Econ 802

Answers to First Midterm

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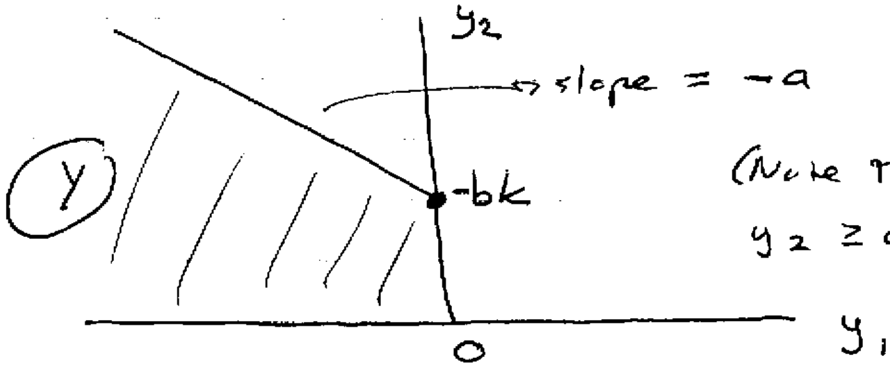
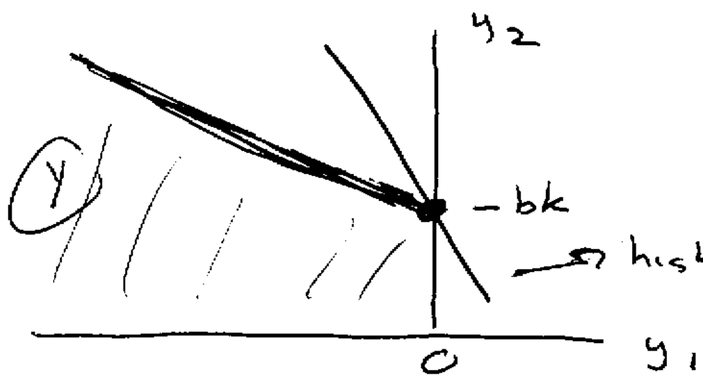
1. (a) In the short run the upper boundary of the production possibilities set is described by $y_2 = -ay_1 - bk$ where $k < 0$, $a, b > 0$



(Note that $y_1 \leq 0$, $y_2 \geq 0$)

The set Y is convex because if y and y' are both in Y then any $y'' = ty + (1-t)y'$ for $0 \leq t \leq 1$ is also in Y (the boundaries of Y are linear). However Y is <u>not</u> strictly convex because the points $(0,0)$ and $(0, -bk)$ are both in Y, but convex combinations of those two points are on the boundary of Y, not in the interior of Y.

(b) The firm wants to maximize $p_1 y_1 + p_2 y_2 + p_3 y_3$ subject to $y \in Y$. But in the short run $y_3 = k$ is fixed, so p_3 is irrelevant and the firm maxes $p_1 y_1 + p_2 y_2$ subject to the short run production possibilities set in (a). The slope of an isoprofit line in (y_1, y_2) space is $-\frac{p_1}{p_2}$. If this is <u>steeper</u> than the upper boundary of Y, the firm operates at the point $(0, -bk)$ ⇒ see graph:



