# Truth Conditions of Quantified Sentences in English

Ling324

# Syntax of a Fragment of English (F2)

(1)
a. S $\longrightarrow$ NP VP
b. S $\longrightarrow$ S conj S
c. S $\longrightarrow$ neg S
d. VP $\longrightarrow$ V$_t$ NP
e. VP $\longrightarrow$ V$_i$
f. VP $\longrightarrow$ V$_{dt}$ NP PP[to]
g. NP $\longrightarrow$ Det N$_c$
h. NP $\longrightarrow$ N$_p$
i. PP[to] $\longrightarrow$ to NP
j. Det $\longrightarrow$ a, some, every
k. N$_p$ $\longrightarrow$ Tim, Joanie, Sarah, Natasha ...
l. N$_c$ $\longrightarrow$ book, fish, man, woman, student ...
m. V$_i$ $\longrightarrow$ is boring, is hungry, is cute ...
n. V$_t$ $\longrightarrow$ likes, hates, reads ...
o. V$_{dt}$ $\longrightarrow$ gives, shows ...
p. conj $\longrightarrow$ and, or
q. neg $\longrightarrow$ it is not the case that

(2) Rule for Quantifier Raising
[$_S$ X NP Y] $\Rightarrow$ [$_S$ NP$_i$ [$_S$ X t$_i$ Y]]

# Semantics of F2

(3) a. If $A$ is a category and $b$ is a trace,
$[\![[_A\ b]]\!]^{M,g} = g(b)$;
otherwise, $[\![[_A\ b]]\!]^{M,g} = V(b)$

b. $[\![[_A\ \text{B}]]\!]^{M,g} = [\![\text{B}]\!]^{M,g}$ for A, B of any category

c. $[\![[_{PP}\ \text{to NP}]]\!]^{M,g} = [\![\text{NP}]\!]^{M,g}$

d. $[\![[_S\ \text{NP VP}]]\!]^{M,g} = 1$ iff $[\![\text{NP}]\!]^{M,g} \in [\![\text{VP}]\!]^{M,g}$

e. $[\![[_S\ \text{S1 conj S2}]]\!]^{M,g} = [\![\text{conj}]\!]^{M,g}(<[\![\text{S1}]\!]^{M,g},[\![\text{S2}]\!]^{M,g}>)$

f. $[\![[_S\ \text{neg S}]]\!]^{M,g} = [\![\text{neg}]\!]^{M,g}([\![\text{S}]\!]^{M,g})$

g. $[\![[_{VP}\ \text{V}_t\ \text{NP}]]\!]^{M,g} = \{x: < x, [\![\text{NP}]\!]^{M,g}> \in [\![\text{V}_t]\!]^{M,g}\}$

h. $[\![[_{VP}\ \text{V}_{dt}\ \text{NP PP}]]\!]^{M,g} = \{x: < x, [\![\text{NP}]\!]^{M,g}, [\![\text{PP}]\!]^{M,g} > \in [\![\text{V}_t]\!]^{M,g}\}$

