Automated Tolerance Optimization Using Feature-driven, Production Operation-based Cost Models

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Abstract

The work addresses two important issues in computer-aided tolerancing: automated generation of design specific cost-to-design-tolerance models and automation of design optimization in tolerance synthesis. These are accomplished by assembling generic, production operation-based, cost-to-manufacturing-tolerance models, based upon the mechanical features of a design and their minimum-cost manufacturing processes. The task is carried out through multiple level optimizations and the application of a knowledge-based intelligent system to form the optimization problems. A typical tolerance design example, under the concurrent engineering principle, is used to illustrate the introduced method.

Keywords: tolerance synthesis, cost modeling, design optimization, concurrent engineering.

Introduction

Tolerance specification is an important part of mechanical design. Design tolerances strongly influence the functional performance and manufacturing costs of a mechanical product. Tighter tolerances normally produce superior components, better performing mechanical systems, and good assemblability with assured exchangeability at the assembly line. However, unnecessarily tight
tolerances also lead to excessive manufacturing costs for a given application. The balance of performance and cost through the identification of optimal design tolerances is a major concern in modern design.

Traditionally, design tolerances are specified in an ad hoc manner based upon the designer’s experience. Computer aided (or software based) tolerance analysis and synthesis programs allow a designer to verify the relations among all design tolerances to produce a consistent and feasible design. As a result of continuous research effort, the functions of these programs have been considerably extended and some computer-aided tolerancing programs have been integrated into the widely used CAD systems (VSA 1993). However, most of these programs and their supporting research focus on the geometric aspects of design. Manufacturing and cost issues were either not addressed, or were only considered for some special cases. The manufacturing consideration-driven tolerance synthesis function is still beyond the reach of a designer.

The essence of tolerance design is to incorporate manufacturing considerations into design. Since no mechanical components can be manufactured with perfect geometry, design tolerances specify the allowed manufacturing errors and assure the design to function without unacceptable performance or assemblability loss. To produce the designed component within the specified tolerances, one need to select appropriate manufacturing operations and their application sequence to form a feasible manufacturing process. The process gradually alters the stock material into the final shape of the designed component. At each manufacturing operation, a manufacturing tolerance is to be specified to assure that the step-by-step produced component satisfies the design
tolerance. In other words, each design tolerance is warranted by a number of manufacturing tolerances imposed on the manufacturing operations used to produce the component.

Mechanical design covers a very broad variety of structures and geometry. Each design can be produced by a variety of manufacturing processes. Their combination leads to an even larger variety, which makes the generalization of manufacturing-consideration-driven tolerance design and optimization a real challenge. However, without this generalization computer-aided tolerancing would not become a general-purpose design tool, and research on tolerance synthesis would be largely limited to an academic pursue.

In this work, a generic method to carry out optimal tolerance design (or tolerance synthesis) is introduced. The work integrates the authors’ earlier work on several related issues, including

- Modeling of cost-to-manufacturing-tolerance relations from empirical production data (Dong et al 1994);
- Identification of the minimum cost manufacturing process for a given design feature (Dong and Hu 1991, Dong 1994); and
- Optimal tolerance design (or tolerance synthesis) for minimum manufacturing costs (Dong and Soom 1990, Dong 1997) or concurrent engineering design (Xue et al 1996, Dong and Wang 1997).

The work presents a systematic approach to automated tolerance synthesis. It allows design tolerances of a common mechanical design to be optimized with automatically constructed objective functions, and provides a means to form the cost-to-design-tolerance model automatically
from the design independent and manufacturing operation-based cost-to-manufacturing-tolerance models. The method and its advantages are demonstrated using a real design example.

**General Background**

In design, various mechanical features, their nominal dimensions and design tolerances, specify mechanical components and their assembly relations. The design tolerances consist of size tolerances and geometric tolerances. Some of these tolerances are often related to accomplish certain functional and assembly relations. These relations are design constraints imposed on a group of tolerances, allowing some tolerances to be looser and others to be tighter depending upon the sensitivity of their values to the design performance and manufacturing costs. Traditionally, these related tolerances are called a “dimension chain.” Recent research regards them as “related tolerances” to include geometric tolerances. The objective of tolerance synthesis is to find the best combination of these related tolerances, satisfying certain design goals, such as minimum manufacturing costs, best functional performance, or a balance of both.

