Relative Clauses, Variables, Variable Binding (Part II)

Heim and Kratzer

Chapter 5

5.3 Multiple Variables

5.3.1 Adding “such that” relatives

- Quine also discussed “such that” relatives, which contain “such” and a pronoun instead of a relative pronoun and a trace, e.g. ”the book such that Joe bought it”.

- We just need to generalize our Traces Rule to cover pronouns, and to rewrite PA so that it treats “such” like a relative pronoun. The “that” is presumably the semantically vacuous complementizer again.

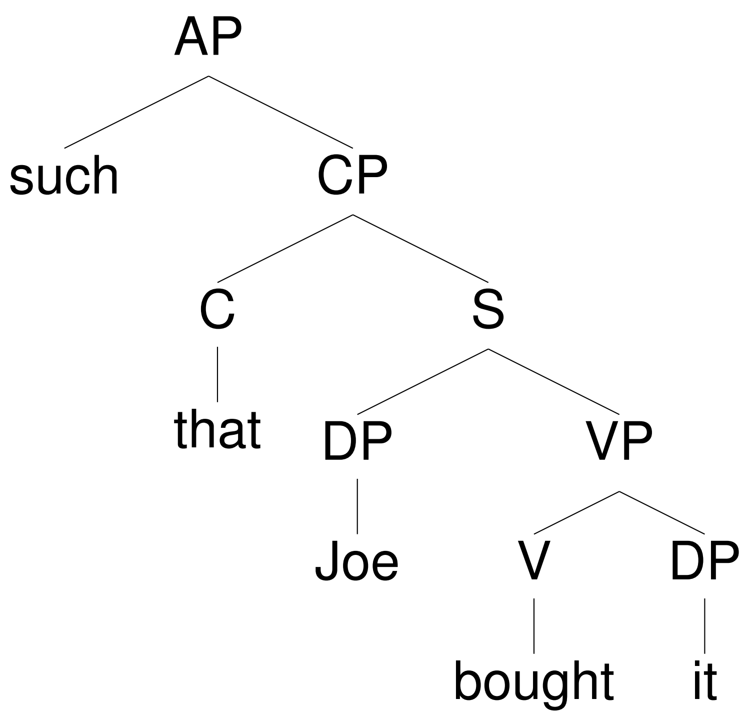
(1) *Pronoun Rule* (new addition)
If α is a pronoun, then for any assignment a $\in$ D (= D_e), $[\![\alpha]\!]^{a}$ = a.

(2) *Predicate Abstraction* (revised)
If α is a branching node and β and γ its daughters, where β is a relative pronoun or β = “such”, then $[\![\alpha]\!] = \lambda x \in D \, . \, [\![\gamma]\!]^{x}$.

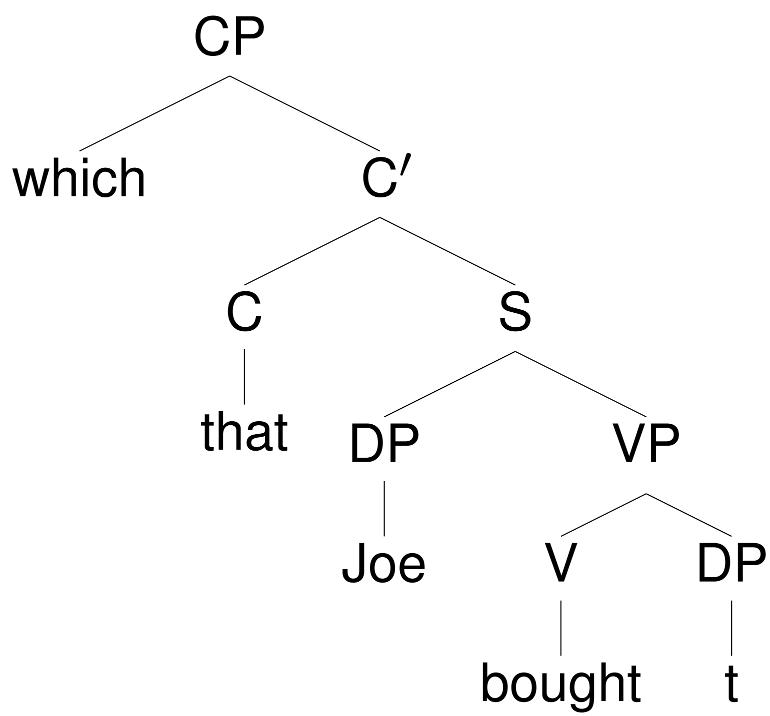
5.3.1 Adding “such that” relatives (cont.)