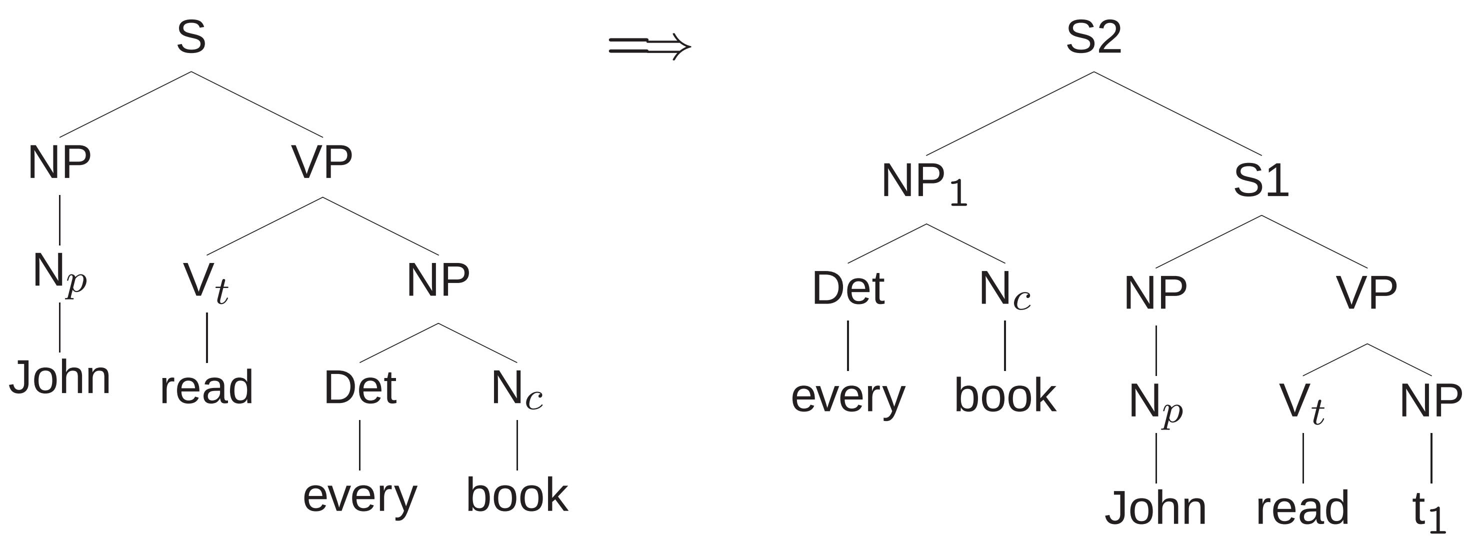
i. $[\![[[\text{every}\ b]_i\ \text{S}]]\!]^{M,g} = 1$ iff
for all $d \in U$, if $d \in [\![b]\!]^{M,g}$, then $[\![\text{S}]\!]^{M,g[d/t_i]} = 1$

j. $[\![[[\text{a}\ b]_i\ \text{S}]]\!]^{M,g} = 1$ iff
for some $d \in U$, $d \in [\![b]\!]^{M,g}$, and $[\![\text{S}]\!]^{M,g[d/t_i]} = 1$

# Compositional Semantics: One Quantifier

(4) John read every book.
$\forall x[\text{book}(x) \rightarrow \text{read}(\text{j}, x)]$

- S-structure $\Longrightarrow$ LF



$[\![t_1]\!]^{M,g} = [\![\text{NP}]\!]^{M,g} = g(t_1)$

$[\![\text{read}]\!]^{M,g} = V(\text{read}) = [\![\text{V}_t]\!]^{M,g} = \{< x, y >: x \text{ read } y \text{ in } M\}$

$[\![\text{VP}]\!]^{M,g} = \{x : x \text{ read } g(t_1) \text{ in } M\}$

$[\![\text{N}_p]\!]^{M,g} = [\![\text{NP}]\!]^{M,g} = \text{j}$

$[\![\text{S1}]\!]^{M,g} = 1$

- iff $[\![\text{NP}]\!]^{M,g} \in [\![\text{VP}]\!]^{M,g}$
- iff $\text{j} \in \{x : x \text{ read } g(t_1) \text{ in } M\}$

$[\![\text{book}]\!]^{M,g} = V(\text{book}) = [\![\text{N}_c]\!]^{M,g} = \{x : x \text{ is a book in } M\}$

$[\![\text{S2}]\!]^{M,g} = 1$

- iff for all $d \in U$, if $d \in \{x : x \text{ is a book in } M\}$ then $[\![\text{S1}]\!]^{M,g[d/t_1]} = 1$
- iff for all $d \in U$, if $d \in \{x : x \text{ is a book in } M\}$ then $\text{j} \in \{x : x \text{ read } d \text{ in } M\}$

$\Longrightarrow \forall x[\text{book}(x) \rightarrow \text{read}(\text{j}, x)]$

## Compositional Semantics: One Quantifier (cont.)

(5) John read some book.
$\exists x$[book$(x) \wedge$ read(j$, x)$]

