A coupled sheet-conduit mechanism for jökulhlaup propagation

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[1] The largest glacier outburst flood (jökulhlaup) ever recorded in Iceland occurred in 1996 and came from subglacial lake Grímsvötn in Vatnajökull ice cap. Among other noteworthy features, this flood was characterized by an unprecedentedly high lake level prior to flood initiation, extremely rapid linear rise in lake discharge, delay between the onset of lake drainage and floodwater arrival at the glacier terminus, formation of short-lived supraglacial fountains, and initially unchannelized outbursts of floodwater at the terminus. Observations suggest that the 1996 flood propagation mechanism was fundamentally different than that of previously observed floods from Grímsvötn. We advance a new model whereby floodwater initially propagates in a turbulent subglacial sheet, which feeds a nascent system of conduits. This model is able to explain key observations made of the 1996 jökulhlaup and may shed light on other outburst floods that do not conform to the standard model. *INDEX TERMS:* 1827 Hydrology: Glaciology (1863); 1821 Hydrology: Floods; 1863 Hydrology: Snow and ice (1827); 9315 Information Related to Geographic Region: Arctic region; 9335 Information Related to Geographic Region: Europe. **Citation:** Flowers, G. E., H. Björnsson, F. Pálsson, and G. K. C. Clarke (2004), A coupled sheet-conduit mechanism for jökulhlaup propagation, *Geophys. Res. Lett.*, *31*, L05401, doi:10.1029/2003GL019088.

1. Introduction

[2] In classical jökulhlaup theory [*Nye*, 1976; *Spring and Hutter*, 1981, 1982; *Clarke*, 1982], lake drainage is controlled by the enlargement of a single ice conduit. Discharge hydrographs predicted by this theory increase exponentially over days to weeks and are identical through any cross-section of the conduit. This theory has satisfactorily explained jökulhlaups from various locations, including Grímsvötn [e.g., *Clarke*, 1982, 2003; *Björnsson*, 1992]. Several lines of evidence suggest that the 1996 jökulhlaup was controlled by fundamentally different processes [*Björnsson*, 1997, 2002; *Jóhannesson*, 2002], most notably the evidence for ice-dam flotation, the rapid rise in discharge from the lake, the delay in floodwater arrival at the terminus, and the numerous signatures of very high basal water pressure. Measured hydrographs and direct observation of floods from other geothermal areas under Vatnajökull and from other ice caps in Iceland provide numerous examples of floods similarly characterized by rapidly rising discharge [see *Björnsson*, 1992; *Jóhannesson*, 2002], yet floods of this type have never been successfully modeled. Based on observations of the 1996 Grímsvötn jökulhlaup, we present a coupled sheet-conduit model of flood propagation that qualitatively explains the unusual features of this event. Below we summarize the salient observations of the 1996 flood, outline our modeling strategy, and present simulation results that illustrate the explanatory power of this approach.

2. Observations and Interpretation

[3] The 1996 jökulhlaup followed the Gjálp subglacial eruption, which occurred within the Grímsvötn subglacial catchment basin. Meltwater produced by the eruption accumulated in lake Grímsvötn for one month, until the lake had risen to a level of 1510 m and began to drain. This is the highest lake level ever recorded for Grímsvötn and corresponds to the calculated value required to lift the ice dam by flotation. Around 0700 h on 5 November, or ~10.5 h after the onset of lake drainage, water emerged from the glacier Skeidarárjökull 50 km downstream (Figure 1). Discharge from Grímsvötn peaked at $\sim 4 \times 10^4$ m^3 s^{-1} after 16 hours, according to the lake discharge hydrograph derived from lake-level measurements and the known lake hypsometry. Peak discharge from the glacier terminus was not measured, and its timing is only constrained as having occurred during darkness, or between approximately 1800 h on 5 November and 0800 h on 6 November.