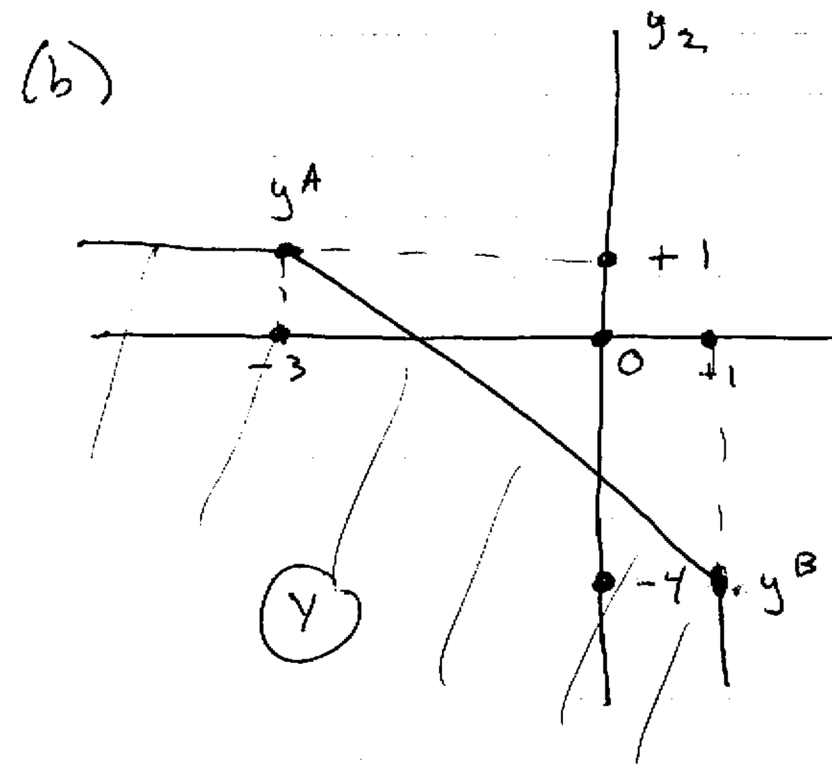
If $-\frac{p_1}{p_2} = -a$ then all points along the upper boundary $y_2 = -ay_1 - bk$ are equally good. In either of those cases the profit max problem has a solution. But if $-\frac{p_1}{p_2} > -a$ or $\frac{p_1}{p_2} < a$ so that the isoprofit lines are ~~steeper~~ flatter than the upper boundary of Y, there is no solution. To see this, plug in $y_2 = -ay_1 - bk$ into the firm's objective function to get

$$p_1 y_1 + p_2[-ay_1 - bk] + p_3(-bk)$$

If $p_1 - ap_2 < 0$, the derivative with respect to y_1 is negative, which means profit can always be increased by making y_1 more negative (using more input).

2. (a) We need $p^A y^A \geq p^A y^B$ and $p^B y^B \geq p^B y^A$. The first inequality is true because $p^A y^A = 0$ and $p^A y^B = 1 - 12 = -11$. The second inequality is true because $p^B y^B = -1$ and $p^B y^A = -9 + 1 = -8$.

(b)



One set that works is the line segment between y^A and y^B plus all points with less of one or both goods.

(This is what Varian calls "the inner bound" to the true Y, or YI)

The origin cannot be in Y, because if it were included zero profit would always be feasible. However, at prices $p^B = (3, 1)$ the firm chooses $y^B = (1, -4)$ which gives profit $= -1$. If zero profit were feasible, this would not be profit-maximizing behavior.

3. (a) We need to show that $\pi(p, w)$ is increasing in p and decreasing in w; that it is homogeneous of degree 1; and that it is continuous.

(i) The function is differentiable so check

$$\frac{\partial \pi}{\partial p} = A\left(\frac{1}{1-\alpha}\right) p^{\frac{1}{1-\alpha}-1} w^{-\frac{\alpha}{1-\alpha}} > 0$$

so it is increasing in p. (note that $\alpha < 1$)

$$\frac{\partial \pi}{\partial w} = A p^{\frac{1}{1-\alpha}} \left[\frac{-\alpha}{1-\alpha}\right] w^{-\frac{\alpha}{1-\alpha}-1} < 0$$

so it is decreasing in w.

(ii) Multiply both prices by $\lambda > 0$ to get

$$\pi(\lambda p, \lambda w) = A\,[\lambda p]^{\frac{1}{1-\alpha}}\,[\lambda w]^{-\frac{\alpha}{1-\alpha}}$$
$$= A\,\lambda^{\frac{1}{1-\alpha}} p^{\frac{1}{1-\alpha}}\,\lambda^{-\frac{\alpha}{1-\alpha}} w^{-\frac{\alpha}{1-\alpha}}$$
$$= \lambda\left[A p^{\frac{1}{1-\alpha}} w^{-\frac{\alpha}{1-\alpha}}\right]$$
$$= \lambda\,\pi(p, w)$$ so it is homog. of degree 1.

(iii) Any differentiable function is automatically continuous.

