

# Truth Conditional Meaning of Sentences

Ling324

Reading: *Meaning and Grammar*, pg. 69-87

# Meaning of Sentences

- A sentence can be true or false in a given situation or circumstance.

(1) The pope talked to Prince Williams between 3 and 4 pm on Feb. 5, 2005.

Although we may not know what the facts are, we know what they ought to be in order to judge the sentence true (i.e., truth conditions).

Conversely, even if we know what the facts are, we cannot use these facts to evaluate whether the sentence is true, if we do not understand what the sentence means.

Thus, the truth condition is a necessary component of sentence meaning, although it may not be a sufficient component.

- $S$  means  $p =_{df} S$  is true in  $V$  iff  $p$ .

$S$  is a structural description of a sentence in a language  $L$ .

$V$  is a situation or a specification of relevant facts.

$p$  describes the conditions that have to obtain for  $S$  to be true in  $V$ .

## Meaning of Sentences (cont.)

- How do we arrive at the truth conditions of sentences?

Can we simply list all possible sentences with the corresponding truth conditions?

No. Because a language contains an infinite number of sentences.

We need a theory of truth that allows us to generate all of the correspondingly infinite number of truth conditions.

This means that the mechanism for specifying truth conditions must be based on a generative device that is similar to the way syntax is characterized.

That is, the truth conditions associated with a sentence must be specified compositionally, by looking at the smaller units and the way they are combined.

- We will study a formal device that can compositionally specify the truth conditions associated with all possible sentences in a language.

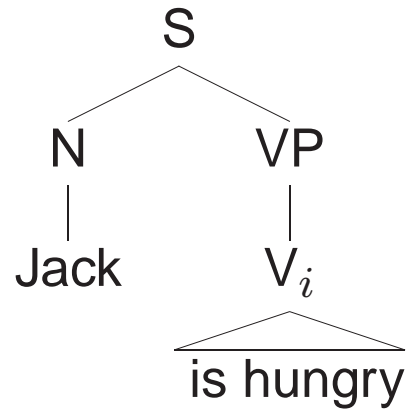
For the time being, we will consider only ordinary declarative sentences, and we will assume that the discourse context in which a sentence is used is fixed.

## Syntax of a Fragment of English (F1)

- (2)
- a.  $S \rightarrow N VP$
  - b.  $S \rightarrow S \text{ conj } S$
  - c.  $S \rightarrow \text{neg } S$
  - d.  $VP \rightarrow V_t N$
  - e.  $VP \rightarrow V_i$
  - f.  $N \rightarrow \text{Jack, Sophia, James}$
  - g.  $V_i \rightarrow \text{is boring, is hungry, is cute}$
  - h.  $V_t \rightarrow \text{likes}$
  - i.  $\text{conj} \rightarrow \text{and, or}$
  - j.  $\text{neg} \rightarrow \text{it is not the case that}$

# Syntax of a Fragment of English (F1)

(3) Jack is hungry.



(4) Sophia likes James.

(5) It is not the case that James is cute.

(6) Jack is hungry, and it is not the case that James likes Jack.

(7) It is not the case that Jack is hungry or Sophia is boring. (2 tree structures)

# Semantics for F1

- If  $\beta$  is a well-formed expression of F1,  $\llbracket \beta \rrbracket^V$  is its semantic value in circumstance  $V$ .

$\llbracket \text{Jack} \rrbracket^V$  stands for the semantic value of *Jack* in circumstance  $V$ .

- If  $S$  is a well-formed sentence of F1,  $\llbracket S \rrbracket^V$  denotes the truth value of  $S$  in circumstance  $V$ .

$\llbracket S \rrbracket^V = 1$  is a shorthand for “ $S$  is true in  $V$ .”

$\llbracket S \rrbracket^V = 0$  is a shorthand for “ $S$  is false in  $V$ .”

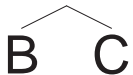
- To be able to specify the truth conditions associated with each sentence in F1, we will specify the semantic values for each lexical expressions and semantic rules for combining them.

Whether  $\llbracket S \rrbracket^V$  is 1 or 0 should depend only on the values in  $V$  of the lexical expressions occurring in  $S$  and the semantic rules applied in interpreting  $S$ .

## Semantics for F1 (cont.)

- Recall that when we interpret a sentence, we are not interpreting strings of words, but syntactic structures assigned to the sentence.