[4] Floodwater first emerged in the river Skeidará at the eastern edge of the glacier terminus (Figure 1), scattering icebergs over the Skeidarársandur outwash plain. Less than 30 minutes later, artesian supraglacial fountains were observed several kilometers upstream from the glacier terminus where the ice is ~350 m thick. These fountains persisted for one or two hours and excavated large blocks of ice [*Roberts et al.*, 2000]. Supraglacial and ice-marginal outbursts of water were observed progressively further west until, around 1600 h, they could be seen across the entire 23 km long glacier margin [*Snorasson et al.*, 1997; *Roberts et al.*, 2000]. An estimated total 3.2 km^3 of water was released during the flood and newly formed crevasses were observed in the ice surface along the length of the floodpath. A depression in the ice surface measuring roughly 6 km long, 1 km wide, and up to 100 m deep was formed by ice collapse, just downstream from Grímsvötn in the vicinity of the ice dam. The energy required to melt this volume

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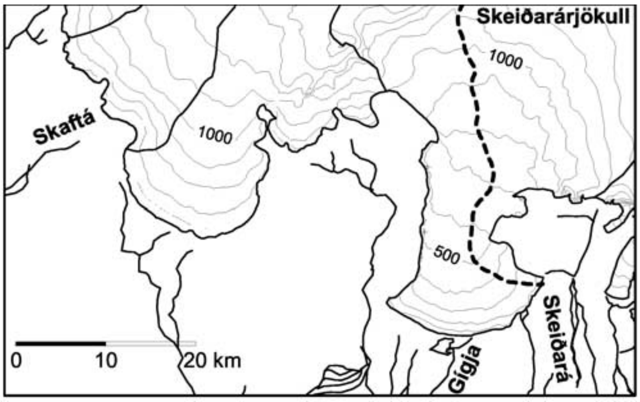

Figure 1. South western Vatnajökull with locations Skeidarárjökull outlet glacier, initial subglacial floodpath (dashed line), and the rivers Skeidará and Gígja.

of ice was sufficient to reduce the water temperature to the pressure melting point [*Björnsson*, 1997, 2002].

[5] Features of the 1996 jökulhlaup relevant to this study can be summarized as follows: (1) the lake reached an unprecedented level, sufficient to float the ice dam, before it began to drain, (2) lake discharge rose rapidly and linearly to its peak of $\sim 4 \times 10^4$ m^3 s^{-1} over 16 hours, (3) floodwater emerged at the terminus of Skeidarárjökull about 10.5 hours after the lake began to drain, (4) supraglacial fountains erupted in areas near the terminus where the ice is several hundred meters thick, (5) the initial outburst was unchannelized and propagated across the glacier margin, and (6) peak discharge from the glacier terminus occurred some time after peak discharge from the lake. We build on the conceptual models of floodwave propagation put forward by *Björnsson* [1997] and *Jóhannesson* [2002] and employ the qualitative and quantitative aspects of the above observations to derive and constrain the model presented below.

3. Modeling Approach

[6] We employ a one-dimensional flowline model that accounts for water transport in a sheet-like subglacial layer and in ice-walled conduits. This formulation allows a water sheet and conduits to coexist and nourish each other. Transport in the sheet is governed by reduced Navier-Stokes equations and conduit physics is patterned after the work of *Spring and Hutter* [1981, 1982], recently revisited by *Clarke* [2003]. Because the thermodynamic formulation in existing jökulhlaup theory is unable to account for the rapid heat transfer documented during the 1996 jökulhlaup and because there is evidence that floodwater temperature in this event was reduced to the pressure melting point within the first 10% of the flowpath [*Björnsson*, 1997, 2002; *Jóhannesson*, 2002], we exclude water temperature as an independent state variable. Rather, we assume that heat transfer at the ice-water interface occurs instantaneously, and hence that the floodwater is maintained at the pressure melting point. We address the implications of this choice later, along with results of modeling water temperature explicitly.

[7] For an incompressible fluid, mass conservation leads to a balance equation for the subglacial water sheet that can be written in local form as

$$\frac{\partial h^s}{\partial t} = -\nabla \cdot Q^s - \phi^{s:c}, \qquad (1)$$

where the dependent variable h^s is the water sheet thickness, Q^s is water flux, and $\phi^{s:c}$ represents water leakage into conduits. In accord with previous modeling work on Vatnajökull hydrology [*Flowers et al.*, 2003], we treat this "water sheet" as a macroporous subglacial layer. Adopting the formulation derived by *Stone and Clarke* [1993] for water flux in a macroporous horizon that accommodates turbulent flow at the ice contact, we write

$$Q^s = -\frac{2 K_s h^s \nabla\psi^s}{\rho_w g}\left(1 + (1 + C\,|\nabla\psi^s|)^{1/2}\right)^{-1}, \qquad (2)$$

