Econ 802

Answers to Midterm #2

Greg Dow

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1. (a) $e(x^*) = \frac{df(tx^*)}{dt} \cdot \frac{t}{f(tx^*)}\Big|_{t=1} = \frac{\sum_{i=1}^{n} \frac{\partial f(x^*)}{\partial x_i} x_i^*}{f(x^*)}$

The Lagrangean for cost min is

$L = \sum_i w_i x_i - \lambda [f(x) - y^*]$

FOC $\Rightarrow$ $w_i = \lambda \frac{\partial f(x^*)}{\partial x_i}$ $\Rightarrow$ $\frac{w_i}{\lambda} = \frac{\partial f(x^*)}{\partial x_i}$

Substitute into $e(x^*)$ to get $e(x^*) = \frac{\frac{1}{\lambda}\sum_{i=1}^{n} w_i x_i^*}{f(x^*)}$

Since $\lambda = MC(y^*)$ this gives

$e(x^*) = \frac{AC(y^*)}{MC(y^*)}$

(b) Let $y = f(x) = A x_1^{\alpha} x_2^{\beta}$ where $\alpha, \beta > 0$

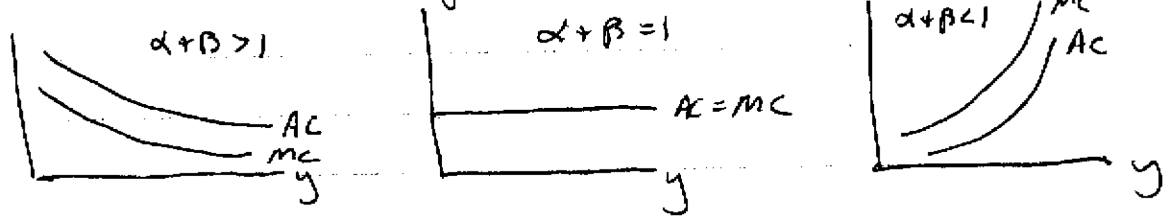
$\frac{\partial f(x)}{\partial x_1} = \alpha A x_1^{\alpha-1} x_2^{\beta}$ and $\frac{\partial f(x)}{\partial x_2} = \beta A x_1^{\alpha} x_2^{\beta-1}$

Therefore $e(x) = \frac{\sum_{i=1}^{2} \frac{\partial f(x)}{\partial x_i} x_i}{f(x)} = \frac{(\alpha+\beta) A x_1^{\alpha} x_2^{\beta}}{A x_1^{\alpha} x_2^{\beta}} = \alpha + \beta$

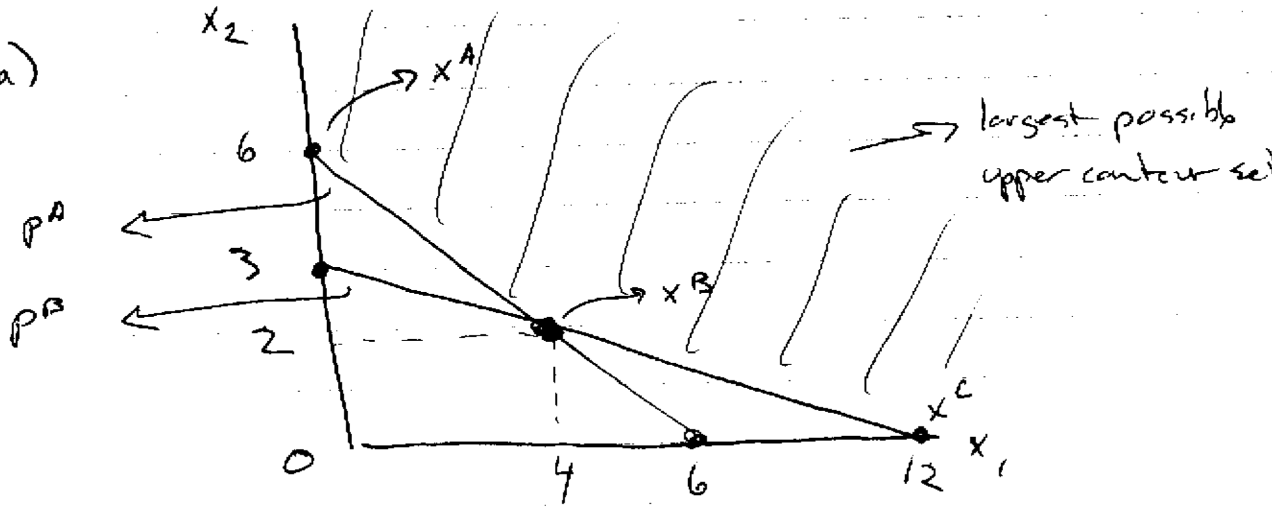
From the result in (a), $e(x^*) = \frac{AC(y^*)}{MC(y^*)} = \alpha + \beta$