- Now we calculate a semantic value for (3) and prove that (3) and (4) have the same denotation.

(3)



(4)



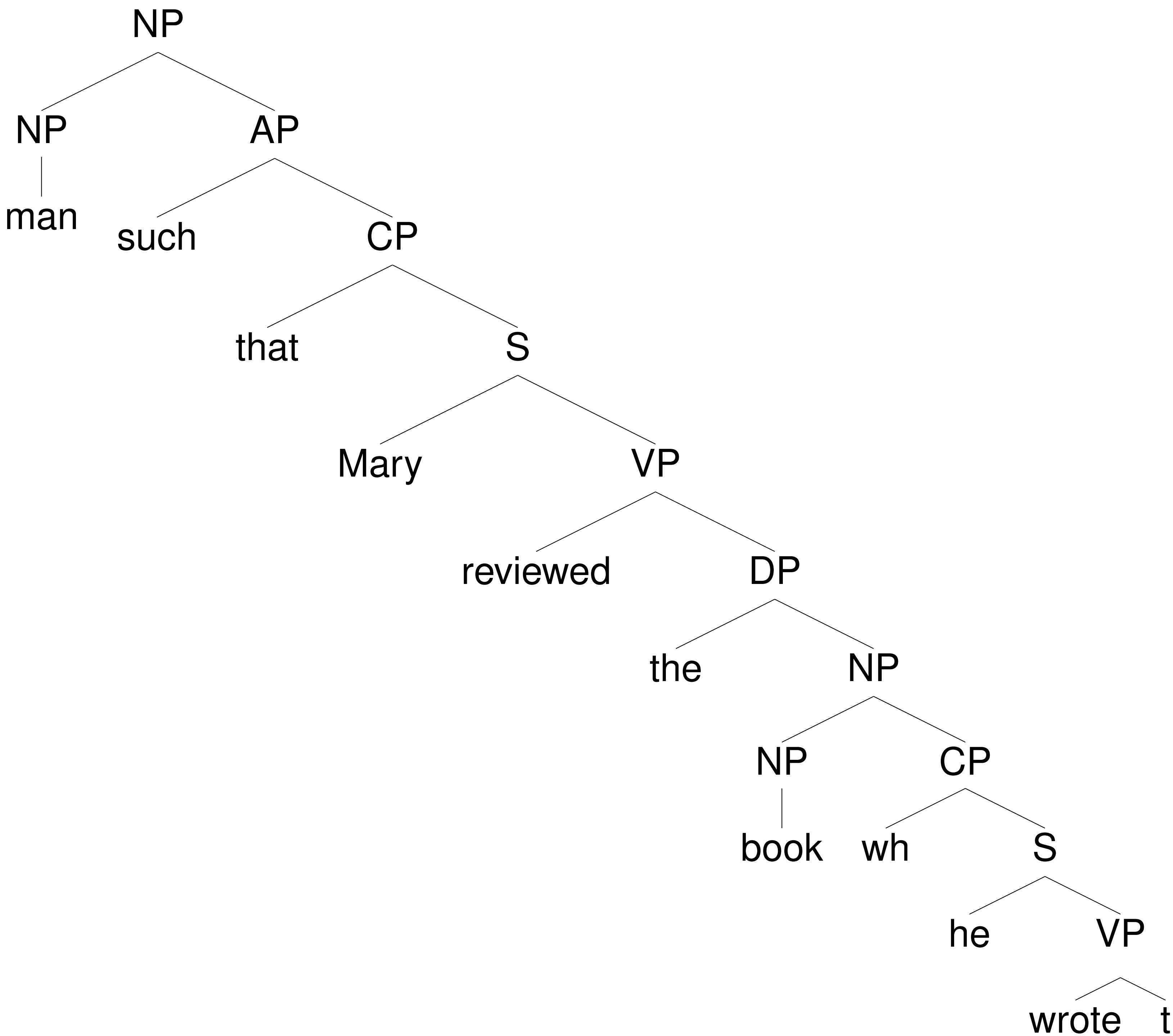
5.3.2 A problem with nested relatives

- One reason for introducing “such that” relatives is that it allows us to look at certain examples where one relative clause is nested inside another, as in (5):

(5) **man such that Mary reviewed the book that he wrote**

- We encounter no problems of interpretability here, but the interpretation we derive under our current assumptions is very wrong. How?

(5’)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5.3.2 A problem with nested relatives (cont.)

- If there isn't a unique book that wrote itself, then (5) has no denotation under any assignment.

- If Mary reviewed the unique book that wrote itself, then $[\![(5)]\!] = [\![\textbf{man}]\!]$.