with hydraulic conductivity K_s, water density $\rho_w = 1000$ kg m^{-3}, $g = 9.81$ m s^{-2}, and fluid potential $\psi^s = p^s + \rho_w\, g\, z_L$, where z_L is the floodpath elevation and p^s is water pressure. Parameter $C = (2880\ K_s^3\ (1 - m)^2/\mathrm{Re}^2\ \mu m^3\ \rho_w\ g^3)^{1/2}$ with porosity m, Reynold's number $\mathrm{Re} = Q^s\ \rho_w/\mu$, and viscosity $\mu = 1.787 \times 10^{-3}$ Pa s is adapted from *Stone and Clarke* [1993]. Basal water pressure p^s can be written as an empirical function of h^s [*Flowers and Clarke*, 2002], $p^s(h^s) = \rho_i\ g\ h_i\ (h^s/h_*)^{7/2}$, with ice density $\rho_i = 910$ kg m^{-3}, ice thickness h_i, and critical water sheet thickness h_*. We parameterize h_* as a function of p^s to account for the effects of hydraulic jacking [*Björnsson*, 1997, 2002; *Jóhannesson*, 2002].

[8] We write the exchange of water between the sheet and a system of conduits as a function of the difference between sheet and conduit water pressures $p^s - p^c$, using the characteristic conduit spacing d_c to provide an appropriate length scale:

$$\phi^{s:c} = \chi^{s:c}\,\frac{K_s\, h^{s:c}}{\rho_w\, g\, d_c^2}\,(p^s - p^c), \qquad (3)$$

where $\chi^{s:c} \in 0, 1$ controls the coupling between sheet and conduit systems. Because the system pressure is multivalued at the sheet-conduit interface where exchange occurs, we define a pressure, $p^{s:c} = (p^s + p^c)/2$, with which $h^{s:c}$ is calculated by inverting $p^s(h^s)$. Note that while water pressures in the sheet and conduit systems are tracked separately, we have assumed that the conduit system can be uniformly characterized by water pressure $p^c(x, t)$.

[9] Evolution of conduit cross-sectional area depends on the respective rates of (1) melting due to viscous dissipation of heat through turbulent water flow and (2) conduit closure by ice deformation. In accordance with our thermodynamic assumptions, we do not include lake sensible heat in the conduit wall melting term. From mass conservation, we write

$$\frac{\partial S}{\partial t} = -\frac{Q^c}{\rho_i L}(\nabla\psi^c - D\nabla p^c) - 2\,S\left(\frac{p_i - p^c}{nB}\right)^n, \qquad (4)$$

for $p_i \geq p^c$, with conduit cross-sectional area S, conduit discharge Q^c, latent heat of fusion $L = 3.34 \times 10^5$ J kg^{-1}, fluid potential $\psi^c = p^c + \rho_w\, g\, z_L$, Glen ice rheology flow-law exponent $n = 3$ and rate factor $B = 5.8 \times 10^7$ N s$^{1/n}$m^{-2}, and $D = c_t\, \rho_w\, c_w$ where $c_t = 7.5 \times 10^{-8}$ K m^3 J^{-1} is the change in melting point temperature with pressure and $c_w = 4.22 \times 10^3$ J kg^{-1} K^{-1} is the heat capacity of water. Water mass balance for a system of conduits can be expressed as

$$\frac{\partial p^c}{\partial t} = -\frac{1}{\beta S}\left(\frac{\partial S}{\partial t} + \nabla \cdot Q^c + \frac{Q^c}{\rho_w L}(\nabla\psi^c - D\nabla p^c) - d_c\, \phi^{s:c}\right), \tag{5}$$

where $\phi^{s:c}$ is given by equation (3) and $\beta = 10^{-9}$ Pa^{-1} is a numerical compressibility parameter [*Clarke*, 2003]. Conduit discharge is a function of fluid potential gradient, and conduit size, shape, and wall roughness [*Röthlisberger*, 1972]. A generalized expression for conduit discharge is

$$Q^c = -\left(\frac{8\,S^3}{P_w\,\rho_w f_R}\right)^{1/2} \frac{\nabla\psi^c}{|\nabla\psi^c|^{1/2}}, \tag{6}$$

with conduit wetted perimeter P_w and Darcy-Weisbach roughness $f_R = 8\ g\, n'^2/R_H^{1/3}$ expressed as a function of Manning roughness n' and hydraulic radius R_H. P_w and R_H are functions of conduit shape (typically circular or semi-circular) and n' may be a composite of values for ice and bed if the conduit perimeter comprises both. Note that while equations (4) and (6) describe physics applied to a single conduit, equations (3) and (5) refer to a system of conduits via the characteristic conduit spacing d_c. In this way, conduit physics is represented individually, while the water balance is computed collectively. This is a convenience that requires assuming identical behavior between conduits.