In order to interpret a syntactic structure compositionally, we need to do two things:

- Each terminal node is assigned with a lexical value.
- For each syntactic rule of the form  $A \rightarrow B C$ , , we define a

semantic rule that specifies the value of the tree whose root is  $A$  in terms of the values of the subtrees rooted in  $B$  and  $C$ .

## Basic Lexical Entries

- (8) For any situation (or circumstance)  $V$ ,
- a.  $\llbracket \text{Jack} \rrbracket^V = \text{Jack}'$
  - b.  $\llbracket \text{Sophia} \rrbracket^V = \text{Sophia}'$
  - c.  $\llbracket \text{James} \rrbracket^V = \text{James}'$
  - d.  $\llbracket \text{is boring} \rrbracket^V = \{x : x \text{ is boring in } V\}.$   
(The set of those individuals that are boring in  $V$ .)
  - e.  $\llbracket \text{is hungry} \rrbracket^V = \{x : x \text{ is hungry in } V\}$
  - f.  $\llbracket \text{is cute} \rrbracket^V = \{x : x \text{ is cute in } V\}$
  - g.  $\llbracket \text{likes} \rrbracket^V = \{ \langle x, y \rangle : x \text{ likes } y \text{ in } V \}$   
(The set of ordered pairs of individuals such that the first likes the second in  $V$ .)



## Logical Connectives

We can think of the semantic values of logical connectives in natural language as functions that map truth values into truth values.

(9) For any situation  $V$ ,

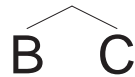
a.  $\llbracket \text{it is not the case} \rrbracket^V = \begin{bmatrix} 1 \rightarrow 0 \\ 0 \rightarrow 1 \end{bmatrix}$

b.  $\llbracket \text{and} \rrbracket^V = \begin{bmatrix} \langle 1, 1 \rangle \rightarrow 1 \\ \langle 1, 0 \rangle \rightarrow 0 \\ \langle 0, 1 \rangle \rightarrow 0 \\ \langle 0, 0 \rangle \rightarrow 0 \end{bmatrix}$

c.  $\llbracket \text{or} \rrbracket^V = \begin{bmatrix} \langle 1, 1 \rangle \rightarrow 1 \\ \langle 1, 0 \rangle \rightarrow 1 \\ \langle 0, 1 \rangle \rightarrow 1 \\ \langle 0, 0 \rangle \rightarrow 0 \end{bmatrix}$

# Interpretive Rules for Each Syntactic Rule

- $[_A B C]$  is equivalent to  $A$



- $\llbracket [_A B C] \rrbracket$  stands for the semantic value of

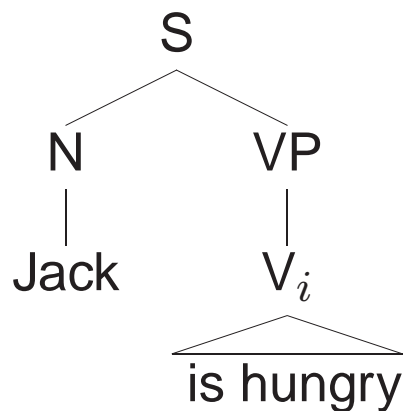


- If  $g$  is a function and  $u$  is a possible argument for  $g$ ,  $g(u)$  indicates the result of applying  $g$  to  $u$ .

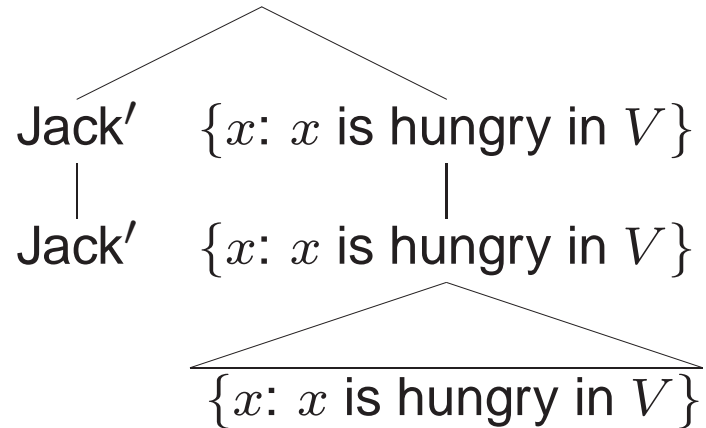
- (10)
- $\llbracket [_S N VP] \rrbracket^V = 1$  iff  $\llbracket N \rrbracket^V \in \llbracket VP \rrbracket^V$  and 0 otherwise.
  - $\llbracket [_S S1 \text{ conj } S2] \rrbracket^V = \llbracket \text{conj} \rrbracket^V (<\llbracket S1 \rrbracket^V, \llbracket S2 \rrbracket^V>)$
  - $\llbracket [_S \text{ neg } S] \rrbracket^V = \llbracket \text{neg} \rrbracket^V (\llbracket S \rrbracket^V)$
  - $\llbracket [_{VP} V_t N] \rrbracket^V = \{x: <x, \llbracket N \rrbracket^V> \in \llbracket V_t \rrbracket^V\}$
  - If  $A$  is a category and  $a$  is a lexical entry or a lexical category and  $\Delta = [_A a]$ , then  $\llbracket \Delta \rrbracket^V = \llbracket a \rrbracket^V$

