

# Introduction to Predicate Logic

Ling324

Reading: *Meaning and Grammar*, pg. 113-141

# Usefulness of Predicate Logic for Natural Language Semantics

- While in propositional logic, we can only talk about sentences as a whole, predicate logic allows us to decompose simple sentences into smaller parts: predicates and individuals.

- (1) a. John is tall.  
b.  $T(j)$

- Predicate logic provides a tool to handle expressions of generalization: i.e., quantificational expressions.

- (2) a. Every cat is sleeping.  
b. Some girl likes David.  
c. No one is happy.

- Predicate logic allows us to talk about variables (pronouns).

The value for the pronoun is some individual in the domain of universe that is contextually determined.

- (3) a. It is sleeping.  
b. She likes David.  
c. He is happy.

# Usefulness of Predicate Logic for Natural Language Semantics (cont.)

- Sentences with quantificational expressions can be divided into two interpretive components.

(4) Every cat is sleeping.

a. (every cat)(it is sleeping)

b. for all  $x$ ,  $x$  is a cat,  $x$  is sleeping

c. = true iff 'it is sleeping' is true for all possible values for 'it' in the domain

– A simple sentence containing a variable/pronoun (a place holder):

It can be evaluated as true or false with respect to some individual contextually taken as a value for the pronoun.

– The quantificational expression:

It instructs us to limit the domain of individuals being considered to a relevant set (e.g., a set of cats), and tells us how many different values of the pronoun we have to consider from that domain to establish truth for the sentence.

## Usefulness of Predicate Logic for Natural Language Semantics (cont.)

- (5) Some girl likes David.
  - a. (some girl)(she likes David)
  - b. for some  $x$ ,  $x$  is a girl,  $x$  likes David
  - c. = true iff 'she likes David' is true for at least one possible value for 'she'
  
- (6) No one is happy.
  - a. (no one)(s/he is happy)
  - b. for all  $x$ ,  $x$  is a person,  $x$  is not happy
  - c. = true iff 's/he is happy' is false for all possible values for 's/he'

# Syntax of Predicate Logic

## 1. Primitive vocabulary

### (a) A set of terms:

A set of individual constants:  $a, b, c, d, \dots$

*John, Mary, Pavarotti, Loren*

A set of individual variables:  $x, y, z, x_0, x_1, x_2, \dots$

*he, she, it*

### (b) A set of predicates: $P, Q, R, \dots$

One-place predicates: *is happy, is boring*

Two-place predicates: *like, hate, love, hit*

Three-place predicates: *introduce, give*

### (c) A binary identity predicate: $=$

### (d) The connectives of propositional logic: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

### (e) Quantifiers: $\forall, \exists$

### (f) brackets: $(, ), [, ]$

## 2. Syntactic rules

- (a) If  $P$  is an  $n$ -place predicate and  $t_1, \dots, t_n$  are all terms, then  $P(t_1, \dots, t_n)$  is an atomic formula.

*John is happy:*  $H(j)$

*John loves Mary:*  $L(j, m)$

*John introduced Mary to Sue:*  $I(j, m, s)$

- (b) If  $t_1$  and  $t_2$  are individual constants or variables, then  $t_1 = t_2$  is a formula.

*John is Bill:*  $j=b$

- (c) If  $\phi$  is a formula, then  $\neg\phi$  is a formula.

- (d) If  $\phi$  and  $\psi$  are formulas, then  $(\phi \wedge \psi)$ ,  $(\phi \vee \psi)$ ,  $(\phi \rightarrow \psi)$ , and  $(\phi \leftrightarrow \psi)$  are formulas too.

- (e) If  $\phi$  is a formula and  $x$  is a variable, then  $\forall x\phi$ , and  $\exists x\phi$  are formulas too.

*Everyone is happy:*  $\forall xH(x)$

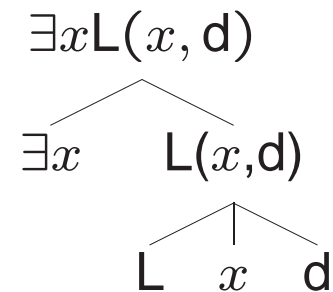
*Someone loves John:*  $\exists xL(x, j)$

*Everyone loves someone:*  $\forall x\exists yL(x, y)$ ,  $\exists y\forall xL(x, y)$

- (f) Nothing else is a formula in predicate logic.