- If Mary didn't review the unique book that wrote itself, then $[\![(5)]\!] = \lambda x \in D \,.\, 0$.

- If we can express the strange predicted meaning in English at all, we have to use quite a different phrase, namely (6).

(6) **man such that Mary reviewed the book which wrote itself**

- In order to rectify this inadequacy, we will have to reconsider the syntax as well as the semantics of variables.

5.3.3 Amending the syntax: co-indexing

- The problem is that the semantics pays no attention to which trace/pronoun is related to which *wh*-word/“such”.

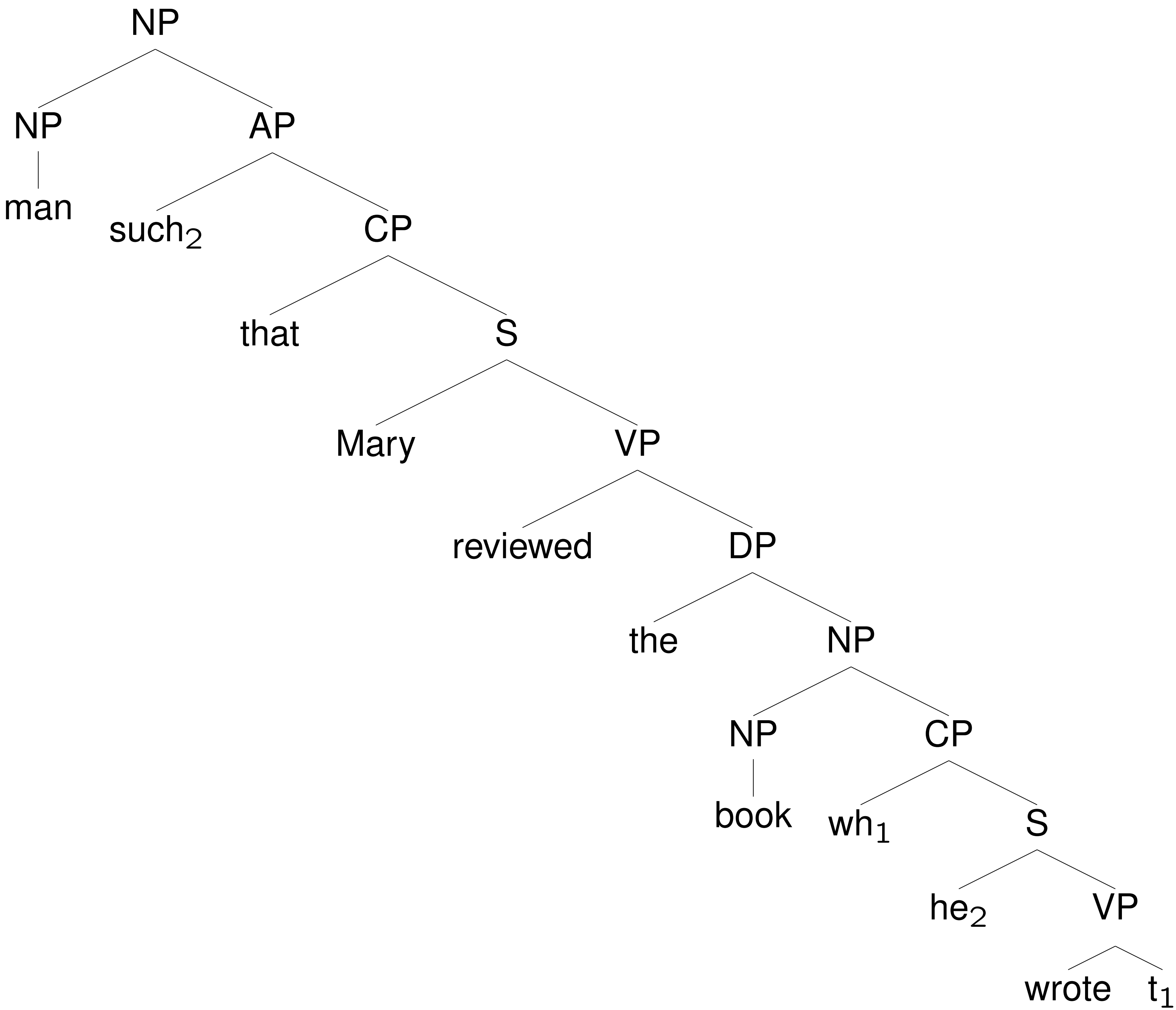
- This suggests that part of the solution might be to employ syntactic representations which explicitly encode which trace or pronoun belongs with which relative pronoun or “such”.