## Compositional Semantics for F1

(11) Jack is hungry.



1 iff  $\text{Jack}' \in \{x: x \text{ is hungry in } V\}$

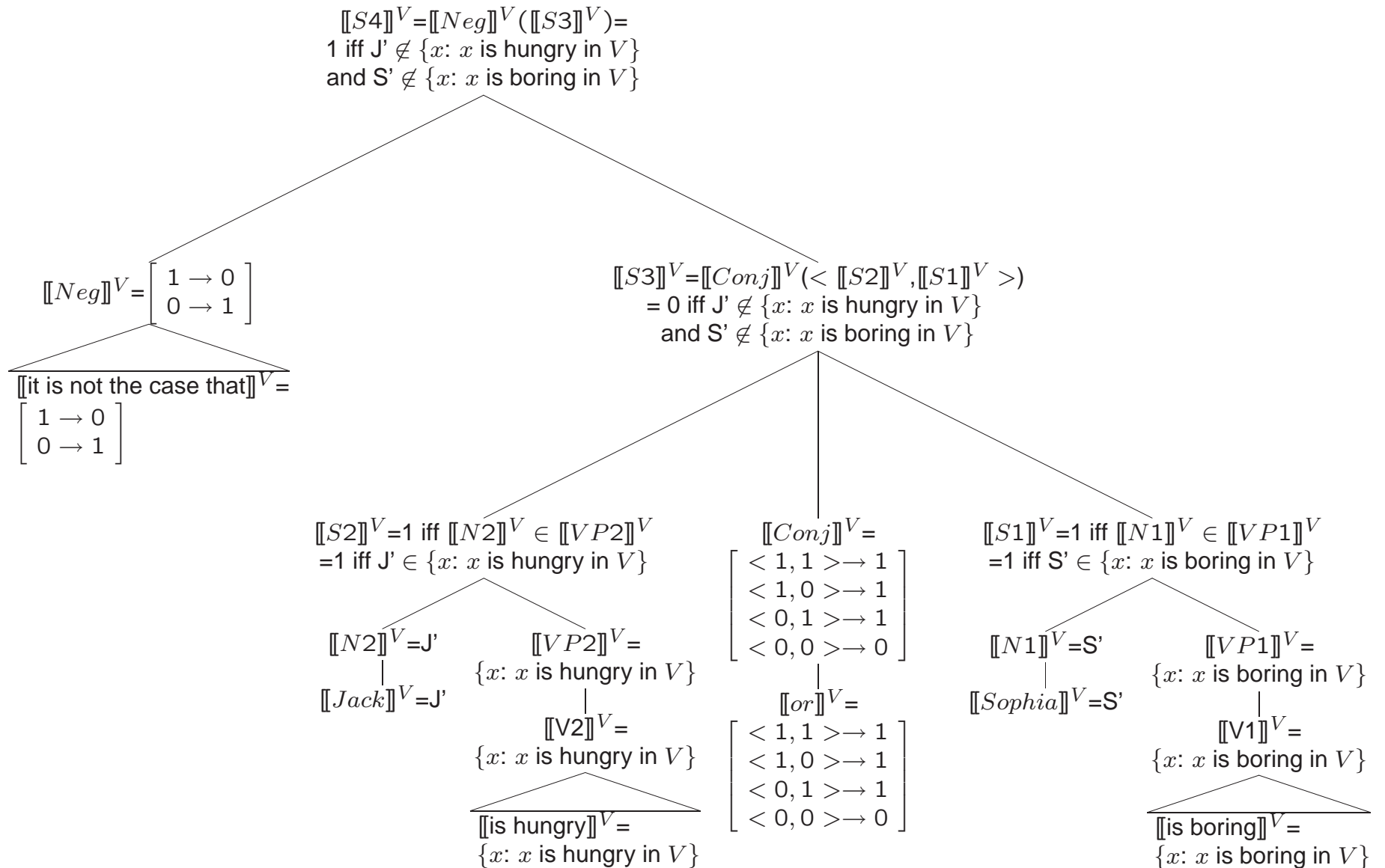


(12) Sophia likes James.