## Syntactic Tree in Predicate Logic

- $\exists x L(x, d)$



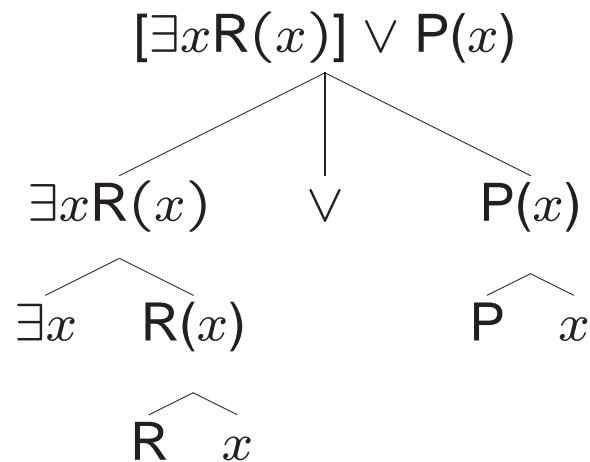
QUESTION: Draw the syntactic tree for the expression in (7) that are well-formed formulas of Predicate Logic.

- (7) a.  $\exists \forall (Qa \rightarrow PR(b)(c))$   
b.  $\forall x (P(x) \rightarrow \exists y Q(x, y))$   
c.  $\exists x_1 \forall x_2 (P(x_1, x_2) \rightarrow (R(x_1) \wedge Q(x_2, a)))$

## Some Syntactic Notions in Predicate Logic

- If  $x$  is a variable and  $\phi$  is a formula to which a quantifier has been attached to produce  $\forall x\phi$ , or  $\exists x\phi$ , then we say that  $\phi$  is the *scope* of the attached quantifier and that  $\phi$  or any part of  $\phi$  *lies in the scope* of that quantifier.

Syntactically, the scope of a quantifier is what it c-commands.



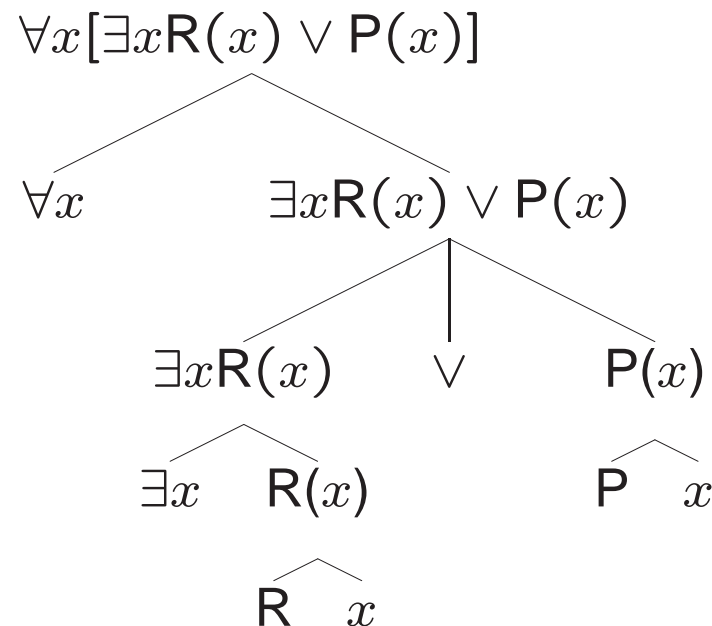
What is the scope of  $\exists x$ ?



## Some Syntactic Notion in Predicate Logic (cont.)

- We say that an occurrence of a variable  $x$  is *bound* if it occurs in the scope of  $\forall x$  or  $\exists x$ . A variable is *free* if it is not bound.

Syntactically, an occurrence of  $x$  is bound by a lowest c-commanding quantifier  $Qx$ .



$x$  in  $R(x)$  is bound by  $\exists x$ ;

$x$  in  $P(x)$  is bound by  $\forall x$ .

- $\forall y[\exists x R(x) \vee P(y)]$  and  $\forall x[\exists x R(x) \vee P(x)]$  are *alphabetic variants* of each other and are semantically equivalent.

## Some Syntactic Notions in Predicate Logic (cont.)

- Formulas with no free variables are called *closed formulas*, simply *formulas*, or *sentences*.

$H(j)$

$\forall x H(x)$

$\exists x L(j, x)$

$\exists x [H(x) \vee L(x, j)]$

Those containing a free variable are called *open formulas*.