[10] We integrate equations (1), (4), and (5) numerically to obtain water sheet thickness h^s, conduit area S, and conduit water pressure p^c as a function of position along the floodpath and time. We employ a staggered grid and scale the numerical equations to improve the matrix condition number. Ice thickness and bed elevation are known along the floodpath, and we take the elevation of the drainage system z_L as that of the bed. Spatial derivatives are defined with respect to the flowline coordinate, in order to account for changes in path length due to variable floodpath elevation. We assume semi-circular bed-floored conduits such that $P_w = (\pi + 2)R$ and $R_H = \pi\, R/(2(\pi + 2))$ as a function of conduit radius R, and we set $n' = 0.032$ m$^{-1/3}$ s based on simulations of other Grímsvötn floods [*Clarke*, 2003]. Assumption of semi-circular versus circular conduits makes little difference to the results. Initial conditions are as follows: $h^s(x, 0) = h_*$, where $h_* = 1$ m [*Flowers et al.*, 2003] represents the areally averaged water content of a saturated macroporous layer ~3 m thick with bulk porosity $m = 0.35$; $S(x, 0) = S_\varepsilon = 0.25$ m^2 is the vestigial conduit area; $p^c(x, 0) = \rho_i\, g\, h_i(x)$. At the glacier terminus, p^s and p^c are fixed equal to atmospheric pressure. At the upstream intake, lake discharge Q_L is prescribed as a boundary condition on the subglacial water sheet as $Q^s(x_L, t) = Q_L/w_s$ where w_s is the characteristic width of the sheet. This boundary condition differs from that of previous models, where the draining lake is pressure-coupled to the water system. Our choice here is necessitated by model limitations, in that flood triggering cannot be properly represented without a detailed understanding of hydraulic uplift in the geometrically complex region of the ice dam.

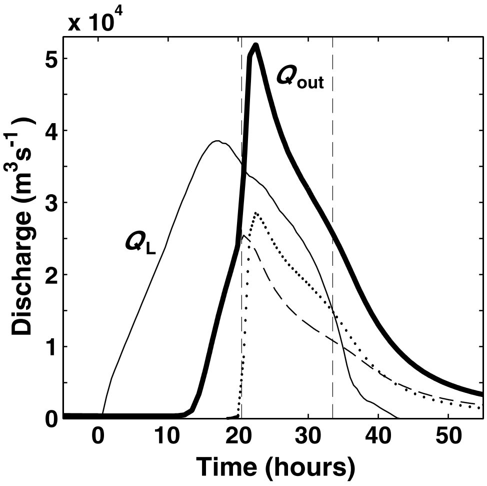

Figure 2. Discharge from Grímsvötn Q_L computed from lake-level measurements and known lake hypsometry, shown with simulated outlet discharge Q_{out}. Q_{out} is the sum of sheet ($w_s Q^s$, dashed line) and conduit ($w_s Q^c/d_c$, dotted line) discharges at the glacier terminus. Simulation parameters are as follows: $K_s = 1.5$ m s^{-1}, $\chi^{s:c} = 0.025$, $d_c = 5.5$ km, $w_s = 10$ km. Although not visible on this scale, Q_{out} increases from its background value at ~10.5 h as required. Vertical dashed lines indicate dusk and dawn, between which observed peak outlet discharge occurred.