(13) It is not the case that James is cute.

# Compositional Semantics for F1 (cont.)

(14) It is not the case that [Jack is hungry or Sophia is boring].



## Compositional Semantics for F1 (cont.)

QUESTION: For the following sentences, compute their truth conditions compositionally and evaluate their semantic values under the specified situations.

(15) Jack is hungry, and it is not the case that James likes Jack.

- Situation  $V'$ : Jack is hungry, James does not like him.
- Situation  $V''$ : Jack is hungry, James likes him.

(16) [It is not the case that Jack is hungry] or [Sophia is boring].

- Situation  $V'''$ : Jack is hungry, Sophia is boring.
- Situation  $V''''$ : Jack is hungry, Sophia is not boring.

# Capturing Semantic Intuitions

- Speakers have the semantic capacity of matching sentences with the situations that they describe.

The truth conditional semantics that we are pursuing is an abstract representation of our semantic capacity.

If  $\llbracket S \rrbracket^V = 1$ , then  $S$  correctly describes situation  $V$ .

If  $\llbracket S \rrbracket^V = 0$ , then  $S$  does not correctly describe situation  $V$ .

- Entailment

$S$  ENTAILS  $S'$  (relative to analyses  $\Delta_S$  and  $\Delta_{S'}$ ) iff for every situation  $V$ , if  $\llbracket \Delta_S \rrbracket^V = 1$ , then  $\llbracket \Delta_{S'} \rrbracket^V = 1$ .

A set of sentences  $\Sigma = \{S_1, \dots, S_n\}$  ENTAILS a sentence  $S$  (relative to analyses  $\Delta_{S_1}, \dots, \Delta_{S_n}$  and  $\Delta_S$ , respectively) iff whenever in any situation  $V$  we have for all  $S' \in \Sigma$ ,  $\llbracket \Delta_{S'} \rrbracket^V = 1$ , we also have that  $\llbracket \Delta_S \rrbracket^V = 1$ .

That is, any situation that makes all of the sentences in  $\Sigma$  true also has to make  $S$  true.

(17) Let us show that  $[_S \text{ it is not the case that } [_S \text{ Jack is hungry or Sophia is boring}]]$  entails  $[_S \text{ it is not the case that Jack is hungry}]$ .

- To show this, we assume that  $[[[_S \text{ neg } [_S \text{ Jack is hungry or Sophia is boring}]]]]^V$  is true, and show that  $[[[_S \text{ Jack is not hungry}]]]^V$  must also be true.
- If  $[[[_S \text{ neg } [_S \text{ Jack is hungry or Sophia is boring}]]]]^V$  is true,  $[[[_S \text{ Jack is hungry or Sophia is boring}]]]^V$  is false, by the semantics for negation.
- If  $[[[_S \text{ Jack is hungry or Sophia is boring}]]]^V$  is false, the  $[[\text{Jack is hungry}]]^V$  is false, and  $[[\text{Sophia is boring}]]^V$  is false, by the semantics for disjunction.
- If  $[[\text{Jack is hungry}]]^V$  is false, then  $[[[_S \text{ Jack is not hungry}]]]^V$  is true, by the semantics for negation.
- Thus,  $[_S \text{ neg } [_S \text{ Jack is hungry or Sophia is boring}]]$  entails  $[_S \text{ it is not the case that Jack is hungry}]$ .

(18) Let us show that  $[_S [_S \text{it is not the case that Jack is hungry}] \text{or } [_S \text{Sophia is boring}]]$  does not entail  $[_S [_S \text{Jack is hungry}] \text{or } [_S \text{it is not the case that Sophia is boring}]]$ .