(b) From Hotelling's Lemma the output supply function is

$$\frac{\partial \pi}{\partial p} = \frac{A}{1-\alpha}\,p^{\frac{\alpha}{1-\alpha}} w^{-\frac{\alpha}{1-\alpha}} = y(p, w)$$

which is increasing in p. The input demand function is

$$-\frac{\partial \pi}{\partial w} = x(p, w) = \frac{\alpha A}{1-\alpha}\,p^{\frac{1}{1-\alpha}}\,w^{-\frac{1}{1-\alpha}}$$

which is decreasing in w. Thus the output supply curve slopes up and the input demand curve slopes down, as expected.

4. (a) Increasing returns to scale. Multiply both inputs by $\lambda > 1$ to get

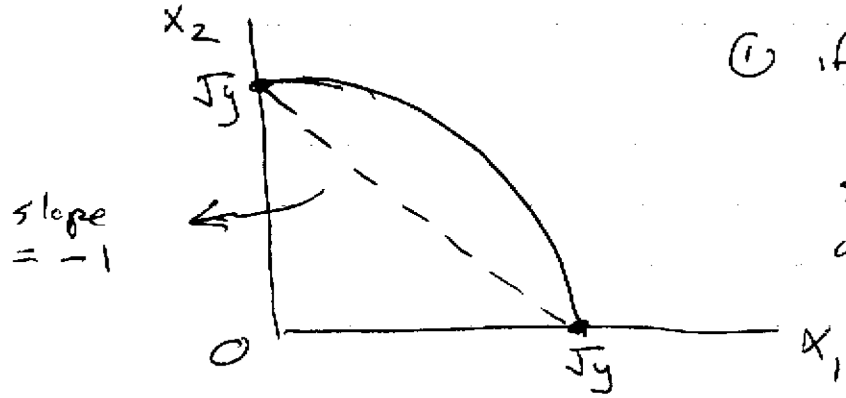
$$f(\lambda x) = (\lambda x_1)^2 + (\lambda x_2)^2 = \lambda^2[x_1^2 + x_2^2] = \lambda^2 f(x).$$

Thus $f(\lambda x) = \lambda^2 f(x) > \lambda f(x)$ as required for IRS.

(b) An isoquant satisfies $y = x_1^2 + x_2^2$ for fixed y.
The RTS is $\frac{MP_1}{MP_2} = \frac{2x_1}{2x_2} = \frac{x_1}{x_2}$

which is the negative of the slope of the isoquant. Thus the isoquant gets <u>steeper</u> as $x_1 \uparrow$ and $x_2 \downarrow$.

The graph of the isoquant looks like this:



① if $w_1 = w_2$ then the isocost lines have slope $= -1$ and there are two optimal points: $x_1 = \sqrt{y}$ and $x_2 = 0$, <u>or</u> $x_1 = 0$, $x_2 = \sqrt{y}$ (see dashed line)

② if $w_1 > w_2$ then input 2 is cheaper and the only optimal point is $x_1 = 0$ $x_2 = \sqrt{y}$

③ if $w_1 < w_2$ then input 1 is cheaper and the only optimal point is $x_1 = \sqrt{y}$, $x_2 = 0$.

5. (a) $L = w_1 x_1 + w_2 x_2 - \lambda[x_1 x_2 - y]$

$$\left.\begin{aligned} \frac{\partial L}{\partial x_1} &= w_1 - \lambda x_2 = 0 \\ \frac{\partial L}{\partial x_2} &= w_2 - \lambda x_1 = 0 \end{aligned}\right\} \Rightarrow \frac{w_1}{w_2} = \frac{x_2}{x_1}$$

$$\frac{\partial L}{\partial \lambda} = -(x_1 x_2 - y) = 0$$

Substitute $x_2 = \frac{w_1 x_1}{w_2}$ into the constraint: $y = \frac{w_1}{w_2} x_1^2$

$$\Rightarrow x_1(w, y) = \sqrt{\frac{y w_2}{w_1}} \qquad x_2(w, y) = \sqrt{\frac{y w_1}{w_2}}$$

(b) Profit is $p x_1 x_2 - w_1 x_1 - w_2 x_2$. Multiply both inputs by $\lambda > 0$ to get $\lambda^2 p x_1 x_2 - \lambda[w_1 x_1 + w_2 x_2]$
This is quadratic in λ and the coefficient of λ^2 is positive. For sufficiently large $\lambda > 0$, this is an increasing function of λ and there is no upper bound on profit.