The tolerance synthesis or design tolerance optimization problem can be illustrated using a simple linear dimension chain of \( n \) design dimensions, \( X_1, X_2, \ldots, X_n \), and their resultant dimension \( X_R \), as shown in Figure 1 (a). The size tolerances associated with these design dimensions form a related tolerance group, \( \delta_1, \delta_2, \ldots, \delta_n \). Through error accumulation their values determine the resultant tolerance, \( \delta_R \), or the actual size of the “gap”, \( X_R \). These related tolerances have to satisfy the consistency requirement:

- For the worse-case approach

\[
\delta_1 + \delta_2 + \cdots + \delta_n = \sum_{i=1}^{n} \delta_i = \delta_R
\]
• For the normal distribution-based statistical approach

$$\sqrt{\delta_1^2 + \delta_2^2 + \cdots + \delta_n^2} = \sqrt{\sum_{i=1}^{n} \delta_i^2} = \delta_R$$

(2)

Each design tolerance is also constrained by a valid range with an upper and lower bounds determined by manufacturing processes.

$$\delta_{\min,i} < \delta_i < \delta_{\max,i} \quad (i = 1, \cdots, n)$$

(3)

To accommodate design intent, we assume that the values of several design tolerances, $\delta_{p+1}, \cdots, \delta_n$, in this related group are pre-determined for certain functional requirements.

If the design objective is to achieve minimum manufacturing costs, the tolerance synthesis problem imposed on these related tolerances can be formulated as the following optimization problem (Dong and Soom 1990):

$$\min_{w.r.t. \delta_j} C(\delta_1, \cdots, \delta_p) = \sum_{i=1}^{p} C_i^0 (\delta_j)$$

subject to

$$\delta_1 + \delta_2 + \cdots + \delta_n = \delta_R$$

(5)

$$\delta_{\min,i} < \delta_i < \delta_{\max,i} < \delta_{i,0} \quad (i = 1, \cdots, p, \ p \leq n)$$

(6)

and

$$F(\vec{d}) \geq F_0 \quad (\vec{d} = [\delta_1, \cdots, \delta_p]^T)$$

(7)
where, $\delta_1, \cdots, \delta_n$ are $n$ component tolerances of a dimension chain, with $\delta_{p+1}, \cdots, \delta_n$ pre-specified to meet certain design requirements. $C(\delta_1, \cdots, \delta_p)$ represents the manufacturing cost for producing the mechanical features associated with the adjustable design tolerances in the dimension chain. $C^D_{i}(\delta_i)$ represents the manufacturing cost that is related to each component design tolerance, $\delta_i$, and is calculated using the cost-to-design-tolerance model, as shown in Figure 1 (b). The stack-up consistency constraint of Eq. (5) is based on stack-up under a worst-case consideration, and a statistical approach could be introduced with minor modifications to this equation. Eq. (6) specifies the lower and upper bounds of each tolerance, and $\delta_{i,0}$ is the blank part tolerance. Eq. (7) represents one or more design functional performance constraints and may not appear explicitly.

In previous research, tolerance synthesis is based upon a number of given cost-to-design-tolerance models. However, these models are very difficult to obtain. First, the model is design-dependent. Each feature-tolerance combination would have a different model. Secondly, in manufacturing, each mechanical feature is produced through a sequence of production or machining operations, called a manufacturing process. Different features with different tolerances require different manufacturing processes. The cost-to-design-tolerance model is a reflection of the cost-to-accuracy relation of all related production operations. At the design stage, without a prior knowledge of the manufacturing process of the part, it is infeasible for a designer to form an
accurate cost-to-accuracy relation model determined by the downstream production operations. The unavailability of the cost-to-design-tolerance models is a severe obstacle to the practical application of tolerance synthesis.

**Related Work**

Earlier research on tolerance synthesis focused on the formulations of a tolerance assignment as an unconstrained optimization problem and their close-form solutions (Speckhart 1972, Spotts 1973, Sutherland and Roth 1975). Based upon the general characteristics of a manufacturing cost-tolerance data curve, several general cost-tolerance relation models, including the exponential, reciprocal squared and the reciprocal powers models, were introduced. The approach suffers from relatively large model fitting errors due to the simple forms of the mathematical models (Wu et al. 1988, Dong et al. 1994). In addition, it fails to consider the valid range of a cost-tolerance curve to avoid infeasible solutions, and requires manual formulation.

Following these earlier efforts, many researchers have significantly improved the method for tolerance specification over the last decade (Roy et al. 1991, Zhang and Huo 1992, Kumor and Raman 1992). Most of the progress was made on the modeling of cost-tolerance relations and the formulation of the optimization problem. Michael and Siddall solved optimal design problems with both design parameters and their tolerances as design variables, and introduced the powers and exponential hybrid model (Michael and Siddall 1982). Parkinson further investigated the optimal design of mechanical tolerances in statistical tolerance assignment (Parkinson 1985). Chase and Greenwood introduced the reciprocal model with better empirical data fitting capability (Chase et al. 1990). Lee and Woo presented a discrete cost-tolerance model and associated tolerance optimization method, using reliability index and integer programming to eliminate
modeling errors (Lee and Woo 1989). Zhang and Wang introduced simulated annealing to discrete
tolerance optimization as a better solution method (Zhang and Wang 1993). Cagan and Kurfess
studied tolerance optimization over multiple manufacturing considerations (Cagan and Kurfess
1992). Turner and Wozny focused on the automated tolerance analysis in a solid modeling system,
and developed a method for representing and analyzing tolerance, using model variations in a
normalized vector space (Turner and Wozny 1990). Wu et al. studied various existing continuous
cost-tolerance models and compared their modeling errors based on a general empirical cost-
tolerance curve (Wu et al. 1988). Dong and Soom have extended the tolerance optimization
formulation to include multiple dimension chains sharing common design tolerances, and
incorporated the valid tolerance range into the formulation (Dong and Soom 1990, 1991). Dong et
al. carried out an in-depth study on the empirical cost-tolerance data from typical production
processes, and introduced several new cost-tolerance models and a hybrid-model tolerance
optimization formulation (Dong et al. 1994). These introduced models and formulation better
represent empirical production data, and provide more reliable results for tolerance synthesis.
Lately, a method that combines a nontraditional optimization method and the Monte Carlo based
tolerance analysis was introduced with improved results on classical examples (Iannuzzi and
Sandgren 1994).