because $MC < AC$ $\Rightarrow$ AC falling and $MC > MC$ $\Rightarrow$ AC rising

But since $\alpha + \beta$ is a constant,
either AC is always above MC (IRS), always equal to it (CRS) or always below (DRS)
None of these cases is compatible with a U-shaped LRAC:



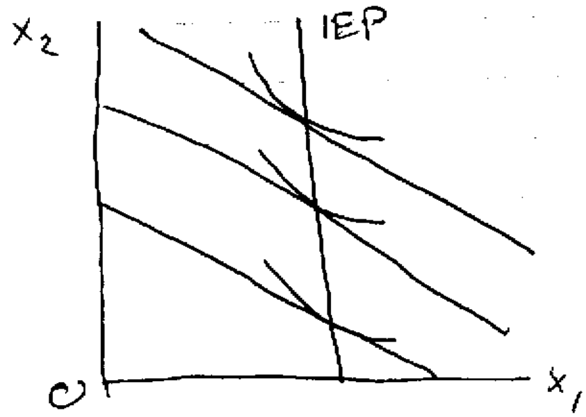
2. (a)

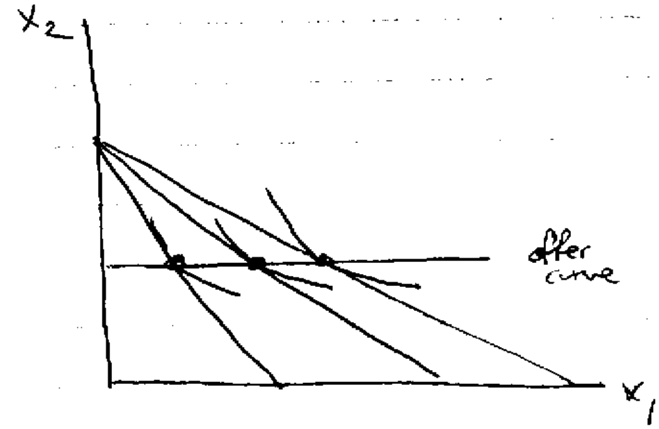


Nothing <u>below</u> the p^A budget line can be in the upper contour set $\{x \geq 0 : u(x) \geq u(x^A)\}$ due to non-satiation. If there were such a point x with $u(x) \geq u(x^A)$ then non-satiation says there is a feasible x' with $u(x') > u(x^A)$ which contradicts the optimality of x^A. For the same reason, nothing below the p^B budget line can be at least as good as x^B. But x^A is revealed to be at least as good as x^B (x^A was chosen when both were feasible). So x^A is better than everything below the p^B budget line. The <u>largest</u> possible upper contour set consists of both budget lines and everything to the northeast (Joe could be indifferent ~~among~~ all points between x^A and x^B, and also between x^B and x^C)

(b) You can't say anything about the sign or size of c because we can always add or subtract a constant from $u(x)$ without affecting behavior. You also can't say anything about the absolute size of a and b because we can multiply by a positive constant. However, we know the indifference curves are linear, and we know Joe chooses an interior point x^B at prices p^B. Thus the indiff curves must have the same slope as the budget line for p^B, which is $-\frac{1}{4}$. Therefore $-\frac{a}{b} = -\frac{1}{4}$ or $\boxed{b = 4a}$. Also Joe chose to be on the budget line rather than below it, so $a > 0$ and $b > 0$. This is enough information to determine the Marshallian demands: Joe spends all his income on x_1 when the budget line is flatter than $-\frac{1}{4}$, all income on x_2 when the budget line is steeper than $-\frac{1}{4}$, and he is indifferent when the slopes are equal, ie $\frac{p_1}{p_2} = \frac{a}{b}$.