$H(x)$

$[\exists x H(x)] \vee L(x, j)$

$\forall x L(x, y)$

## Translations in Predicate Logic

- (8) a. Every student is happy.  
b.  $\forall x [\text{student}(x) \rightarrow \text{happy}(x)]$   
c. **wrong:**  $\forall x [\text{student}(x) \wedge \text{happy}(x)]$
- (9) a. Some students are happy.  
b.  $\exists x [\text{student}(x) \wedge \text{happy}(x)]$   
c. **wrong:**  $\exists x [\text{student}(x) \rightarrow \text{happy}(x)]$
- (10) a. No student complained.  
b.  $\forall x [\text{student}(x) \rightarrow \neg \text{complained}(x)]$   
c.  $\neg \exists x [\text{student}(x) \wedge \text{complained}(x)]$
- (11) a. Not every student complained.  
b.  $\neg \forall x [\text{student}(x) \rightarrow \text{complained}(x)]$   
c.  $\exists x [\text{student}(x) \wedge \neg \text{complained}(x)]$

## Translations in Predicate Logic (cont.)

QUESTION: Translate the following English sentences into predicate logic formula.

- a. John likes Susan.
- b. John has a cat.
- c. A whale is a mammal.
- d. Barking dogs don't bite.
- e. Either every fruit is bitter or every fruit is sweet.
- f. Every student heard some news. (possibly different news for each student)
- g. There is some news that every student heard.
- h. No student likes any exams.

# Semantics of Predicate Logic: Trial 1

1. If  $\alpha$  is a constant, then  $\llbracket \alpha \rrbracket$  is specified by a function  $V$  (in the model  $M$ ) that assigns an individual object to each constant.

$$\llbracket \alpha \rrbracket^M = V(\alpha)$$

If  $P$  is a predicate, then  $\llbracket P \rrbracket$  is specified by a function  $V$  (in the model  $M$ ) that assigns a set-theoretic objects to each predicate.

$$\llbracket P \rrbracket^M = V(P)$$

2. If  $\alpha$  is a variable, then ???
3. If  $P$  is an  $n$ -ary predicate and  $t_1, \dots, t_n$  are all terms (constants or variables), then for any model  $M$ ,  $\llbracket P(t_1, \dots, t_n) \rrbracket^M = 1$  iff  $\langle \llbracket t_1 \rrbracket^M, \dots, \llbracket t_n \rrbracket^M \rangle \in \llbracket P \rrbracket^M$

4. If  $\phi$  and  $\psi$  are formulas, then for any model  $M$ ,

$$\llbracket \neg \phi \rrbracket^M = 1 \text{ iff } \llbracket \phi \rrbracket^M = 0$$

$$\llbracket \phi \wedge \psi \rrbracket^M = 1 \text{ iff } \llbracket \phi \rrbracket^M = 1 \text{ and } \llbracket \psi \rrbracket^M = 1$$

$$\llbracket \phi \vee \psi \rrbracket^M = 1 \text{ iff } \llbracket \phi \rrbracket^M = 1 \text{ or } \llbracket \psi \rrbracket^M = 1$$

$$\llbracket \phi \rightarrow \psi \rrbracket^M = 1 \text{ iff } \llbracket \phi \rrbracket^M = 0 \text{ or } \llbracket \psi \rrbracket^M = 1$$

$$\llbracket \phi \leftrightarrow \psi \rrbracket^M = 1 \text{ iff } \llbracket \phi \rrbracket^M = \llbracket \psi \rrbracket^M$$

5. If  $\phi$  is a formula, and  $v$  is a variable, then, for any model  $M$ ,

$$\llbracket \forall v \phi \rrbracket^M = 1 \text{ iff } \llbracket [c/v] \phi \rrbracket^M = 1 \text{ for all constants } c.$$

$$\llbracket \exists v \phi \rrbracket^M = 1 \text{ iff } \llbracket [c/v] \phi \rrbracket^M = 1 \text{ for some constant } c.$$

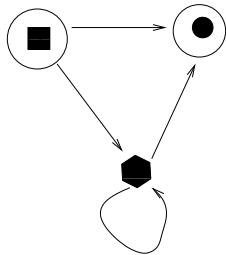
$[c/v]\phi$ : the formula resulting from having the constant  $c$  instead of the variable  $v$  in  $\phi$ .

## Semantics of Predicate Logic: Trial 1 (cont.)

- An example model

Let us take the model  $M_1$ , depicted below. Let us take a language in Predicate Logic such that the constants  $a$ ,  $b$ , and  $c$  denote the individuals dark box, dark circle and dark trapezoid, respectively, the unary predicate  $A$  denotes the set of individuals with a circle around, and the binary predicate  $R$  denotes the relation encoded by the arrows.