The optimization-based tolerance synthesis method has also been extended to directly involve
functional performance consideration to accomplish concurrent engineering design in the authors’
recent work. In this balanced performance and cost tolerance synthesis method, the functional
performance measures of a design are embedded in the objective function of the optimization,
rather than treated as the design constraints (Xue et al. 1996). The optimized tolerances lead to the best overall performance of the designed product.

Another emerging research area in tolerance analysis and synthesis is computer automation and interface to CAD systems (Bjorke 1989, Roy et al. 1991, Zhang and Huo 1992, Shah 1991). Dong and Soom first developed a method for automated tolerance analysis and synthesis in conventional CAD environments and automated formulation of tolerance optimization using an intelligent system (Dong and Soom 1986, 1990, 1991). The method was later extended to a feature-based CAD environment (Dong 1992, Xue and Dong 1993) as well as integrated concurrent engineering design (Xue and Dong 1994, Xue et al. 1996). Martino and Gabriele developed a method for analyzing conventional, statistical and some geometric tolerances of a part using solid models and variational geometry (Martino and Gabriele 1989). Software tools for automated tolerance analysis were also made available (VSA 1993).

**Relation of Manufacturing Tolerance and Design Tolerance**

Design tolerances, their functions in mechanical design, and the formulation of tolerance synthesis as an optimization problem have been discussed previously. These design tolerances are ensured through a manufacturing process that consists of several production operations. The selection and arrangement of these production operations as well as the machining parameters used in these operations will all influence the final manufacturing errors of the part, which are constrained by the design tolerances.

In manufacturing, each mechanical feature of a designed part is modified from its raw material form to the designed shape and accuracy through a manufacturing process. The geometric
accuracy and surface finish of a part are continually improved by the selected production operations applied in tandem. Based upon this fact, a method for calculating the tolerance-related manufacturing costs by using a cost-to-design tolerance model obtained by fitting several cost-to-manufacturing-tolerance models of all involved production operations, was developed (Dong and Hu 1991, Dong 1994). The approach opened a new venue to bring manufacturing information into design. However, considerable errors are introduced during this second level curve fitting and the curve fitting itself is a computation intensive task in tolerance optimization.

In this work, a new approach to directly use the design independent, production operation-based, cost-to-manufacturing-tolerance models in tolerance optimization is introduced. Constructed directly from machine shop empirical data, these models serve as basic building blocks for calculating tolerance-related manufacturing costs. The models are only associated with production operations and are independent to any mechanical designs. It is this characteristic of these models that allows the approach to be generic to all designs. A typical cost-to-manufacturing-tolerance model represented as $c^m_k(\delta)$ for the $k$th production operation. The operation can be considered as a part of the $j$th feasible manufacturing process for producing design tolerance $\delta_i$ and mechanical feature $i$, as illustrated in Figure 2. This production operation improves the manufacturing error of a mechanical feature from an initially looser manufacturing tolerance, $\delta_{ij,k-1}$, to a slightly better manufacturing tolerance, $\delta_{ijk}$. A tolerance-related manufacturing cost, $\Delta c^m_k$, is paid to achieve this accuracy improvement. This tolerance related manufacturing cost is calculated using the cost-to-manufacturing-tolerance model, $c^m_k(\delta)$. The model is independent to any design and is solely determined by the production (or machining) operation.
The cost-to-manufacturing-tolerance models are built directly from the empirical data of commonly used machining operations. These data can be collected directly from machine shop (Truck 1976). Since these models only reflect the cost-to-accuracy improvement capability of a specific production operation, they are can be easily modeled with accurate results (Dong, et al. 1994). A model library can also be established to cover all commonly used production operations.

In manufacturing, production operations are used in tandem to improve the accuracy of a mechanical feature step-by-step to satisfy given design tolerance. The collective manufacturing costs of these production operations can thus be calculated using several cost-to-manufacturing-tolerance models associated with these production operations. In terms of manufacturing tolerance, the starting point is the very loose tolerance of the stock and the result must be a tolerance value less or equal to the given design tolerance. The process for changing a mechanical
feature from its raw material state with a considerably larger error to the finished part with the designed tolerance can be modeled as shown in Figure 3.