- The standard notational device for this purpose is co-indexing by means of numerical subscripts, as in (5”) below.

- There are infinitely many other indexings that would work, a point which we will return to below, but with this indexing, it receives intuitively the right interpretation.

- Now we must amend the semantic rules so that they can “see” the indices.

(5”)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5.3.4 Amending the semantics

- In order to write an appropriately index-sensitive set of semantic rules, we must first redefine what we mean by an "assignment".

- So far, an assignment was just an element of D (=D_e). This worked for examples in which each variable was contained in at most one relative clause, but now we need a richer notion of assignment, as in (7). Examples are shown in (8).

(7) A *(variable) assignment* is a partial function from |N (the set of natural numbers) into D..

(8) $\begin{bmatrix} 1 \to \text{John} \\ 2 \to \text{Mary} \end{bmatrix}$ $\begin{bmatrix} 1 \to \text{John} \\ 2 \to \text{John} \end{bmatrix}$ $\begin{bmatrix} 1 \to \text{John} \end{bmatrix}$

$\begin{bmatrix} 2 \to \text{John} \\ 5 \to \text{Mary} \\ 7 \to \text{Ann} \end{bmatrix}$

5.3.4 Amending the semantics (cont.)

- Given definition (7), $\emptyset$ (the empty set) comes out as an assignment too. $\emptyset$ is the (only) function with $\emptyset$ as its domain.

- Any function from $\emptyset$ into D would have to be subset of $\emptyset \times$ D, the Cartesian product of $\emptyset$ and D.

- Since $\emptyset \times$ D = $\emptyset$, the only subset of $\emptyset \times$ D is $\emptyset$.

- Since $\emptyset \subseteq |$N, $\emptyset$ qualifies as a partial function from |N into D, hence as an assignment.

5.3.4 Amending the semantics (cont.)

- Since assignments are now functions, it makes sense to speak of their "domain". We write "dom(a)" to abbreviate "the domain of the assignment a".

- Assignments now assign potentially different individuals to different numerical indices. We now replace our old rules for (unindexed) traces and pronouns by a new rule (9) that is sensitive to the index. (9) makes the predictions shown in (10).

(9) *Traces and Pronouns Rule*
If α is a pronoun or trace, a is a variable assignment, and i $\in$ dom(a), then $[\![\alpha_i]\!]^{\text{a}}$ = a(i).