$M_1$



$$U(\text{niverse}) = \{a, b, c\}$$

$$A = \{a, b\}$$

$$R = \{ \langle a, b \rangle, \langle a, c \rangle, \langle c, b \rangle, \langle c, c \rangle \}$$

Determine the truth value of the following formulas in  $M_1$ .

- (12)
- $R(a, b) \wedge R(b, b)$
  - $\neg A(c) \rightarrow R(a, c)$
  - $\forall x [R(x, x)]$
  - $\forall x [R(x, x) \leftrightarrow \neg A(x)]$
  - $\exists x \exists y \exists z [R(x, y) \wedge A(y) \wedge R(x, z) \wedge \neg A(z)]$

## Semantics of Predicate Logic: Trial 1 (cont.)

- Problem with the semantic rule having to do with quantifiers in Trial 1:

In the semantic rule in Trial 1, quantifiers are ranging over constants, individuals that have names. But it could be that there is an individual that does not have a name. Then, this individual will be excluded from being considered when evaluation for truth.

This suggests that when we are interpreting quantifiers, we need to range over individuals, not names.



## Semantics of Predicate Logic: Trial 2

1. If  $\alpha$  is a variable, then  $\llbracket \alpha \rrbracket$  is specified by a variable assignment function  $g$  (in the model  $M$ ) that assigns an individual object to each variable.

$$\llbracket \alpha \rrbracket^{M,g} = g(\alpha)$$

$g$ : set of variables  $\rightarrow$  universe of individuals

Let  $g_1$  be an assignment function such that  $g_1(x_1) = \text{John}$ ,  $g_1(x_2) = \text{Mary}$ , and for all  $n \geq 3$ ,  $g_1(x_n) = \text{Pete}$ .

$$g_1 = \left[ \begin{array}{l} x_1 \rightarrow \text{John} \\ x_2 \rightarrow \text{Mary} \\ x_n \rightarrow \text{Pete} \end{array} \right] \text{ where } n \geq 3$$

Let  $g_2$  be an assignment function such that  $g_2(x_1) = \text{Mary}$ ,  $g_2(x_2) = \text{John}$ , and for all  $n \geq 3$ ,  $g_2(x_n) = \text{Pete}$ .

$$g_2 = \left[ \begin{array}{l} x_1 \rightarrow \text{Mary} \\ x_2 \rightarrow \text{John} \\ x_n \rightarrow \text{Pete} \end{array} \right] \text{ where } n \geq 3$$

$$\llbracket \text{He}_1 \text{ is happy} \rrbracket^{M,g_1} = \llbracket \text{happy}(x_1) \rrbracket^{M,g_1} = \text{John is happy.}$$

$$\llbracket \text{He}_3 \text{ is happy} \rrbracket^{M,g_2} = \llbracket \text{happy}(x_3) \rrbracket^{M,g_2} = \text{Pete is happy.}$$

2. If  $\alpha$  is a constant, then  $\llbracket \alpha \rrbracket$  is specified by a function  $V$  (in the model  $M$ ) that assigns an individual object to each constant.

$$\llbracket \alpha \rrbracket^{M,g} = V(\alpha)$$

If  $P$  is a predicate, then  $\llbracket P \rrbracket$  is specified by a function  $V$  (in the model  $M$ ) that assigns a set-theoretic objects to each predicate.

$$\llbracket P \rrbracket^{M,g} = V(P)$$

3. If  $P$  is an  $n$ -ary predicate and  $t_1, \dots, t_n$  are all terms (constants or variables), then for any model  $M$  and an assignment function  $g$ ,  $\llbracket P(t_1, \dots, t_n) \rrbracket^{M,g} = 1$  iff  $\langle \llbracket t_1 \rrbracket^{M,g}, \dots, \llbracket t_n \rrbracket^{M,g} \rangle \in \llbracket P \rrbracket^{M,g}$

4. If  $\phi$  and  $\psi$  are formulas, then for any model  $M$  and an assignment function  $g$ ,

$$\llbracket \neg \phi \rrbracket^{M,g} = 1 \text{ iff } \llbracket \phi \rrbracket^{M,g} = 0$$

$$\llbracket \phi \wedge \psi \rrbracket^{M,g} = 1 \text{ iff } \llbracket \phi \rrbracket^{M,g} = 1 \text{ and } \llbracket \psi \rrbracket^{M,g} = 1$$

$$\llbracket \phi \vee \psi \rrbracket^{M,g} = 1 \text{ iff } \llbracket \phi \rrbracket^{M,g} = 1 \text{ or } \llbracket \psi \rrbracket^{M,g} = 1$$

$$\llbracket \phi \rightarrow \psi \rrbracket^{M,g} = 1 \text{ iff } \llbracket \phi \rrbracket^{M,g} = 0 \text{ or } \llbracket \psi \rrbracket^{M,g} = 1$$

$$\llbracket \phi \leftrightarrow \psi \rrbracket^{M,g} = 1 \text{ iff } \llbracket \phi \rrbracket^{M,g} = \llbracket \psi \rrbracket^{M,g}$$