![Figure 3: Cost-to-process-tolerance Model.](image)

It is assumed that the production process consists of three production operations: rough turning \((k=r)\), semi-finish turning \((k=sf)\), and finish grinding \((k=f)\). The manufacturing costs of this process, which are to be minimized, can be calculated by

\[
C_i^m(\delta_i) = \min_{\delta_{ij}} C_i^m(\delta_{ij0}, \delta_{ij1}, \delta_{ij2}, \delta_{ij3}) = \min \sum_{k=1}^{q_j} \Delta c_i^m = \min \{\Delta c_r^m + \Delta c_{sf}^m + \Delta c_f^m\}
\]

where, \(q_j\) is the number of production operations of the \(j\)th feasible manufacturing process; \(\Delta c_r^m, \Delta c_{sf}^m, \Delta c_f^m\) are the relative manufacturing costs for improving the mechanical feature \(i\) (or tolerance \(\delta_i\)) through a feasible manufacturing process \(j\) of three production operations: rough turning \((r)\), semi-finish turning \((sf)\), and finish grinding \((f)\); and \(\delta_{ij0}\) is the tolerance of the blank part. \(\delta_i = \delta_{ij3}\) indicates that the design tolerance equals the final manufacturing tolerance accomplished by the manufacturing process. \(c_r^m(\delta), c_{sf}^m(\delta), c_f^m(\delta)\) are the cost-to-
manufacturing-tolerance models for the selected rough, semi-finish and finish production operations. A more complex model considering set-up errors is given in (Dong 1994).

Given a designed feature and its expected accuracy, all feasible manufacturing processes for producing the feature and the process cost models can be automatically generated, by assembling the elementary production operations and adding their cost-to-manufacturing-tolerance models. In addition, the minimum-cost manufacturing process and its process tolerances can be obtained by comparing the minimized manufacturing costs of all feasible processes.

**Automated Formation of the Cost-to-design-tolerance Models**

In the previous section, the method for forming a cost-to-design-tolerance model from a number of related cost-to-manufacturing-tolerance models is discussed. The automated formulation of the cost-to-design-tolerance model for a design tolerance, however, is a no trivial task. It involves the identification of the manufacturing process that truly reflects the manufacturing cost needed to accomplish the specified design tolerance. This identification can only be made after a comparison on the minimum manufacturing costs of all feasible manufacturing processes that can be used to produce the mechanical feature to the design tolerance. The minimum manufacturing cost of a feasible manufacturing process can only be calculated through the cost minimization on a number of related cost-to-manufacturing-tolerance models.

The task is accomplished through the qualitative reasoning of a knowledge-based intelligent system and the optimization on manufacturing tolerances. The function of the knowledge-based system that automatically forms all feasible manufacturing processes according to each design feature and tolerance. This is accomplished by first identify the connection between the design tolerance and
the mechanical feature that it specify. The task is more accomplishable with today’s feature-based, computer geometric modeling system. The knowledge associates each common mechanical feature with a large number of potential production operations. Through an AND-OR tree search, the intelligent system can identify all feasible manufacturing processes for the mechanical feature and its design tolerance (Dong 1994). Each of these manufacturing process consists of several production operations in a specific order. Machine availability can be considered during this process.

Based upon the listed production operations of these feasible manufacturing processes, the cost-to-manufacturing-tolerance models associated with each of those production operations are retrieved from the database and put together to form the manufacturing cost models for these manufacturing processes. An example manufacturing process cost model was illustrated previously in Figure 3. The objective functions for minimizing tolerance-related manufacturing costs are then formed as in Eq. (8) for a given design tolerance. This lower-level optimization calculates the minimum manufacturing costs for achieving the design tolerance through all identified feasible manufacturing processes. The manufacturing process that has the least “minimum manufacturing cost” is the process that truly reflects the tolerance-related manufacturing cost. The manufacturing cost model of this process is then used as the cost-to-design-tolerance model for the given design tolerance, i.e.

$$C_i^D(\delta_i) = \min \left\{ C_{ij}^m(\delta_i) \right\}; \quad j = 1, \cdots, s$$

where, the minimized manufacturing cost of the $j$th manufacturing process, $C_{ij}^m(\delta_i)$, is given in Eq. (8). The approach is further illustrated by Figure 4. The figure illustrates how the previously discussed functional modules work together to form a mechanical feature-driven and
manufacturing process-based tolerance synthesis method. It also demonstrates the relations among the top-level optimization that carries out tolerance design, the lower-level optimization that identifies the cost model for the *cost-to-design-tolerance* relation, and the intelligent system that generates all feasible manufacturing processes and their tolerance-related cost models. The method of concurrent engineering design will be discussed in the following section.
Figure 4: Function Modules of Integrated Concurrent Tolerance Design Method.
This method directly uses several “discrete” cost-to-manufacturing-tolerance models of related production operations as the cost-to-design-tolerance model. The approach avoided any large modeling error by forming design tolerance-related mathematical model directly from machine shop production data, as well as the secondary modeling error caused when forming the cost-to-design-tolerance model from several cost-to-manufacturing-tolerance models. The most important advantage of the approach lies on its use of modular, production operation-based cost-tolerance models. This approach makes the mathematical models and tolerance synthesis method independent to specific designs and general applicable to all common mechanical components.

