degree one in $(p, w)$ and the cost function $c(w, y)$ is homogenous of degree one in $w$ (with $y$ held constant).

4. A firm's production plan is written $y = (y_1, y_2)$ where $y_1 \leq 0$ is an input and $y_2 \geq 0$ is an output. Price vectors $p = (p_1, p_2)$ are always positive. An economist has made a large number of observations of the form $(p^t, y^t)$ for $t = 1 \,.\,.\, T$ where $p^t$ is the price vector in period $t$ and $y^t$ is the production plan in period $t$.

(a) Suppose that in all periods when $p_1^t < p_2^t$ the firm chose $y^t = (-2, +2)$ and in all periods when $p_1^t \geq p_2^t$ the firm chose $y^t = (0, 0)$. Show that these choices are consistent with the Weak Axiom of Profit Maximization (WAPM).

(b) Find the smallest convex monotonic production set YI that is consistent with the information in part (a), show it graphically, and explain your reasoning.

(c) Describe a new observation $(p^{T+1}, y^{T+1})$ that would not violate WAPM and would prove the true production set Y is larger than the set YI in part (b). Explain using a graph.

5. Consider the production function $f(x_1 \,.\,.\, x_n) = (x_1 x_2 x_3 \,.\,.\, x_n)^{1/n}$ where $x_i \geq 0$ for all $i = 1 \,.\,.\, n$.

(a) Compute the local elasticity of output with respect to scale $e(x)$ at some given input bundle $(x_1 \,.\,.\, x_n) > 0$. Interpret your answer using economic concepts.

(b) For the case $n = 2$, compute the elasticity of substitution $\sigma$ at a given price vector $(w_1, w_2) > 0$. Interpret your answer using economic concepts.

(c) For the case $n = 2$, suppose $w_2$ rises while $w_1$ remains unchanged. Would the firm's expenditure on input 1 as a fraction of total cost rise, fall, or stay the same? Explain your reasoning carefully.