(10) $[\![\textbf{he}_2]\!]^{[1\rightarrow\text{Sue},2\rightarrow\text{Joe}]} = \begin{bmatrix} 1 \rightarrow \text{Sue} \\ 2 \rightarrow \text{Joe} \end{bmatrix}(2) = \text{Joe}$

$[\![\textbf{t}_1]\!]^{[1\rightarrow\text{Sue},2\rightarrow\text{Joe}]} = \begin{bmatrix} 1 \rightarrow \text{Sue} \\ 2 \rightarrow \text{Joe} \end{bmatrix}(1) = \text{Sue}$

5.3.4 Amending the semantics (cont.)

- (9) also implies that a given trace or pronoun will not have a well-defined semantic value under just any assignment.

- For instance **he**$_2$ is not in the domain of $[\![\ \ ]\!]^{[1 \to \text{Joe}]}$, $[\![\ \ ]\!]^{[3 \to \text{Joe}]}$, or $[\![\ \ ]\!]^{\emptyset}$,

- With regard to $[\![\ \ ]\!]^{\emptyset}$, *no* pronoun or trace is in its domain. Are any expressions at all? Yes, but only those which are in the domain of $[\![\ \ ]\!]^{\text{a}}$ for *every* a, e.g. the lexical items.

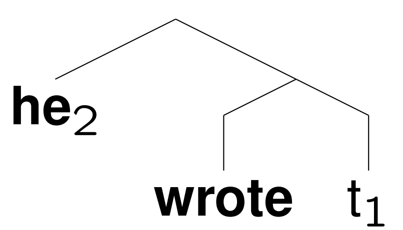
- Our earlier convention stated that whenever an expression is in the domain of $[\![\ \ ]\!]$, it is also in the domain of $[\![\ \ ]\!]^{\text{a}}$ for all a. We will now think of $[\![\ \ ]\!]$ as simply an abbreviation for $[\![\ \ ]\!]^{\emptyset}$,

5.3.4 Amending the semantics (cont.)

(11) For any tree α, $[\![\alpha]\!]$= $[\![\alpha]\!]^{\emptyset}$.

- To have a semantic value *simpliciter* means nothing more and nothing less than to have a semantic value under the empty assignment.

- Our rules can stay the same, but the meaning of “assignment” has now changed. We still pass assignment dependency up the tree. We can now prove (12):

(12) $[\![$ 

$]\!]^{[1\rightarrow \text{“Barriers”}, 2\rightarrow \text{Joe}]}$ = 1 iff Joe wrote “Barriers”

5.3.4 Amending the semantics (cont.)

- Before we modify the Predicate Abstraction Rule, we must define a further technical concept, that of a modified (variable) assignment function.

(13) Let a be an assignment, $i \in |N$, and $x \in D$. Then $a^{x/i}$ (read "a modified so as to assign x to i") is the unique assignment which fulfills the following conditions:
(i) $dom(a^{x/i}) = dom(a) \cup \{i\}$,
(ii) $a^{x/i}(i) = x$, and
(iii) for every $j \in dom(a^{x/i})$ such that $j \neq i$: $a^{x/i}(j) = a(j)$.

- Clause (i) adds i to the domain of the assignment function if it is not there already. Clause (ii) states that i now maps to x. Suppose a(5) = Mary. Then $a^{Ann/5}$ = Ann, Clause (iii) says that everything not changed stays the same.

5.3.4 Amending the semantics (cont.)

- Modified assignments can be modified further, so the notation can be iterated, as in (15):

(15) $[\,[1 \rightarrow \text{John}]^{\text{Mary/2}}]^{\text{Sue/1}} = \begin{bmatrix} 1 \rightarrow \text{John} \\ 2 \rightarrow \text{Mary} \end{bmatrix}^{\text{Sue/1}} = \begin{bmatrix} 1 \rightarrow \text{Sue} \\ 2 \rightarrow \text{Mary} \end{bmatrix}$

- In practice, we will write the following, but it must be understood that this refers to the result of first modifying [1 → John] so as to assign Mary to 1, and then modifying *the result of that* so as to assign Sue to 1:

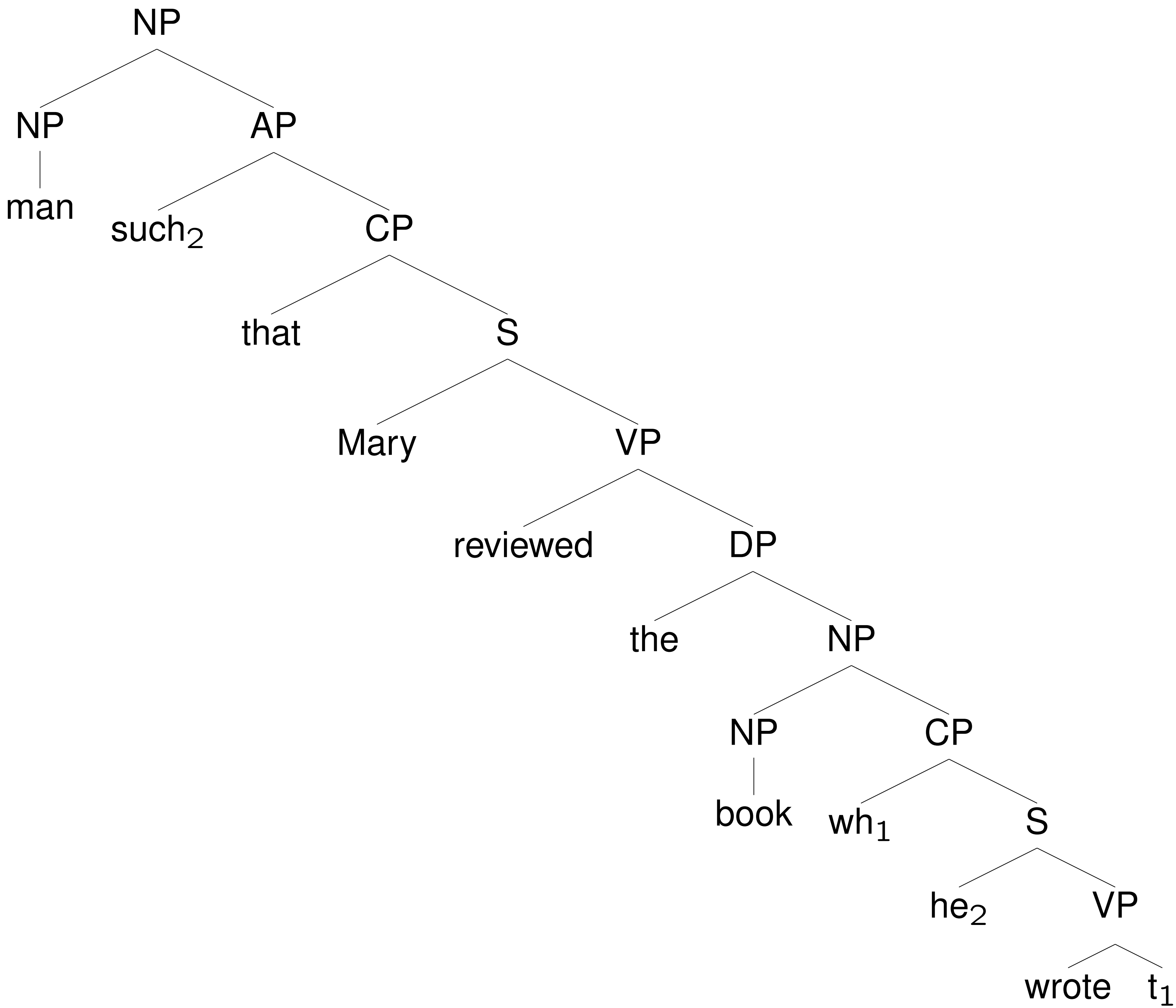
$[1 \rightarrow \text{John}]^{[\text{Mary/2, Sue/1}]}$

5.3.4 Amending the semantics (cont.)

- Reference to modified assignments allows us to manipulate the values of certain designated variables (i.e. traces or pronouns) while leaving the values of all other variables intact. The usefulness of this device can be seen when we apply our new rule of Predicate Abstraction:

(16) *Predicate Abstraction Rule* (PA)
If α is a branching node whose daughters are β_i and γ, where β is a relative pronoun or "such", and i $\in$ |N, then for any variable assignment a, $[\![\alpha]\!]^{a}$ = λx $\in$ D . $[\![\gamma]\!]^{a^{x/i}}$.