- To show this, we construct a situation  $V'$ , such that  $[_S [_S \text{it is not the case that Jack is hungry}] \text{or } [_S \text{Sophia is boring}]]$  is true in  $V'$ , but  $[_S [_S \text{Jack is hungry}] \text{or } [_S \text{it is not the case that Sophia is boring}]]$  is false in  $V'$ .
- In order for  $[[[_S [_S \text{Jack is hungry}] \text{or } [_S \text{Sophia is not boring}]]]^{V'}$  to be false, it must be the case that  $[[[_S \text{Jack is hungry}]]^{V'}$  is false, and  $[[[_S \text{Sophia is not boring}]]^{V'}$  is false, by the semantics for disjunction.
- By the semantics for negation, this means that  $[[[\text{Sophia is boring}]]^{V'}$  is true and  $[[[\text{Jack is not hungry}]]^{V'}$  is true.
- By the semantics for disjunction, this means that  $[[[_S [_S \text{Jack is not hungry}] \text{or } [_S \text{Sophia is boring}]]]^{V'}$  is true.
- Thus, we have constructed a situation  $V'$  where  $[_S [_S \text{it is not the case that Jack is hungry}] \text{or } [_S \text{Sophia is boring}]]$  is true, but  $[_S [_S \text{Jack is hungry}] \text{or } [_S \text{it is not the case that Sophia is boring}]]$  is false, and hence the former does not entail the latter.



## Capturing Semantic Intuitions (cont.)

- Contradiction

$S$  is CONTRADICTORY (relative to analysis  $\Delta_S$ ) iff there is no situation  $V$ , such that  $\llbracket \Delta_S \rrbracket^V = 1$ .

A set of sentences  $\Sigma = \{S_1, \dots, S_n\}$  is CONTRADICTORY (relative to analyses  $\Delta_{S1}, \dots, \Delta_{Sn}$ ) iff there is no situation  $V$  such that for all  $S \in \Sigma$ ,  $\llbracket \Delta_S \rrbracket^V = 1$ .

(19) Let us show that  $\llbracket \text{Jack is boring} \rrbracket$  and  $\llbracket \text{it is not the case that Jack is boring} \rrbracket$  to be contradictory.

- Assume that there exists a  $V$  such that  $\llbracket \llbracket \text{Jack is boring} \rrbracket \text{ and } \llbracket \text{Jack is not boring} \rrbracket \rrbracket^V$  is true.
- By the semantics for conjunction,  $\llbracket \llbracket \text{Jack is boring} \rrbracket \rrbracket^V$  is true and  $\llbracket \llbracket \text{Jack is not boring} \rrbracket \rrbracket^V$  is true.
- But by the semantics for negation,  $\llbracket \llbracket \text{Jack is boring} \rrbracket \rrbracket^V$  ends up being both true and false. This is a contradiction. Thus, the original assumption is wrong.
- Therefore, there does not exist a  $V$  such that  $\llbracket \llbracket \text{Jack is boring} \rrbracket \text{ and } \llbracket \text{Jack is not boring} \rrbracket \rrbracket^V$  is true.

## Capturing Semantic Intuitions (cont.)

- Validity

$S$  is LOGICALLY TRUE (VALID) (relative to analysis  $\Delta_S$ ) iff there is no situation  $V$ , such that  $\llbracket \Delta_S \rrbracket^V = 0$ .

(20) Let us show that  $\llbracket [\text{Jack is boring}] \text{ or } [\text{it is not the case that Jack is boring}] \rrbracket$  to be valid.

- Assume that there exists  $V$  such that  $\llbracket \llbracket [\text{Jack is boring}] \text{ or } [\text{Jack is not boring}] \rrbracket \rrbracket^V$  is false.
- By the semantics for disjunction  $\llbracket [\text{Jack is boring}] \rrbracket^V$  is false and  $\llbracket [\text{Jack is not boring}] \rrbracket^V$  is false.
- By the semantics for negation  $\llbracket [\text{Jack is boring}] \rrbracket^V$  ends up being both true and false. This is contradiction. Thus, our original assumption is wrong.
- Therefore, there does not exist  $V$  such that  $\llbracket \llbracket [\text{Jack is boring}] \text{ or } [\text{Jack is not boring}] \rrbracket \rrbracket^V$  is false.

## Capturing Semantic Intuitions (cont.)

- Logical Equivalence

$S$  is LOGICALLY EQUIVALENT to  $S'$  (relative to analyses  $\Delta_S$  and  $\Delta_{S'}$ ) iff  $S$  entails  $S'$  (relative to  $\Delta_S$  and  $\Delta_{S'}$ ) and  $S'$  entails  $S$  (relative to  $\Delta_S$  and  $\Delta_{S'}$ ).