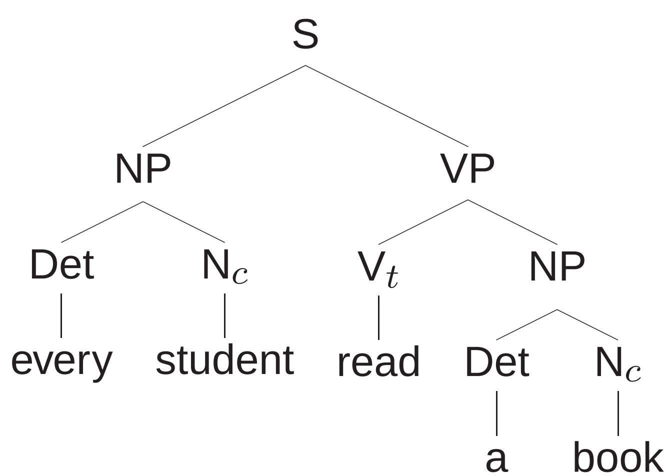
# Compositional Semantics: Two Quantifiers

(6) Every student read a book.

a. $\forall x[\text{student}(x) \rightarrow \exists y[\text{book}(y) \wedge \text{read}(x, y)]]$

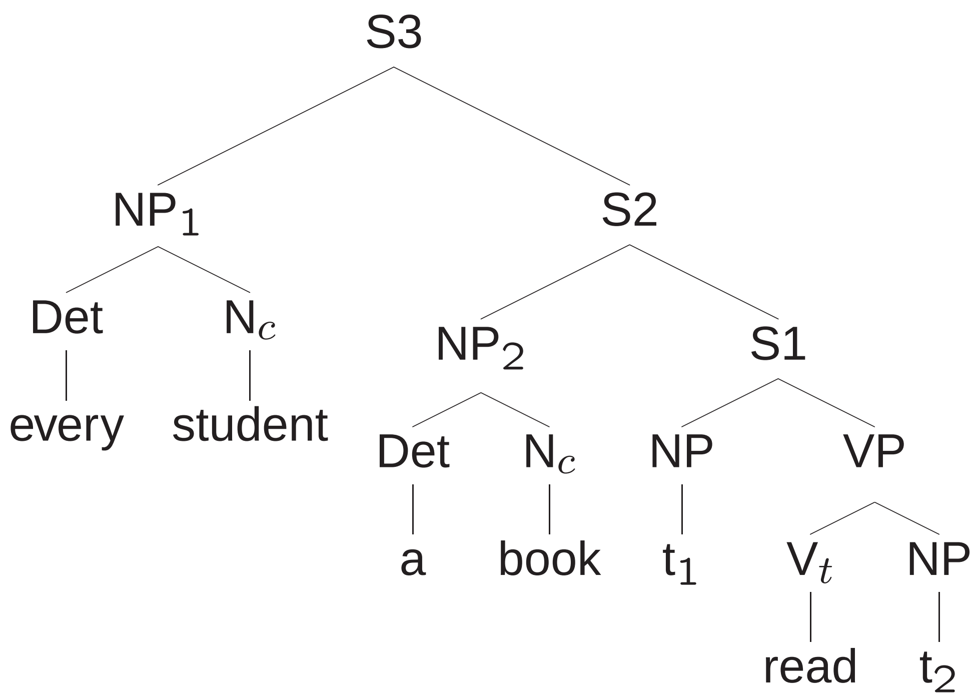
b. $\exists y[\text{book}(y) \wedge \forall x[\text{student}(x) \rightarrow \text{read}(x, y)]]$

- S-structure



# Compositional Semantics: Two Quantifiers (cont.)

- LF 1: $\forall > \exists$



$[\![t_2]\!]^{M,g} = [\![\text{NP}]\!]^{M,g} = g(t_2)$

$[\![\text{read}]\!]^{M,g} = V$(read) = $[\![\text{V}_t]\!]^{M,g}$ = $\{< x, y >$: $x$ read $y$ in $M\}$