- Every application of PA targets one variable (identified by its index) in an expression γ, and defines a function of type $< e, t >$ by manipulating the value assigned to that particular variable. Let's calculate for (5").


NP
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book
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S
he2
VP
wrote
t1

5.3.4 Amending the semantics (cont.)

$[\![$**such**$_2$ **that Mary reviewed the book wh**$_1$ **he**$_2$ **wrote t**$_1]\!]$
= (by convention (11))

$[\![$**such**$_2$ **that Mary reviewed the book wh**$_1$ **he**$_2$ **wrote t**$_1]\!]^{\emptyset}$
= (by PA)

λx $\in$ D . $[\![$**that Mary reviewed the book wh**$_1$ **he**$_2$ **wrote t**$_1]\!]^{\emptyset^{x/2}}$
= (by definition (13) (assignment modification))

λx $\in$ D . $[\![$**that Mary reviewed the book wh**$_1$ **he**$_2$ **wrote t**$_1]\!]^{[2 \rightarrow x]}$
= (by vacuity of **that** and three applications of FA)

λx $\in$ D . $[\![$**reviewed**$]\!]^{[2 \rightarrow x]}$
($[\![$**the**$]\!]^{[2 \rightarrow x]}$($[\![$**book wh**$_1$ **he**$_2$ **wrote t**$_1]\!]^{[2 \rightarrow x]}$))($[\![$**Mary**$]\!]^{[2 \rightarrow x]}$)
= by convention (9) of section 5.2 and lexical entries for **review, the, Mary**)

λx $\in$ D . Mary reviewed the unique y such that
$[\![$**book wh**$_1$ **he**$_2$ **wrote t**$_1]\!]^{[2 \to x]}$(y) = 1
= (by PM and lexical entry for **book**)

λx $\in$ D . Mary reviewed the unique y such that y is a book and
$[\![$**wh**$_1$ **he**$_2$ **wrote t**$_1]\!]^{[2 \to x]}$(y) = 1
= (by PA)

λx $\in$ D . Mary reviewed the unique y such that y is a book and
λz $\in$ D. $[\![$**he**$_2$ **wrote t**$_1]\!]^{[2 \to x]^{z/1}}$ (y) = 1
= (by definition (13))

λx $\in$ D . Mary reviewed the unique y such that y is a book and
λz $\in$ D. $[\![$**he**$_2$ **wrote t**$_1]\!]^{[2 \to x,\ 1 \to z]}$(y) = 1
= (by two applications of FA)

λx $\in$ D . Mary reviewed the unique y such that y is a book and
λz $\in$ D. $[\![$ **wrote**$]\!]^{[2 \to x,\ 1 \to z]}$ ($[\![$**t**$_1]\!]^{[2 \to x,\ 1 \to z]}$)
($[\![$**he**$_2]\!]^{[2 \to x,\ 1 \to z]}$) (y) = 1

= (by convention (9), and traces and pronouns rule)

λx $\in$ D . Mary reviewed the unique y such that y is a book and
λz $\in$ D. ⟦ **wrote**⟧ (z) (x) (y) = 1
= (by lexical entry for **wrote**)

λx $\in$ D . Mary reviewed the unique y such that y is a book and
λz $\in$ D. [λw $\in$ D. [λv $\in$ D. v wrote w]] (z) (x) (y) = 1
= (by lambda conversion)

λx $\in$ D . Mary reviewed the unique y such that y is a book and
λz $\in$ D. x wrote z] (y) = 1
= (by lambda conversion)

λx $\in$ D . Mary reviewed the unique y such that y is a book and x wrote y