**Concurrent Engineering Design in Tolerance Synthesis**

Balanced functional performance and manufacturing cost design is aimed at identifying the best trade-off between functional performance and manufacturing costs, subject to all functional and cost constraints. To merge the two “performance” measures of distinct nature, the functional performance and manufacturing costs are transformed into a comparable form – functional performance index, \( I^{(F)}(\bar{d}) \), and manufacturing cost index, \( I^{(C)}(\bar{d}) \), respectively. A design with balanced functional performance and manufacturing costs can be accomplished by:

\[
\begin{align*}
\min_{\bar{d}} & -I(\bar{d}) = -\omega_f I^{(F)}(\bar{d}) + \omega_c I^{(C)}(\bar{d}) \\
\omega_f + \omega_c &= 1
\end{align*}
\]

where \( I(\bar{d}) \) is the overall design performance rating; and \( \omega_f, \omega_c \) are application dependent weighting factors. The formulation is a good representation of the concurrent engineering principle, especially when more life-cycle aspects are considered, and it puts more control on the hand of designers. An ideal design can be accomplished by improving the design with more functional performance improvement and less manufacturing cost increase, rather than by using one of the two extremes (Xue et al. 1996).
The overall manufacturing cost increase, or manufacturing cost index, can be calculated by

\[
I^{(C)}(\tilde{d}) = \frac{C(\tilde{d}) - C(\tilde{d}_0)}{C(\tilde{d}_0)}
\]  

where, \( C(\tilde{d}) \) is tolerance-related manufacturing cost and its minimization is carried out using Eq. (4); and \( C(\tilde{d}_0) \) is the manufacturing cost of a reference design. These costs are calculated using the cost-to-design-tolerance models, which are automatically formed through lower level optimizations. A dimensionless cost reading – relative cost increase, or decrease, is used here to make the cost reading comparable to the performance reading.

The functional performance increase in the \( l \)th functional performance aspect, \( I^{(F)}_l(\tilde{d}) \), and the overall functional performance increase of the design are defined and measured by

\[
I^{(F)}_l(\tilde{d}) = \frac{F_i(\tilde{d}) - F_i(\tilde{d}_0)}{|F_i(\tilde{d}_0)|}, \quad l = 1, 2, \ldots, r
\]

\[
I^{(F)}(\tilde{d}) = \frac{\sum_{l=1}^{r} \alpha_l I^{(F)}_l(\tilde{d})}{\sum_{l=1}^{r} \alpha_l}, \quad \alpha_1 + \alpha_2 + \cdots + \alpha_r = 1
\]  

where, \( F_i(\tilde{d}_0) \) is the functional performance of a reference design, \( F_i(\tilde{d}) \) and \( F_i(\tilde{d}_0) \) are calculated using the \( l \)th functional performance aspect model, and \( \alpha_1, \alpha_2, \ldots, \alpha_r \) are coefficients for weighting the inputs from all related functional performance aspects (Dong 1997). The modeling of the functional performance is case dependent. The critical factors that determine the functional performance normally include the design geometry, the accuracy of design geometry (tolerance), the material of the part, and its manufacturing methods. Many related issues were addressed in references (Xue and Dong 1993, 1994, and 1996).
Implementation of Integrated Concurrent Tolerance Design

The function modules and their interactions of this introduced approach are illustrated in Figure 4. In this figure, index $i$ denotes the $i$th design tolerance and mechanical feature; $j$ denotes the $j$th feasible manufacturing process; and the $k$ denotes the $k$th production operation. Thus $c^m_k(\delta_{0k})$ represents the manufacturing cost of a specific production operation $k$. This production operation is one element of the $j$th feasible manufacturing process for producing the $i$th mechanical feature to its design tolerance $\delta_i$. The tolerance synthesis module is illustrated at the top center of the figure.

The optimization of tolerance design takes inputs from: given functional requirements of tolerance design, valid tolerance range constraints, dimension chain consistency constraints (or error stack-up constraints), the implicit cost-to-design-tolerance models of all related tolerances, and other tolerance-related functional performance evaluations. The cost-to-design-tolerance model formation part is illustrated in the lower part of the figure. An iterative process is used to form the correct models and to carry out tolerance optimization.