$[\![\text{VP}]\!]^{M,g}$ = $\{x$ : $x$ read $g(t_2)$ in $M\}$

$[\![t_1]\!]^{M,g} = [\![\text{NP}]\!]^{M,g} = g(t_1)$

$[\![\text{S1}]\!]^{M,g}$ = 1

- iff $[\![\text{NP}]\!]^{M,g} \in [\![\text{VP}]\!]^{M,g}$
- iff $g(t_1) \in \{x$ : $x$ read $g(t_2)$ in $M\}$

$[\![\text{book}]\!]^{M,g} = V$(book) = $[\![\text{N}_c]\!]^{M,g}$ = $\{x$ : $x$ is a book in $M\}$

$[\![\text{S2}]\!]^{M,g}$ = 1

- iff for some $d \in U$, $d \in \{x$ : $x$ is a book in $M\}$ and $[\![\text{S1}]\!]^{M,g[d/t_2]}$ = 1
- iff for some $d \in U$, $d \in \{x$ : $x$ is a book in $M\}$ and $g(t_1) \in \{x$ : $x$ read $d$ in $M\}$

$[\![\text{student}]\!]^{M,g} = V(\text{student}) = [\![\text{N}_c]\!]^{M,g} = \{x$ : $x$ is a student in $M\}$

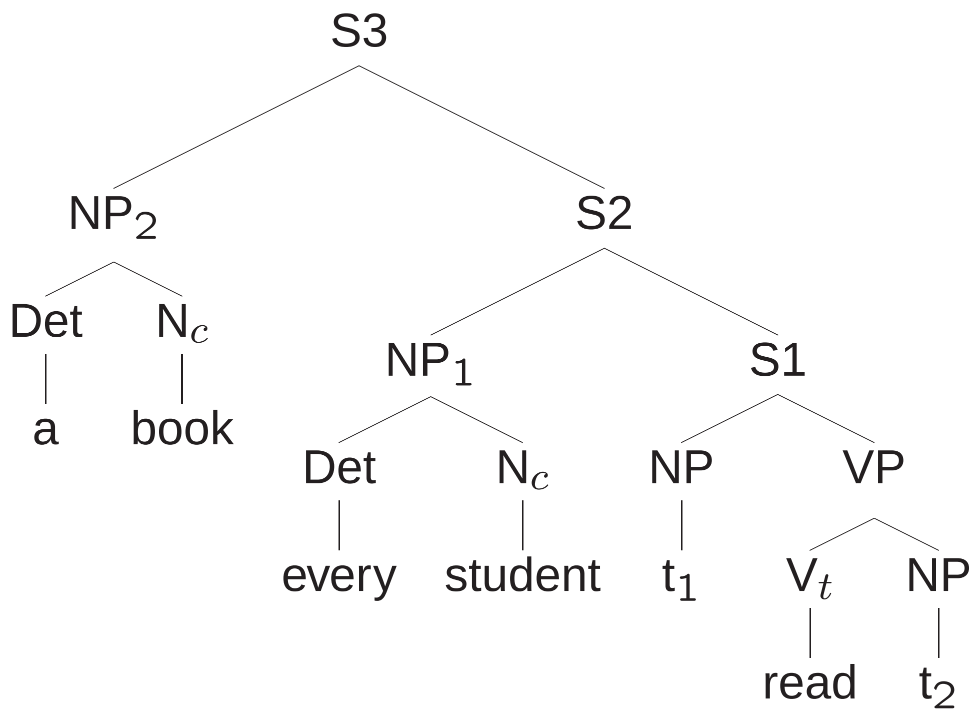
$[\![\text{S3}]\!]^{M,g}$ = 1

- iff for all $d' \in U$, if $d' \in \{x$ : $x$ is a student in $M\}$ then $[\![\text{S2}]\!]^{M,g[d'/t_1]}$ = 1
- iff for all $d' \in U$, if $d' \in \{x$ : $x$ is a student in $M\}$ then for some $d \in U$, $d \in \{x$ : $x$ is a book in $M\}$ and $[\![\text{S1}]\!]^{M,g[[d'/t_1]d/t_2]}$ = 1
- iff for all $d' \in U$, if $d' \in \{x$ : $x$ is a student in $M\}$ then for some $d \in U$, $d \in \{x$ : $x$ is a book in $M\}$ and $d' \in \{x$ : $x$ read $d$ in $M\}$

