INPUT - OUTPUT ANALYSIS

Input - output analysis is a method of calculating income and employment multipliers which takes account of differences in technology between industries and of the linkages between industries. The data required is the input-output accounts for the region often referred to as the transactions matrix.

Example: the numbers are billions of $

Sales		interm,	demand	_Purchases_ final	demand		
	sector	manuf.	nonmanuf	cons.	govt.	expt.	total
<u>inter.</u>	manuf,	18	18	40	14	30	120
<u>inputs</u>	nonman	20	37	43	7	18	125
	hshlds	56	52	16	9	27	160
<u>prim.</u>	govt.	6	7	20	22	0	55
<u>inputs</u>	impt.	20	11	41	3	5	80
	total	120	125	160	55	80	

Here just two industrial sectors are used. Their output and the income and employment generated in each sector is determined by the final demand for the output of each sector. Input-output analysis is a method for estimating how changes in final demand will affect these outputs, incomes and employment.

Calculation of Gross Domestic Product

1. Use Y = household income + indirect taxes + CCA
    $$= 160 + (6 + 7) + 0$$
    $$= 173$$

2. Use Y = C + I + G + X - M
    $$= (160 - 20) + 0 + (55 - 22) + 80 - 80$$
    $$= 173$$

Technical Coefficients Matrix

I - O analysis assumes fixed coefficients in production. That is that in each sector the ratios of the various inputs to the output of the sector do not depend on the scale of production. Thus in the example, the inputs needed to produce one unit of output in each of the sectors is given by

sector	manufacturing	nonmanufacturing
manufacturing	.15	.14
nonmanufacturing	.17	.30
households	.47	.42
government	.05	.06
imports	.17	.09
total	1.00	1.00

The top 2 x 2 portion of the table is the matrix of technical coefficients which will be referred to as **A** . The other coefficients will also be needed in the calculation of impacts.

Calculation of Impacts

Suppose there is an increase in final demand for the nonmanufacturing sector of 1. A round by round approach can be used to calculate the impact this will have on the outputs of each of the two sectors. The additional requirements are:

	manufacturing	nonmanufact.	
direct effect	0	1	direct req.
first round of spending	.14	.30	inputs needed to produce dr
second round of spending	.15*.14+ .14*.30 = .06	.17*.14+ .30*.30 = .11	inputs needed to produce fr
third round of spending	.15*.06+ .14*.11 = .02	.17*.06+ .30*.11 = .04	inputs needed to produce sr
fourth round of spending	.15*.02+ .14*.04 = .01	.17*.02+ .30*.04 = .02	inputs needed to produce tr
total outputs	.23	1.47	

The totals are the additional outputs manufacturing and nonmanufacturing must produce in order to have one more unit of nonmanufacturing output available for sale to final demand. They are called **output multipliers**.

The Income and Employment Multipliers

The calculation of the various effects of an increase in final demand for nonmanufacturing is done in the following table.

effects	direct	dir. +indir.	indirect
man. output	0	.23	.23
nonman. out.	1.0	1.47	.47
hsld. income	.42	.23*.47+ 1.47*.42=.73	.31
indirect tax	.06	.23*.05+ 1.47*.06=.10	.04
gdp	.42+.06=.48	.73+.10=.83	.35
imports	.09	.23*.17+ 1.47*.09=.17	.08

The **gdp multiplier .83** is the most interesting of these numbers. It means that if final demand for nonmanufacturing increases by $1 then regional gdp will increase by $0.83 or if final demand for nonmanufacturing increases by $1.5 million then regional gdp will increase by $1.245 million. The gdp multipliers of the two sectors here can be compared. These multipliers cannot however be compared to the Keynesian multipliers because the induced effects have not been calculated here.

Other uses of the numbers calculated above are:
(1) if G increases by $1 spent on nonmanufacturing goods, the deficit would increase only by $0.90
(2) if X increases by $1 spent on nonmanufacturing goods, the balance of payments deficit would fall by $0.83.
(3) **employment multipliers** can be calculated if one has data on the marginal products of labour in both industries. The formula is $k = .23/mpl_m + 1.47/mpl_{nm}$.