The approach overcomes a major drawback of the automated tolerance optimization method, introduced in the authors’ previous work (Dong and Wang 1997). In this previous “bottom-up” optimization method, the minimum-cost manufacturing process is identified through the optimization on each assembled cost model of all feasible manufacturing processes. This minimum-cost manufacturing process, its process cost model, and its optimized manufacturing tolerances are then used to form the cost-to-design-tolerance model through curve fitting for the design tolerance at the top level tolerance analysis. Using this “bottom-up” approach, the top-level design tolerances have to be specified to obtain their minimum-cost manufacturing process. The lower level optimization is thus performed with respect to various feasible manufacturing processes.
and the manufacturing tolerances of these processes. The obtained minimum-cost manufacturing process is therefore only corresponding to a fixed point in the design space of the top-level design tolerance optimization. The search of the design tolerance optimization might move away from this point and violate the formed cost model. In addition, formation of the cost-to-design-tolerance model through fitting of several cost-to-manufacturing-tolerance models introduces considerable modeling errors.

In this work, an iterative and top-down optimization method is used. During the $r$th iteration of the top-level design optimization, design tolerances are given certain trial values, $\delta_i^r, \ldots, \delta_p^r$. In tolerance-related manufacturing the cost calculation, these trial values are inputs to the lower level optimization. For each given design tolerance, $\delta_i^r$, the mechanical feature $(i)$ that it associates is first identified. The knowledge-based intelligent system produces a list all production operations that can be applied to produce the mechanical feature to the design tolerance and combines these productions into all feasible manufacturing processes. The cost-to-manufacturing-tolerance models of all involved production operations are then assembled to for the cost model for each feasible manufacturing process. At this stage the problems of the lower level optimizations are formulated. The optimizations are carried out to calculate the minimum manufacturing cost of each feasible manufacturing process. The manufacturing process with the least minimized manufacturing cost, as the process that truly represents the tolerance-related manufacturing cost of the design tolerance, is identified. The cost model of this manufacturing process is then used as the cost-to-design-tolerance model for the design tolerance. The top-level design optimization can then be carried out based upon the cost-to-design-tolerance models of the design tolerances. The calculated design tolerances will replace the initial values of the design tolerances and go through the following
iteration. Due to the nature of a tolerance modeling and design problem, the calculation can converge after a few iterations. The method directly uses the elementary empirical cost-to-manufacturing-tolerance models, and eliminates the need of the “human-made” cost-to-design-tolerance models. Automation of design tolerance optimization is accomplished.

A Design Example

To demonstrate the functionality and advantages of the proposed method, the approach is illustrated using a concurrent tolerance design example, used in the authors’ previous publication (Dong 1997(a)). The example is about the design of a multiple spindle drill head. For ease of illustration, we focus on only two design variables of the drill head, the size tolerance, $\delta_d$, and location tolerance, $\delta_{xy}$, of the hole on the drill head case for the spindle, as illustrated in Figure 5.

![Figure 5: Spindle Hole of a Multiple-spindle Drill Head.](image)

The design objective is to find the optimal values of these two tolerances, which lead to the best life-cycle performance of the drill head. In this example, the lower-level optimizations, which
search for the minimum-cost manufacturing process using stored cost-to-manufacturing-tolerance models, were integrated into the global optimization of the two design tolerances for best product life-cycle performance.

Modeling of Functional Performance

The considered life-cycle performance of the spindle hole is limited to the design and manufacturing aspects. The size tolerance of the spindle hole, $\delta_d$, influences the clearance between the spindle and the journal bearings. Two functional performance measures, power loss variation, $\Delta PL$, and spindle case working temperature variation, $\Delta T$, are related to the clearance change. Both the location tolerance of the spindle hole, $\delta_{xy}$, and the size tolerance of the spindle hole, $\delta_d$, influence the alignment of spindle and other shafts. Misalignments between two shafts with mating gears will change the distribution of the contact stress on the tooth of the gear. Stress concentration will reduced the designed lifetime of the gears. The detailed formulation of the design problem is given in (Dong 1997 (a)). A simplified problem description is given in the following section.

![Figure 6: Hole-shaft Clearance vs. Power Loss and Temperature.](image-url)
The functional performance indices, including power loss variation, temperature variation, and gear life, are modeled based upon the data curves from the mechanical design handbooks (Faires 1965, Shigley 1989). The original curves of: (a) clearance vs. power loss; (b) clearance vs. temperature variation; and (c) gear life vs. maximum stress are illustrated in Figure 6 and Figure 7.

Select the minimum clearance, \( \Delta_{cr} \), as 0.05 mm. The maximum clearance is

\[
\Delta_{max} = \frac{3}{2} \delta_D + \Delta_{cr}
\]

A larger tolerance \( \delta_D \) can introduce a larger variation of clearance, thereby increasing the variations of power loss and temperature, and leading to poor product quality in mass production. The functional performance measures are defined as
Power Loss Variation: \[ \Delta PL(\delta_d) = -|PL(\Delta_{\text{max}}) - PL(\Delta_{\text{min}})| \] (14)

Temperature Variation: \[ \Delta T(\delta_d) = -|T(\Delta_{\text{max}}) - T(\Delta_{\text{min}})| \] (15)