$\Longrightarrow \forall x[\text{student}(x) \rightarrow \exists y[\text{book}(y) \wedge \text{read}(x, y)]]$

# Compositional Semantics: Two Quantifiers (cont.)

- LF 2: $\exists > \forall$



$[\![t_2]\!]^{M,g} = [\![\text{NP}]\!]^{M,g} = g(t_2)$

$[\![\text{read}]\!]^{M,g} = [\![\text{V}_t]\!]^{M,g} = V$(read) = $\{< x, y >$: $x$ read $y$ in $M\}$

$[\![\text{VP}]\!]^{M,g}$ = $\{x$ : $x$ read $g(t_2)$ in $M\}$

$[\![t_1]\!]^{M,g} = [\![\text{NP}]\!]^{M,g} = g(t_1)$

$[\![\text{S1}]\!]^{M,g}$ = 1

- iff $[\![\text{NP}]\!]^{M,g} \in [\![\text{VP}]\!]^{M,g}$
- iff $g(t_1) \in \{x$ : $x$ read $g(t_2)$ in $M\}$

$[\![\text{student}]\!]^{M,g} = [\![N_c]\!]^{M,g} = V$(student) $= \{x$ : $x$ is a student in $M\}$

$[\![\text{S2}]\!]^{M,g}$ = 1

- iff for all $d \in U$, if $d \in \{x$ : $x$ is a student in $M\}$ then $[\![\text{S1}]\!]^{M,g[d/t_1]}$ = 1
- iff for all $d \in U$, if $d \in \{x$ : $x$ is a student in $M\}$ then $d \in \{x$ : $x$ read $g(t_2)$ in $M\}$

$[\![\text{book}]\!]^{M,g} = [\![\text{N}_c]\!]^{M,g} = V$(book) $= \{x$ : $x$ is a book in $M\}$

$[\![\text{S3}]\!]^{M,g}$ = 1

- iff for some $d' \in U$, $d' \in \{x$ : $x$ is a book in $M\}$ and $[\![\text{S2}]\!]^{M,g[d'/t_2]}$ = 1
- iff for some $d' \in U$, $d' \in \{x$ : $x$ is a book in $M\}$ and for all $d \in U$, if $d \in \{x$ : $x$ is a student in $M\}$ then $[\![\text{S1}]\!]^{M,g[[d'/t_2]d/t_1]}$ = 1
- iff for some $d' \in U$, $d' \in \{x$ : $x$ is a book in $M\}$ and for all $d \in U$, if $d \in \{x$ : $x$ is a student in $M\}$ then $d \in \{x$ : $x$ read $d'$ in $M\}$

$\Longrightarrow \exists y$[book$(y) \wedge \forall x$[student$(x) \rightarrow$ read$(x, y)$]]

# Pronouns: Free or Bound

- Free pronouns

(7) a. John likes **her**.
b. **He** talked to **her**.
c. **She** thinks that every student is hard-working.

- Bound pronouns

(8) a. Every boy loves **his** mother.
b. Every linguist thinks **he** is smart.
c. Every man hates **himself**.

# Pronouns: Free or Bound (cont.)

- We will interpret pronouns just as we interpreted variables in predicate logic and traces in F2. To do this, we will need to add a syntactic rule to F2 for pronouns, and modify the corresponding semantic rules to handle pronouns.
- Pronouns inherently have an unpronounced arbitrary numerical index.

(9) N$_{pro}$ $\rightarrow$ he$_n$, she$_n$, it$_n$, him$_n$, her$_n$, himself$_n$, herself$_n$, itself$_n$, for arbitrary number $n$

- Free pronouns are interpreted w.r.t. a specified assignment function $g$. Bound pronouns are interpreted w.r.t. possible modified assignment functions.