5. If  $\phi$  is a formula, and  $v$  is a variable, then, for any model  $M$  and an assignment function  $g$ ,

$\llbracket \forall v \phi \rrbracket^{M,g} = 1$  iff  $\llbracket \phi \rrbracket^{M,g[d/v]} = 1$  for all individuals  $d \in U$ .

$\llbracket \exists v \phi \rrbracket^{M,g} = 1$  iff  $\llbracket \phi \rrbracket^{M,g[d/v]} = 1$  for some individual  $d \in U$ .

$g[d/v]$ : the variable assignment  $g'$  that is exactly like  $g$  except (maybe) for  $g(v)$ , which equals the individual  $d$ .

$$g_1 = \left[ \begin{array}{l} x_1 \rightarrow \text{John} \\ x_2 \rightarrow \text{Mary} \\ x_3 \rightarrow \text{Pete} \\ x_n \rightarrow \text{Pete} \end{array} \right] \text{ where } n \geq 4$$

$$g_1[\text{John}/x_3] = \left[ \begin{array}{l} x_1 \rightarrow \text{John} \\ x_2 \rightarrow \text{Mary} \\ x_3 \rightarrow \text{John} \\ x_n \rightarrow \text{Pete} \end{array} \right] \text{ where } n \geq 4$$

$$g_1[[\text{John}/x_3]\text{Pete}/x_1] = \left[ \begin{array}{l} x_1 \rightarrow \text{Pete} \\ x_2 \rightarrow \text{Mary} \\ x_3 \rightarrow \text{John} \\ x_n \rightarrow \text{Pete} \end{array} \right] \text{ where } n \geq 4$$

## Semantics of Predicate Logic: Trial 2 (cont.)

QUESTION: Complete the equivalences assuming:  $g(x) = \text{Mary}$ , and  $g(y) = \text{Susan}$ .

1.  $g[\text{Paul}/x](x) =$

2.  $g[\text{Paul}/x](y) =$

3.  $g[[\text{Paul}/x]\text{Susan}/x](x) =$

4.  $g[[\text{Paul}/x]\text{Susan}/x](y) =$

5.  $g[[\text{Paul}/x]\text{Susan}/y](x) =$

6.  $g[[\text{Paul}/x]\text{Susan}/y](y) =$

# Compositional Interpretation

- Assume  $M_2$ , lexical meanings and assignment function  $g'$  specified below:

$$U(\text{niverse}) = \{a, b, c\}$$

$$\llbracket \text{Allan} \rrbracket^{M_2, g'} = V(\text{Allan}) = a; \llbracket \text{Betty} \rrbracket^{M_2, g'} = V(\text{Betty}) = b$$

$$\llbracket H \rrbracket^{M_2, g'} = V(H) = \{b, c\}$$

$$\llbracket L \rrbracket^{M_2, g'} = V(L) = \{ \langle a, a \rangle, \langle a, b \rangle, \langle b, a \rangle, \langle c, b \rangle \}$$

Assignment  $g'$ :

$$\llbracket x \rrbracket^{M_2, g'} = g'(x) = a; \llbracket y \rrbracket^{M_2, g'} = g'(y) = b.$$

- Syntax

$$\begin{array}{c} \exists x [H(x)] \\ \swarrow \quad \searrow \\ \exists x \quad H(x) \\ \quad \swarrow \quad \searrow \\ \quad H \quad x \end{array}$$

- Semantic Interpretation

$$\begin{aligned} \llbracket \exists x H(x) \rrbracket^{M_2, g'} = 1 & \text{ iff for some } d \in U, \llbracket H(x) \rrbracket^{M_2, g'[d/x]} = 1 \\ & = 1 \text{ iff for some } d \in U, d \in \{b, c\} \end{aligned}$$

$$\begin{array}{c} \swarrow \quad \searrow \\ \llbracket H(x) \rrbracket^{M_2, g'} = 1 \text{ iff } g'(x) \in \{b, c\} \\ \swarrow \quad \searrow \\ \llbracket H \rrbracket^{M_2, g'} = V(H) = \{b, c\} \quad \llbracket x \rrbracket^{M_2, g'} = g'(x) \end{array}$$

If we do the same computation with respect to any other assignment  $g$ ,  $\llbracket \exists x H(x) \rrbracket^{M_2, g} = 1$ . Hence,  $\llbracket \exists x H(x) \rrbracket^{M_2} = 1$ .