Both the size and location tolerances, \( \delta_d \) and \( \delta_{xy} \), contribute to the alignment of the shafts, and influence the lifetime of the gear. The misalignment of shafts changes the equal distribution of contact stress on the surface of the gear tooth cross the width of gear. Higher contact stresses will be imposed on certain areas of the tooth. This part of the tooth will then experience earlier failure and shorter lifetime than expected. The maximum stress is calculated as

\[ \sigma_{\text{max}} = \sigma_0 + \Delta \sigma \] (16)

where

\[
\sigma_0 = \sqrt{\frac{2W_t}{\pi \cos \phi} \left( \frac{1}{d_{p1} \sin \phi} + \frac{1}{d_{p2} \sin \phi} \right) \left( \frac{1}{E_1} + \frac{1}{E_2} \right) + \frac{2l}{\pi n L} \frac{E_1 E_2}{E_1 + E_2} \left( 0.5 \delta_{xy} + 0.5 \Delta_{\text{max}} \right)}
\]

\[ \Delta \sigma = \frac{2l}{\pi n L} \frac{E_1 E_2}{E_1 + E_2} \left( 0.5 \delta_{xy} + 0.5 \Delta_{\text{max}} \right) \] (17)

\( W_t \): tangential load applied to the gear surface (given as 1200 N);
\( l \): width of the gear (given as 25.4 mm);
\( L \): length of the spindle head (given as 0.762 m);
\( \phi \): pressure angle (given as 20 degree);
\( d_{p1}, d_{p2} \): pitch diameters of the gears (given as 0.1016 m and 0.3175 m, respectively);
\( \nu_1, \nu_2 \): Poisson’s ratio parameters of the two gears (given as 0.292 and 0.211, respectively);
\( E_1, E_2 \): two elasticity module parameters of the two gears (given as 207.0*10^9 and 100.0*10^9 Pa);
\( m \): module of the two gears (given as 0.004 m);
The mapping from the size and location tolerances to the gear lifetime functional performance can be accomplished by:

\[ \text{Gear Lifetime: } N(\delta_x, \delta_y) = N_0(\sigma_{\text{max}}) \]  

(18)

Modeling of Manufacturing Costs

The production costs for machining the spindle hole will vary according to the size and location tolerances specified in the design. High accuracy and small tolerance need more manufacturing effort and require higher costs. The production cost-tolerance relations were obtained from machine shop and experiments (Trucks 1976), and modeled in the authors' earlier work (Dong 1994). The feasible manufacturing processes for producing the spindle hole and the locating the hole position are generated based on the product geometric features and the manufacturing capability of a plant. For instance, the spindle hole can be produced by any of the manufacturing processes listed in Table 1.

Table 1. Feasible Hole Making Processes

<table>
<thead>
<tr>
<th>Manufacturing Process</th>
<th>Production Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>drilling, broaching, fine grinding, and high accuracy boring</td>
</tr>
<tr>
<td>2</td>
<td>drilling, grinding, fine grinding, and high accuracy boring</td>
</tr>
<tr>
<td>3</td>
<td>general boring, high accuracy boring, and special equipment</td>
</tr>
<tr>
<td>4</td>
<td>grinding, semi-finish grinding, and high accuracy boring</td>
</tr>
<tr>
<td>5</td>
<td>grinding, semi-finish grinding, fine grinding, and special equipment</td>
</tr>
<tr>
<td>6</td>
<td>…</td>
</tr>
</tbody>
</table>
For each manufacturing process, the manufacturing cost can be calculated by Eq. (7). For the global life-cycle tolerance design, specifically for this example, the hierarchical concurrent optimization problem is formulated as below:

\[
\begin{align*}
\min_{\delta_D, \delta_{sy}} & - I(\delta_D, \delta_{sy}) = w_1 I^{(C)}(\delta_D, \delta_{sy}) - w_2 I^{(F)}(\delta_D, \delta_{sy}) \\
I^{(C)}(\delta_D, \delta_{sy}) &= \frac{C(\delta_D) - C(\delta_{0})}{C(d_0)} \\
I^{(F)}(\delta_D, \delta_{sy}) &= \frac{1}{3}[I^{(F)}_{\delta D}(\delta_D) + I^{(F)}_w(\delta_D) + I^{(F)}_{N}(\delta_D, \delta_{sy})] \\
I^{(F)}_{\delta D}(\delta_D) &= \frac{\Delta P L(\delta_D) - \Delta P L_0}{|\Delta P L_0|}; \quad I^{(F)}(\delta_D) = \frac{\Delta T(\delta_D) - \Delta T_0}{|\Delta T_0|}; \quad I^{(F)}_{N}(\delta_D, \delta_{sy}) = \frac{N(\delta_D, \delta_{sy}) - N_0}{|N_0|} \\
C(d) &= 4[C^{D}(\delta_D) + C^{D}(\delta_{sy})]; \\
C^{D}(\delta_D) &= \min_j \left\{ \min_{\delta, \delta_0} \{ [c_r(\delta_{D,j}) - c_r(\delta_{D,0})] + [c_f(\delta_{D,j}) - c_f(\delta_{D,0})] \} \right\} \\
C^{D}(\delta_{sy}) &= \min_j \left\{ \min_{\delta, \delta_0} \{ [c_r(\delta_{sy,j}) - c_r(\delta_{sy,0})] + [c_f(\delta_{sy,j}) - c_f(\delta_{sy,0})] \} \right\} \\
\text{subject to} & \Delta P L \leq 4 \text{ kw} \\
& \Delta T \leq 13^\circ \\
& N \geq 5 \times 10^4 \\
& \delta_D, \delta_{sy} \in [0.02 \text{ mm}, 0.5 \text{ mm}] \\
\end{align*}
\]