(10) If $A$ is a category and $b$ is a trace or a pronoun,
$[\![[_A\ b]]\!]^{M,g} = g(b)$;
otherwise, $[\![[_A\ b]]\!]^{M,g} = V(b)$

(11) $[\![[[\text{every } b]_i\ S]]\!]^{M,g} = 1$ iff for all $d \in U$, if $d \in [\![b]\!]^{M,g}$, then $[\![S]\!]^{M,g[d/t_i]}$ = 1, where $t_i$ is a trace or a pronoun.

(12) $[\![[[\text{a } b]_i\ S]]\!]^{M,g} = 1$ iff for some $d \in U$, $d \in [\![b]\!]^{M,g}$, and $[\![S]\!]^{M,g[d/t_i]}$ = 1, where $t_i$ is a trace or a pronoun.

# Pronouns: Free or Bound (cont.)

- Assume that $g$ is specified as follows:

  We will assume that pronouns with same index map onto the same individual regardless of their forms.

  We will also assume that a pronoun and a trace $t$ with the same index map onto the same individual.

$$
g = \begin{bmatrix}
he_1 \longrightarrow \text{Jack} \\
himself_1 \longrightarrow \text{Jack} \\
she_2 \longrightarrow \text{Yoshiko} \\
herself_2 \longrightarrow \text{Yoshiko} \\
her_2 \longrightarrow \text{Yoshiko} \\
she_3 \longrightarrow \text{Natasha} \\
herself_3 \longrightarrow \text{Natasha} \\
t_1 \longrightarrow \text{Jack} \\
t_2 \longrightarrow \text{Yoshiko} \\
t_3 \longrightarrow \text{Natasha} \\
t_4 \longrightarrow \text{Fido} \\
. \\
. \\
.
\end{bmatrix}
$$

# Pronouns: Free or Bound (cont.)

- Interpreting free pronouns

(13) a. Natasha likes $\text{her}_2$.
b. $\text{He}_1$ likes $\text{himself}_1$.

$[\![\text{S}]\!]^{M,g}$ = 1 iff $[\![\text{NP}]\!]^{M,g} \in [\![\text{VP}]\!]^{M,g}$.
- iff Natasha$' \in \{x : x$ likes $g(\text{her}_2)$ in $M\}$.
- iff Natasha$' \in \{x : x$ likes Yoshiko in $M\}$

$[\![\text{NP}]\!]^{M,g}$ = Natasha$'$

$[\![\text{N}_p]\!]^{M,g}$ = Natasha$'$

$[\![\text{Natasha}]\!]^{M,g} = V(\text{Natasha})$ = Natasha$'$

$[\![\text{VP}]\!]^{M,g} = \{x : x$ likes $g(\text{her}_2)$ in $M\}$

$[\![\text{V}_t]\!]^{M,g} = \{< x, y >: x$ likes $y$ in $M\}$

$[\![\text{likes}]\!]^{M,g} = V(\text{likes}) = \{< x, y >: x$ likes $y$ in $M\}$