(21) Let us show that  $[_S \text{ it is not the case that } [_S \text{ Jack is hungry or Sophia is boring}]]$  and  $[_S [_S \text{ it is not the case that Jack is hungry}]]$  and  $[\text{it is not the case that Sophia is boring}]]$  are logically equivalent.

First we need to show that  $[_S \text{ it is not the case that } [_S \text{ Jack is hungry or Sophia is boring}]]$  entails  $[_S [_S \text{ it is not the case that Jack is hungry}]]$  and  $[\text{it is not the case that Sophia is boring}]]$ .

- To show this, we assume that  $[[[\text{neg } [\text{Jack is hungry or Sophia is boring}]]]]^V$  is true, and show that  $[[[\text{Jack is not hungry}]] \text{ and } [\text{Sophia is not boring}]]]^V$  must also be true.
- If  $[[[\text{neg } [\text{Jack is hungry or Sophia is boring}]]]]^V$  is true,  $[[[\text{Jack is hungry or Sophia is boring}]]]^V$  is false, by the semantics for negation.

- If  $\llbracket \text{[Jack is hungry or Sophia is boring]} \rrbracket^V$  is false, then  $\llbracket \text{[Jack is hungry]} \rrbracket^V$  is false, and  $\llbracket \text{[Sophia is boring]} \rrbracket^V$  is false, by the semantics for disjunction.
- By the semantics for negation,  $\llbracket \text{[Jack is not hungry]} \rrbracket^V$  is true, and  $\llbracket \text{[Sophia is not boring]} \rrbracket^V$  is true.
- By the semantics for conjunction,  $\llbracket \text{[[Jack is not hungry] and [Sophia is not boring]]} \rrbracket^V$  is true.
- Thus,  $[_S \text{ neg } [_S \text{ Jack is hungry or Sophia is boring}]]$  entails  $[_S [_S \text{ it is not the case that Jack is hungry}]]$  and  $[_S \text{ it is not the case that Sophia is boring}]]$

We then need to show that  $[_S [_S \text{ it is not the case that Jack is hungry}]]$  and  $[_S \text{ it is not the case that Sophia is boring}]]$  entails  $[_S \text{ it is not the case that } [_S \text{ Jack is hungry or Sophia is boring}]]$ .

- We assume that  $\llbracket \text{[[Jack is not hungry] and [Sophia is not boring]]} \rrbracket^V$  is true and show that  $\llbracket \text{[neg [Jack is hungry or Sophia is boring]]} \rrbracket^V$  must also be true.

- If  $\llbracket \llbracket \text{Jack is not hungry} \rrbracket \text{ and } \llbracket \text{Sophia is not boring} \rrbracket \rrbracket^V$  is true,  $\llbracket \llbracket \text{Jack is not hungry} \rrbracket \rrbracket^V$  is true and  $\llbracket \llbracket \text{Sophia is not boring} \rrbracket \rrbracket^V$  is true, by the semantics for conjunction.
- By the semantics for negation,  $\llbracket \llbracket \text{Jack is hungry} \rrbracket \rrbracket^V$  is false and  $\llbracket \llbracket \text{Sophia is boring} \rrbracket \rrbracket^V$  is false.
- By the semantics for disjunction,  $\llbracket \llbracket \text{Jack is hungry or Sophia is boring} \rrbracket \rrbracket^V$  is false.
- By the semantics for negation,  $\llbracket \llbracket \text{neg } \llbracket \text{Jack is hungry or Sophia is boring} \rrbracket \rrbracket \rrbracket^V$  is true.
- Thus,  $[_S [_S \text{ it is not the case that Jack is hungry} ] \text{ and } [\text{it is not the case that Sophia is boring} ]]$  entails  $[_S \text{ it is not the case that } [_S \text{ Jack is hungry or Sophia is boring} ]]$ .

Therefore,  $[_S \text{ it is not the case that } [_S \text{ Jack is hungry or Sophia is boring} ]]$  and  $[_S [_S \text{ it is not the case that Jack is hungry} ] \text{ and } [\text{it is not the case that Sophia is boring} ]]$  are logically equivalent.

## Capturing Semantic Intuitions (cont.)

- EXERCISE: (from *Meaning and Grammar*, Exercise 3, pg. 85)
  1. Prove that ‘Jack is hungry and Sophia is boring’ entails ‘Sophia is boring.’
  2. Prove that (22a) does not entail (22b).
    - (22) a. [[Jack is hungry] or [it is not the case that Sophia is boring]]
    - b. [[it is not the case that Jack is hungry] or [Sophia is boring]]