4. Results and Discussion

[11] In order to discriminate between models, we make use of several observational constraints: floodwater arrival at the glacier terminus ~10.5 hours after lake-drainage onset, delay in channelization of floodwater at the terminus, and peak outlet discharge 20–35 h after lake drainage onset. Figure 2 presents one model realization that satisfies these constraints. As compared to classical jökulhlaup theory where conduits enlarge over days to weeks in response to water injection from the lake, conduits in the sheet-coupled system are able to develop rapidly because the source water is distributed along the length of the glacier by the initial sheet flood. This allows conduit development near the glacier margin, which provides a positive feedback on growth by reducing back-pressure in the system. Common to many simulations that satisfied the evaluation criteria was an early conduit development stage where the sheet feeds the conduit system near the lake, while the conduits discharge water back into the sheet where they become overpressurized downstream. In this way, conduit growth is facilitated by the sheet playing differential roles of source and sink along the floodpath.

[12] The suite of model simulations that satisfies the observational constraints suggests possible bounds on flood characteristics that were not measured. Simulated peak outlet discharge was typically in the range of 4–6 $\times$ 10^4 m^3 s^{-1}, thus generally higher than peak lake discharge. Conduit cross-sectional area reached its maximum several kilometers upstream from the glacier terminus in the

simulations. Maximum values fell between 450 and 1000 m^2 depending on the prescribed latent conduit spacing d_c. Simulated subglacial sheet thickness h^s reached a maximum of 7–11 m in the first several kilometers of the flowpath, in rough agreement with *Björnsson*'s [1997] postulation of a 10 m-thick basal water layer. Simulated water pressures in the sheet and conduits exceeded flotation along much of the flowpath but were dramatically reduced upon full development of the conduit system. Successful simulations hinged upon high values of K_s ($\geq$1 m s^{-1}) to produce rapid floodfront propagation, $\chi_{s:c} \in (0.01, 0.04)$ to ensure stable conduit development, and values of the characteristic sheet width w_s = 6–10 km and conduit spacing d_c = 4–8 km. The actual spacing between major hydraulic outlets along the margin of Skeidarárjökull is roughly 4–5 km.

[13] Many things have been left unaccounted for in this simple model. It is a one-dimensional, laterally-integrated representation of two idealized interacting systems, so it does not account for flood progression across the width of the glacier and its attendant geometrical complexity. Neither does it include the physics of ice hydrofracture, which was clearly an important process in this event. This also raises the question as to whether the ice deformation term in equation (4) should be applied for conduit overpressure, or $p^c > p_i$. Numerical experiments along these lines suggest that allowing water-pressure induced ice deformation does not substantially alter the simulated outlet hydrograph, except to somewhat reduce peak discharge. Finally, we have assumed instantaneous ice-water heat transfer such that the system remains at the local pressure melting temperature. In doing do, we have not attempted to reconcile the jökulhlaup thermodynamic theory of *Spring and Hutter* [1981, 1982] and *Clarke* [2003] with the 1996 observations. However, in numerical experiments following *Clarke* [2003], we have treated water temperature as an independent state variable and expressed ice-water heat transfer as a function of the difference between ice and water temperatures. This analysis includes both turbulent and lake sensible heat. Simulations yield outlet hydrographs similar in form and timing to that pictured in Figure 2, with peak outlet discharges up to 15% lower, irrespective of the assumed initial lake temperature. Simulated outlet water temperature is overestimated, peaking at 2–3°C, emphasizing a persistent discrepancy between observations and the predictions of jökulhlaup thermodynamic theory [*Clarke*, 2003]. That this alternative formulation does not strongly affect the simulated outlet hydrograph attests to the controlling nature of sheet dynamics in this particular event.

5. Conclusion

[14] Based on observations of the 1996 Grímsvötn jökulhlaup, we have developed a simple model that provides a plausible mechanism for flood propagation and evolution. This model combines a turbulent subglacial water sheet and a system of conduits. For a modest range of parameters, simulation results are qualitatively and quantitatively consistent with the observations, namely: the simulated arrival of floodwater at the glacier terminus lags the onset of lake drainage by ~10.5 h hours and precedes the simulated arrival of floodwater conveyed in conduits, simulated outlet discharge peaks after the discharge maximum from Grímsvötn, and simulated subglacial water pressures exceed ice flotation in the early flood stages. This modeling suggests that unusually rapid conduit growth was facilitated by the distribution of source water along the length of the floodpath. Limitations are inherent in our rudimentary approach where we have omitted some processes and simplified others, yet this framework provides a hopeful starting point for understanding other floods that have eluded classical jökulhlaup theory.

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