$[\![\text{NP}]\!]^{M,g} = g(\text{her}_2)$

$[\![\text{N}_{pro}]\!]^{M,g} = g(\text{her}_2)$

$[\![\text{her}_2]\!]^{M,g} = g(\text{her}_2)$

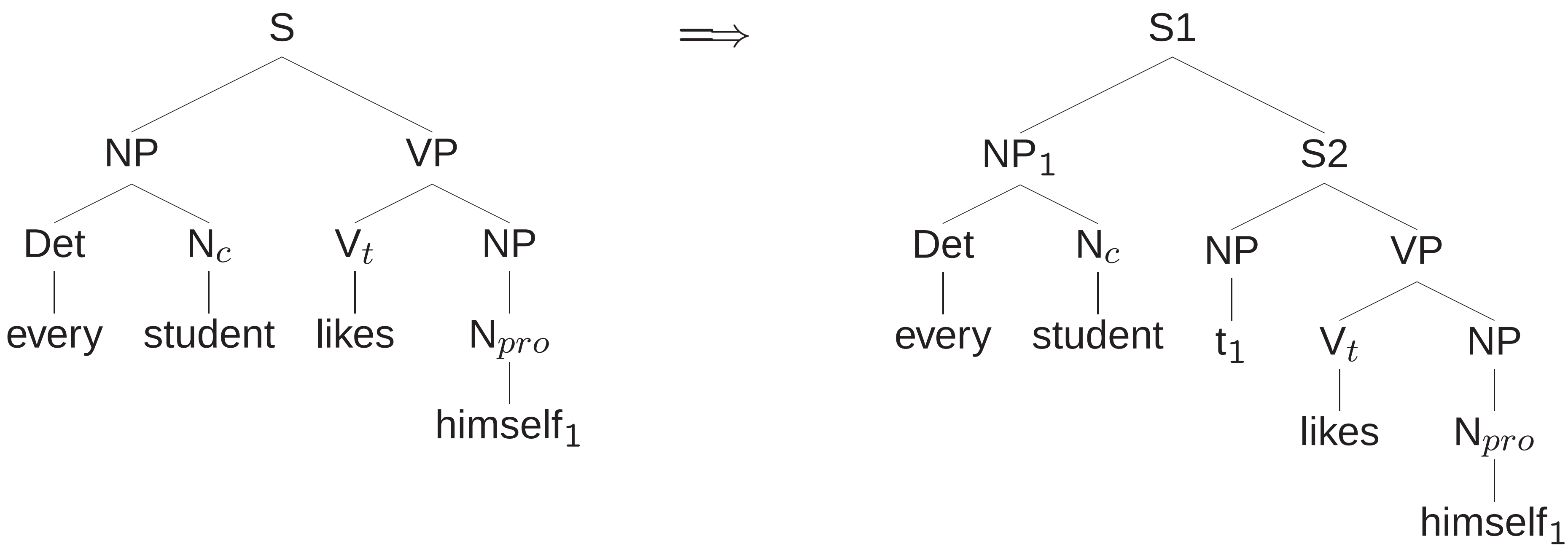
$\Rightarrow$ likes(Natasha, Yoshiko)

# Pronouns: Free or Bound (cont.)

- Interpreting pronouns that are bound by quantifiers

(14) a. Every student likes himself$_1$.

b. Some girl hit herself$_2$.



$[\![\text{himself}_1]\!]^{M,g} = g(\text{himself}_1)$

$[\![\text{N}_{pro}]\!]^{M,g}$ = $[\![\text{NP}]\!]^{M,g}$ = $g(\text{himself}_1)$

$[\![\text{likes}]\!]^{M,g} = V(\text{likes})$ = $[\![\text{V}_t]\!]^{M,g}$ = $\{< x, y >: x$ likes $y$ in $M\}$

$[\![\text{VP}]\!]^{M,g}$ = $\{x : x$ likes $g(\text{himself}_1)$ in $M\}$

$[\![\text{t}_1]\!]^{M,g} = [\![\text{NP}]\!]^{M,g}$ = $g(t_1)$

$[\![\text{S2}]\!]^{M,g}$ = 1 iff $[\![\text{NP}]\!]^{M,g} \in [\![\text{VP}]\!]^{M,g}$ iff $g(t_1) \in \{x : x$ likes $g(\text{himself}_1)$ in $M\}$

$[\![\text{student}]\!]^{M,g} = V(\text{student})$ = $[\![\text{N}_c]\!]^{M,g}$ = $\{x : x$ is a student in $M\}$

$[\![\text{S1}]\!]^{M,g}$ = 1

- iff for all $d \in U$, if $d \in \{x : x$ is a student in $M\}$, then $[\![\text{S2}]\!]^{M,g[d/t_1]}$ = 1
- iff for all $d \in U$, if $d \in \{x : x$ is a student in $M\}$, then $d \in \{x : x$ likes $d$ in $M\}$

$\Longrightarrow \forall x[\text{student}(x) \rightarrow \text{likes}(x, x)]$