The Formula for the Impact Matrix

The **output multipliers** can be calculated from the technical coefficients matrix **A** using the following formula. Write

$$\mathbf{A} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} .15 & .14 \\ .17 & .30 \end{vmatrix}$$

Then the matrix of direct and indirect requirements or the impact matrix is given by:

$$\mathbf{(I-A)^{-1}} = 1/|I-A| \begin{vmatrix} 1-a_{22} & a_{12} \\ a_{21} & 1-a_{11} \end{vmatrix} = \begin{vmatrix} 1.23 & .25 \\ .30 & 1.49 \end{vmatrix}$$

where $|I-A| = (1-a_{11})(1-a_{22})-a_{12}a_{21}$. The first column of this table gives the **output multipliers** for sector 1 and the second column gives the **output multipliers** for sector 2. Thus the direct + indirect effect on household income of a $1 increase in manufacturing final demand is $1.23*.47 + .30*.42 = 0.70, on indirect taxes is $1.23*.05 + .30*.06 = 0.08 and on gdp is $0.78.

Closing the Input-Output Model

The calculations above do not capture the induced effects of an expansion of final demand, i.e., households are not allowed to spend their additional income. The open input-output model above is closed by adding a third endogenous sector, namely households whose inputs are given by the consumption column in the transactions matrix. The technical coefficients matrix is then:

sector	manuf.	nonman.	cons.
manuf.	.15	.14	.25
nonman.	.17	.30	.27
hshlds.	.47	.42	.10
govt.	.05	.06	.12
imports	.17	.09	.26
total	1.00	1.00	1.00

The associated impact matrix can be shown to be:

$$(I-A)^{-1} = \begin{vmatrix} 1.72 & .77 & .71 \\ .93 & 2.16 & .91 \\ 1.34 & 1.41 & 1.91 \end{vmatrix}$$

The first column of this table gives the **output multipliers** for sector 1, the second column gives the **output multipliers** for sector 2 and the third gives the **output multipliers** for a direct payment of $1 to households. The third row gives the household income multipliers including the induced effects. The induced effects on household income are $1.41 - $0.73 for a $1 expansion of final demand for nonmanufacturing and $1.34 - $0.70 = $0.64 for manufacturing. The total effects on taxes are 1.72*.05 + .93*.06 + 1.34*.12 = .30 for manufacturing and .77*.05 + 2.16*.06 +1.41*.12 = .34 for nonmanufacturing. The total effects on imports can be calculated in a similar manner. The induced effects on gdp are (1.34 + 1.72*.05 + .93*.06) - .78 = $0.70 and (1.41 + .77*.05 + 2.16*.06) - .83 = $0.75 for manufacturing and nonmanufacturing respectively. Note that only the indirect taxes are considered in calculating gdp effects.

Round by Round including the Induced Effects

Suppose there is an increase in final demand for the manufacturing sector of 1. A round by round approach can be used to calculate the impact this will have on the outputs of each of the two sectors and on household income. The additional requirements are:

	manuf.	nonmanuf.	hsld. inc.	
dir. effect	1	0	0	direct req.
first round of spending	.15	.17	.47	inputs for prod. of dr
second round of spending	.15*.15+ .14*.17+ .25*.47 = .16	.17*.15+ .30*.17+ .27*.47 = .20	.47*.15+ .42*.17+ .10*.47 = .19	inputs for prod. of fr
third round	.15*.16+ .14*.20+ .25*.19 = .10	.17*.16+ .30*.20+ .27*.19 = .14	.47*.16+ .42*.20+ .10*.19 = .18	inputs for prod. of sr
fourth round	.08	.11	.12	for tr
fifth round	.06	.08	.10	for fr
total up to fifth round	1.55	.70	1.06	

The fourth and fifth rounds effects have been calculated in the same way as the second and third round effects although the details are not shown. If more rounds are calculated (it takes about 15) the total row will be close to the first column of the impact matrix.

Type I and Type II Multipliers

The multipliers discussed above are all applied to increases in exogenous final demand. Some believe it is misleading to compare multipliers for units of final demand for different sectors as the multipliers will have different units of measurement attached to them. They prefer to make their comparisons in terms of type I or type II multipliers which have no units of measurement attached to them. The formulas are:

$$\text{type I} = (\text{direct} + \text{indirect})/\text{direct}$$
$$\text{type II} = (\text{direct} + \text{indirect} + \text{induced})/\text{direct}$$

Here the type I gdp multipliers are $.78/(.47 + .05) = 1.50$ for manufacturing and $.83/(.42 + .06) = 1.73$ for nonmanufacturing. The type II gdp multipliers for these two sectors are $1.48/(.47 + .05) = 2.85$ and $1.58/(.42 + .06) = 3.29$ respectively. Both calculations indicate that nonmanufacturing is more strongly linked to the rest of the economy than manufacturing.