where, \(w_1, w_2\), are selected as \(1/3\) and \(2/3\), respectively. In this example, the design with the minimum manufacturing costs is selected as the reference design, (to obtain the reference point, only needs to the change the top level optimization objective to the cost function.)

\[
\delta_{0} = (\delta_{D,0}, \delta_{sy,0})^T = (0.1272 \text{ mm}, 0.0973 \text{ mm})^T. \tag{21}
\]

*Cost-to-process- tolerance* models vary from plant to plant, machine to machine. In our research,
these models are generated based on the known models (Dong 1994). In Figure 8, hole machining cost-to-processes-tolerance curves are based on the Turning on Lathe curve; and the positioning curves are based on the Hole Position curve (Dong 1994). In each figure, three curves represent the manufacturing process: rough machining, semi-finish machining and finish machining. For different processes, for instance, processes in Table 1, those curves will be different. Then the design optimization result will thus be different. By comparing the design optimums based on various operation sequences, one can identify the best manufacturing process for each feature, which leads to the optimum life-cycle performance of the design. The corresponding operational tolerances are also obtained through the hierarchical optimization. In our example, due to the unavailability of practical cost-to-manufacturing-tolerance curves, the above process curves in Figure 8 are generated based on available resources in reference (Dong 1994) to demonstrate the method per se. The elementary cost-to-manufacturing-curves, however, are rather easy to obtain in each individual manufacturing unit.
By executing Eq. (17), the design optimum is at \( \mathbf{d} = (\delta_D, \delta_{xy})^T = (0.1046\,mm, 0.0769\,mm)^T \). For the spindle hole size, the rough machining tolerance is 0.381 mm; the semi-finish machining tolerance is 0.163 mm. For positioning, the rough positioning tolerance is 0.3204 mm; the semi-finish positioning tolerance is 0.1905 mm. The contour map plot of the product life-cycle performance optimum is illustrated in Figure 9.

![Contour Plot and Global Optimum of the Example.](image)

**Figure 9: Contour Plot and Global Optimum of the Example.**

As we know, the present design practice follows a quite different approach from the discussed life-cycle performance optimization approach. In general, a designer first specifies a rough design target in terms of functional performance. Based on the determined design objective, the values of design parameters are determined according to the recommendations of design handbooks and/or experience. Problems with this design practice are well illustrated in reference (Dong 1997 (a)). For the multiple spindle drill head design, the values of the two considered design variables, the spindle hole size tolerance and spindle hole location tolerance, are chosen as \((\delta_D, \delta_{xy})^T = (0.0840\,mm, 0.115\,mm)^T\), according to the recommendations of the ANSI tolerance grades IT10 and IT9, respectively.
The functional performance, manufacturing cost, and life-cycle performance readings of the manual design and from design optimizations of various design objectives are listed in Table 2. The manual design is by no means targeted at either the peak functional performance or the minimum manufacturing cost. The peak functional performance-oriented design and the minimum manufacturing cost-oriented design produce strong bias with poor life-cycle performance readings as well. From the comparison we know, the manual approach is unable to reach the best-balanced design, while the integrated concurrent design optimization yields the best life-cycle performance design.

Table 2 Performance Comparison of Different Design Schemes

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$I^F$ (Rank)</td>
<td>0.4031 (2)</td>
<td>0 (4)</td>
<td>0.911 (1)</td>
<td>0.289 (3)</td>
</tr>
<tr>
<td>$I^C$ (Rank)</td>
<td>0.866 (3)</td>
<td>0.00088 (1)</td>
<td>3.680 (4)</td>
<td>0.389 (2)</td>
</tr>
<tr>
<td>I (Rank)</td>
<td>-0.02 (3)</td>
<td>0.00029 (2)</td>
<td>-0.619 (4)</td>
<td>0.060 (1)</td>
</tr>
</tbody>
</table>

Summary

A new generic tolerance design method is introduced. The method automatically forms the cost-to-design-tolerance models from several production operation-based, cost-to-manufacturing-tolerance models, through a hierarchical optimization procedure. This approach significantly changed traditional tolerance synthesis practice by eliminating the critical barrier to the industrial application of tolerance synthesis: lack of general reliable cost-to-design-tolerance models. The approach allows empirical cost data of low-level production operations be used in a high-level design activity -- tolerance synthesis before manufacturing of the part is launched. The method
serves as a platform for further tolerance synthesis research, and the framework for a computer automated tolerance synthesis software tool.

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**References**