3. (a) <u>Income expansion path</u>





x_1 doesn't depend on m, so as we shift the budget line out in a parallel way, x_1 stays constant $\Rightarrow$ <u>vertical</u> IEP.

x_2 doesn't depend on p_1, so as we vary this price, x_2 remains constant $\Rightarrow$ <u>horizontal</u> offer curve.

(b) Use the Slutsky equation: $\dfrac{\partial x_i}{\partial p_j} = \dfrac{\partial h_i}{\partial p_j} - \dfrac{\partial x_i}{\partial m} x_j$

$$\Rightarrow \frac{\partial h_i}{\partial p_j} = \frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial m} x_j$$

(i) set $i = 2,\ j = 1 \Rightarrow \dfrac{\partial h_2}{\partial p_1} = 0 + \left(\dfrac{1}{p_2}\right)\left(\dfrac{p_2}{p_1}\right) = \dfrac{1}{p_1}$

(ii) set $i = 1,\ j = 2 \Rightarrow \dfrac{\partial h_1}{\partial p_2} = \dfrac{1}{p_1} + 0\left(\dfrac{m}{p_2} - 1\right) = \dfrac{1}{p_1}$

as required.

4. (a) solve $u(x) = \min_{p} v(p,1)$ s.t. $px = 1$

$$L = \ln\left(\tfrac{1}{2}\right) - \tfrac{1}{2}(\ln p_1 + \ln p_2) - \lambda(p_1 x_1 + p_2 x_2 - 1)$$

$$\left.\begin{aligned} \frac{\partial L}{\partial p_1} &= -\frac{1}{2p_1} - \lambda x_1 = 0 \\ \frac{\partial L}{\partial p_2} &= -\frac{1}{2p_2} - \lambda x_2 = 0 \end{aligned}\right\} \Rightarrow \frac{p_2}{p_1} = \frac{x_1}{x_2} \Rightarrow p_2 = \frac{p_1 x_1}{x_2}$$

Use the constraint: $p_1 x_1 + x_2\left(\frac{p_1 x_1}{x_2}\right) = 1 \Rightarrow 2p_1 x_1 = 1 \Rightarrow \boxed{\begin{aligned} p_1 &= \tfrac{1}{2x_1} \\ p_2 &= \tfrac{1}{2x_2} \end{aligned}}$

So $u(x) = \ln\left(\frac{1}{2}\right) - \frac{1}{2}\ln\left(\frac{1}{2x_1}\right) - \frac{1}{2}\ln\left(\frac{1}{2x_2}\right)$

This simplifies to: $\ln\left(\frac{1}{2}\right) - \frac{1}{2}\ln\left(\frac{1}{2}\right) - \frac{1}{2}\ln\left(\frac{1}{2}\right) - \frac{\ln\left(\frac{1}{x_1}\right)}{2} - \frac{\ln\left(\frac{1}{x_2}\right)}{2}$

or $u(x) = \frac{1}{2}\left[\ln x_1 + \ln x_2\right] = \ln\left[x_1^{1/2} x_2^{1/2}\right]$ which is the log version of a Cobb Douglas utility function.

(b) Use Roy's identity to get person i's demand:

$$x_i^k(p, m_i) = -\frac{\frac{\partial v(p,m_i)}{\partial p_k}}{\frac{\partial v(p,m_i)}{\partial m_i}} = -\frac{\left[-\frac{1}{2p_k}\right]}{\frac{2}{m_i}\cdot\frac{1}{2}} = \frac{m_i}{2p_k}$$

So $X^k(p, m_1 \dots m_n) = \sum_{i=1}^{n} x_i^k(p, m_i) = \frac{\sum_{i=1}^{n} m_i}{2p_k} = \frac{M}{2p_k} \quad k = 1, 2$

depends only on total income $M = \sum m_i$, not how it is distributed. This is not surprising because the indirect utility function for each person has the Gorman form $v(p,m_i) = a_i(p) + b(p) m_i$ if we carry out a suitable positive monotonic transformation. Rewrite ~~above~~ $v(p,m_i) = \ln\left[\frac{m_i}{2p_1^{1/2} p_2^{1/2}}\right]$ and transform this using

$w(p,m_i) = e^{v(p,m_i)} = \frac{m_i}{2p_1^{1/2} p_2^{1/2}}$. This is in the Gorman form where $a_i(p) \equiv 0$ for all i and $b(p) = \frac{1}{2p_1^{1/2} p_2^{1/2}}$

(*) [Note: The last part is hard, don't deduct any points if they don't get it.]