## Compositional Interpretation (cont.)

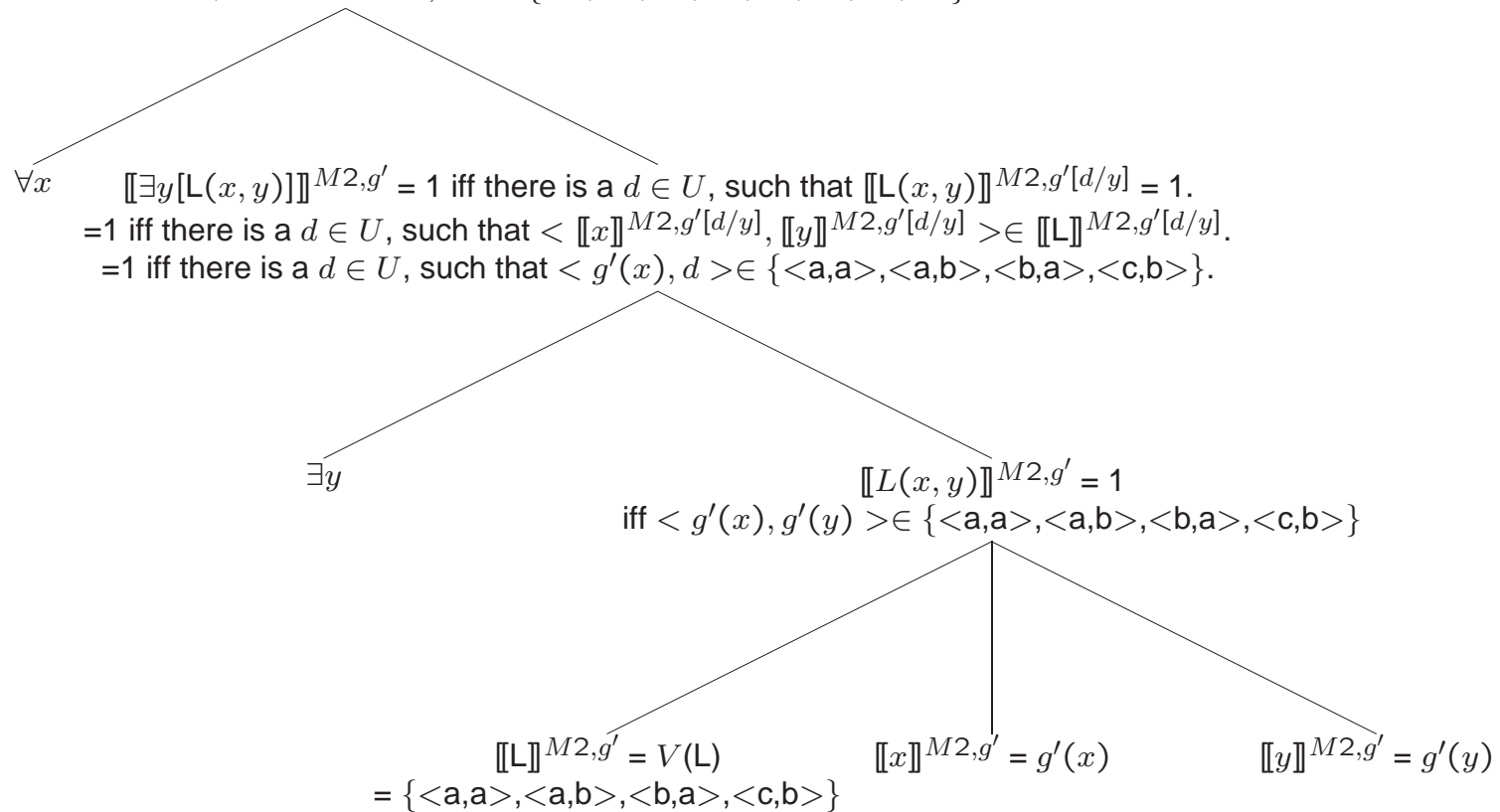
QUESTION: Draw the syntactic trees and spell out the compositional semantic interpretation of the following predicate logic formulas, against M2.

