Executing the Fregean Program (Part II)

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Chapter 2.5

Defining functions in the λ-notation

- So far we have defined functions according to the following format:

  (1) F_{+1} = f : |N $\rightarrow$ |N
  For every x $\in$ |N, f(x) = x + 1

- We will now switch to a format like:

  (2) F_{+1} = [λx : x $\in$ |N . x + 1]

  "The (smallest) function which maps every x such that x is in |N to x + 1."

- Here is a schema for constructing λ-terms:

  (3) $[\lambda\alpha : \phi \,.\, \gamma]$

  α is the *argument variable*, ϕ the *domain condition* and γ the *value description*

Defining functions in the λ-notation (cont.)

- This is to be read as follows:

  (3') the smallest function which maps every α such that ϕ to γ

  We will typically omit "smallest", but it is always understood, and is strictly speaking necessary. Why?

- Like other function terms, a λ-term can be followed by argument terms:

  [λx : x $\in$ |N . x + 1](5) = 5 + 1 = 6.

Defining functions in the λ-notation (cont.)

- the λ-notation just introduced doesn't work for functions which involve a distinction between two or more cases, such as in (5) and crucially (6):

  (5) G = f : $\mathbb{N} \rightarrow \mathbb{N}$

  For every x $\in \mathbb{N}$, f(x) = 2, if x is even, and f(x) = 1 otherwise

  (6) $[\![$**smoke**$]\!]$:= f : D $\rightarrow \{0, 1\}$

  For all x $\in$ D, f(x) = 1 iff x smokes.

- Why not?

Defining functions in the λ-notation (cont.)

- We need a new way of reading our schema $[\lambda\alpha : \phi \, . \, \gamma]$ when γ is a sentence instead of a noun phrase.

- Namely, "the function which maps every α such that ϕ to 1 if γ and to 0 otherwise"

- Now we can abbreviate our lexical item:

  (7) ⟦**smoke**⟧ := [λx : x $\in$ D . x smokes]

- This reads "let ⟦**smoke**⟧ be the function which maps every x in D to 1 if x smokes, and to 0 otherwise"

Defining functions in the λ-notation (cont.)

- We will write:

  (8') $[\![$**smoke**$]\!]$(Ann) = [λx : x $\in$ D . x smokes](Ann) = 1 iff Ann smokes.

- We stipulate:

  (9) Read "[$\lambda\alpha$: ϕ . γ]" as either (i) or (ii) whichever makes sense.
  (i) "the function which maps every α such that ϕ to γ"
  (ii) "the function which maps every α such that ϕ to 1, if γ, and to 0 otherwise"

Defining functions in the λ-notation (cont.)

- The conciseness of the λ-notation makes it especially handy for the description of function-valued functions, such as transitive verb lexical entries:

  (10) $[\![$**love**$]\!]$:= [λx : x $\in$ D . [λy : y $\in$ D . y loves x]]

- Which clauses of (9) do we read this by?

- It is also useful for functions that take functions as arguments:

  (11) [λf : f $\in$ $D_{<e,t>}$. there is some x $\in$ D_e such that f(x) = 1]

Defining functions in the λ-notation (cont.)

- Apply this to (12) and we get (13):

(12) [λy : y $\in$ D_e . y stinks]

(13) [λf : f $\in$ $D_{<e,t>}$. there is some x $\in$ D_e such that f(x) = 1](λy : y $\in$ D_e . y stinks]) = 1
iff there is some x $\in$ D_e such that [λy : y $\in$ D_e . y stinks](x) = 1
iff there is some x $\in$ D_e such that x stinks.

Defining functions in the λ-notation (cont.)

- We will sometimes abbreviate by omitting the outermost brackets when the λ-expression is not embedded in a larger expression.

- We will contract the domain condition when it is of the form "$\alpha \in \beta$", as shown in (14a).

- We will leave out the domain condition altogether when it happens to be "x $\in$ D", as shown in (14b).

  (14a) $[\![$**love**$]\!]$:= λx $\in$ D . [λy $\in$ D . y loves x]
  (14b) $[\![$**love**$]\!]$:= λx . [λy . y loves x]

Defining functions in the λ-notation (cont.)

- When λ-terms are followed by argument terms, you have to be careful about omitting crucial parentheses. (15a) is not legitimate because it is ambiguous between (15b) and (15c), which are not equivalent.

(15a) λx $\in$ D. [λy $\in$ D . y loves x](Sue)
(15b) [λx $\in$ D. [λy $\in$ D . y loves x](Sue)] = λx $\in$ D . Sue loves x
(15c) [λx $\in$ D. [λy $\in$ D . y loves x]](Sue) = λy $\in$ D . y loves Sue

Defining functions in the λ-notation (cont.)

- The abstraction notation for sets and the λ-notation for functions are closely connected.

- For example, the characteristic function of the set $\{x \in |N : x \neq 0\}$ is $[\lambda x \in |N . x \neq 0]$

- See the textbook on p 39 for practicing the correspondence between set talk and function talk.