- (13)
- a.  $\forall y[H(y)]$
  - b.  $\exists y[H(\text{Allan})]$
  - c.  $\forall y[L(y, x)]$
  - d.  $\forall x\exists y[L(x, y)]$
  - e.  $\exists y\forall x[L(x, y)]$

# Compositional Interpretation: Examples

(13d)  $\forall x \exists y [L(x, y)]$

$\llbracket \forall x \exists y [L(x, y)] \rrbracket^{M2, g'} = 1$  iff for all  $d' \in U$ ,  $\llbracket \exists y [L(x, y)] \rrbracket^{M2, g' [d'/x]} = 1$ .  
 $= 1$  iff for all  $d' \in U$ , there is a  $d \in U$ , such that  $\llbracket L(x, y) \rrbracket^{M2, g' [d'/x] d/y} = 1$ .  
 $= 1$  iff for all  $d' \in U$ , there is a  $d \in U$ , such that  $\langle d', d \rangle \in \{ \langle a, a \rangle, \langle a, b \rangle, \langle b, a \rangle, \langle c, b \rangle \}$ .



$\implies \llbracket \forall x \exists y [L(x, y)] \rrbracket^{M2, g'} = 1$

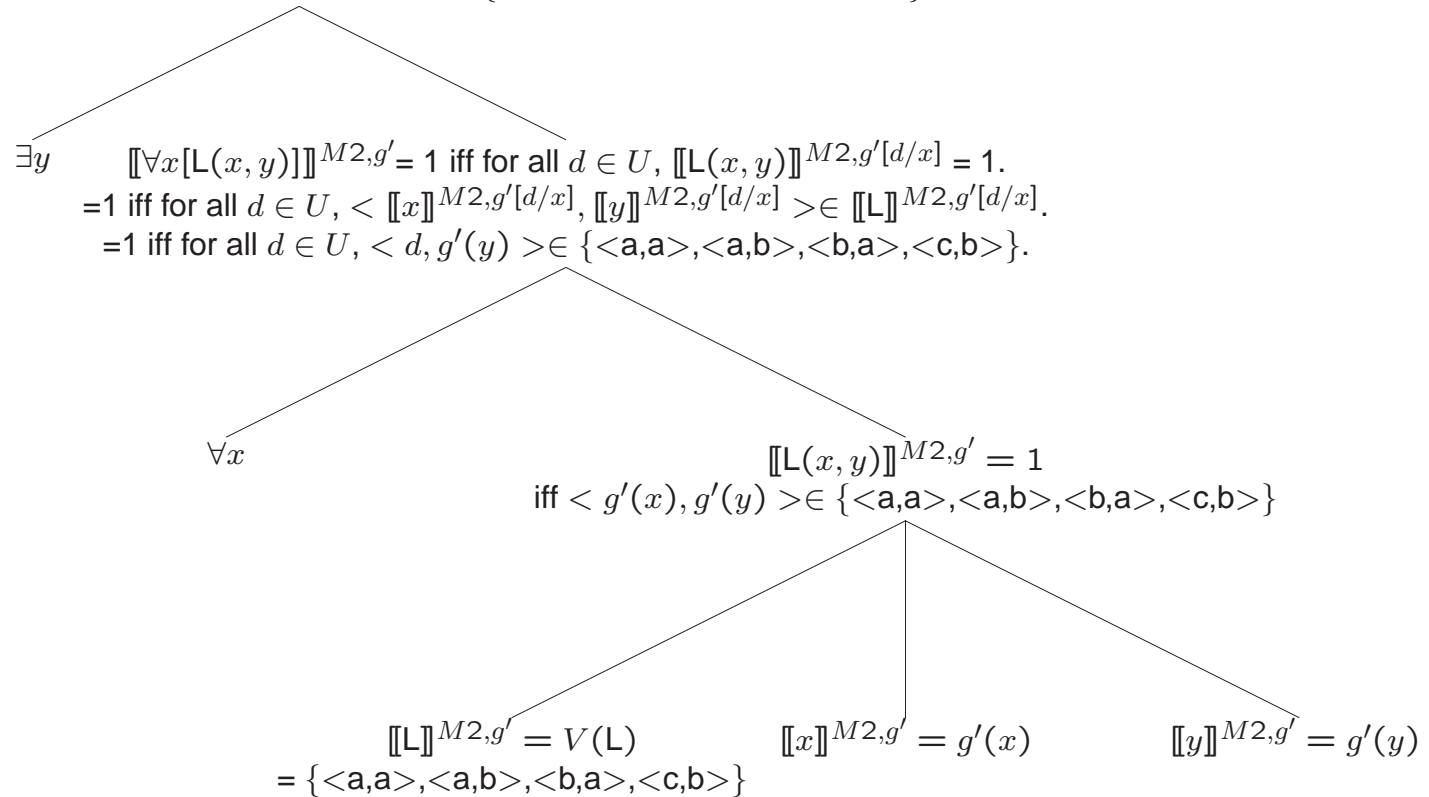
# Compositional Interpretation: Examples (cont.)

(13e)  $\exists y \forall x [L(x, y)]$

$\llbracket \exists y \forall x [L(x, y)] \rrbracket^{M2, g'} = 1$  iff there exists a  $d' \in U$ , such that  $\llbracket \forall x [L(x, y)] \rrbracket^{M2, g' [d'/y]} = 1$ .

=1 iff there is a  $d' \in U$ , such that for all  $d \in U$ ,  $\llbracket L(x, y) \rrbracket^{M2, g' [d'/y] d/x} = 1$ .

=1 iff there is a  $d' \in U$ , such that for all  $d \in U$ ,  $\langle d, d' \rangle \in \{\langle a, a \rangle, \langle a, b \rangle, \langle b, a \rangle, \langle c, b \rangle\}$ .



$\implies \llbracket \exists y \forall x [L(x, y)] \rrbracket^{M2, g'} = 0$



## Entailment, Logical Equivalence, Contradiction, Validity (cont.)

- A formula  $\phi$  entails a formula  $\psi$  iff for every model  $M$  such that  $\llbracket \phi \rrbracket = 1$ ,  $\llbracket \psi \rrbracket = 1$ .

$$\exists x \forall y [R(x, y)] \Rightarrow \forall y \exists x [R(x, y)]$$

- A formula  $\phi$  is valid iff for every model  $M$ ,  $\llbracket \phi \rrbracket = 1$ .
- A formula  $\phi$  is contradictory iff for every model  $M$ ,  $\llbracket \phi \rrbracket = 0$ .

## Entailment, Logical Equivalence, Contradiction, Validity

- A formula  $\phi$  is logically equivalent to a formula  $\psi$  iff they entail each other (i.e., they are true in exactly the same models).

$$\neg\forall x[P(x)] \Leftrightarrow \exists x[\neg P(x)]$$

$$\forall x[P(x)] \Leftrightarrow \neg\exists x[\neg P(x)]$$

$$\neg\forall x[\neg P(x)] \Leftrightarrow \exists x[P(x)]$$

$$\forall x[\neg P(x)] \Leftrightarrow \neg\exists x[P(x)]$$

# Entailment, Logical Equivalence, Contradiction, Validity (cont.)

Proof of  $\forall x[\neg P(x)] \Leftrightarrow \neg \exists x[P(x)]$ .

(i) Proof that  $\forall x[\neg P(x)]$  entails  $\neg \exists x[P(x)]$

- Assume that  $\llbracket \forall x[\neg P(x)] \rrbracket^{M,g} = 1$  for any model  $M$  and any assignment  $g$ .
- For all  $d \in U$ ,  $\llbracket \neg P(x) \rrbracket^{M,g[d/x]} = 1$ , by the semantics for  $\forall$ .
- For all  $d \in U$ ,  $\llbracket P(x) \rrbracket^{M,g[d/x]} = 0$ , by the semantics for  $\neg$ .
- There is no  $d \in U$  such that  $\llbracket P(x) \rrbracket^{M,g[d/x]} = 1$ .
- $\llbracket \exists x P(x) \rrbracket^{M,g} = 0$ , by the semantics of  $\exists$ .
- $\llbracket \neg \exists x P(x) \rrbracket^{M,g} = 1$ , by the semantics of  $\neg$ .

(ii) Proof that  $\neg \exists x[P(x)]$  entails  $\forall x[\neg P(x)]$

- Assume that  $\llbracket \neg \exists x[P(x)] \rrbracket^{M,g} = 1$  for any model  $M$  and any assignment  $g$ .
- $\llbracket \exists x[P(x)] \rrbracket^{M,g} = 0$ , by the semantics of  $\neg$ .
- There is no  $d \in U$  such that  $\llbracket P(x) \rrbracket^{M,g[d/x]} = 1$ , by the semantics of  $\exists$ .
- For all  $d \in U$ ,  $\llbracket P(x) \rrbracket^{M,g[d/x]} = 0$ .
- For all  $d \in U$ ,  $\llbracket \neg P(x) \rrbracket^{M,g[d/x]} = 1$ , by the semantics of  $\neg$ .
- $\llbracket \forall x \neg P(x) \rrbracket^{M,g} = 1$ , by the semantics of  $\forall$ .