Quantifiers: Their Semantic Type

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Chapter 6

- We have treated proper names, definite descriptions, pronouns and traces as denoting individuals, elements of D_e.

- There are many other kinds of DPs: those made up of determiners like “this”, ‘that”, “every”, “no”, “many”, “few”, “most”; and yet other types like “only John”.

- What denotations should we assign? What types of denotations?

- We will show that at least some DPs do not denote individuals, and also do not denote sets of individuals (or their characteristic functions).

6.1 Problems with individuals as DP-denotations

- Individuals work well for the types of DPs we have dealt with so far.

- individuals would work for demonstrative phrases like “this book” and for free pronouns like “he” and “I” if we allow their denotations to vary from occasion to occasion.

- “I” denotes on each occasion the individual who utters it.

- But individuals don’t seem to be the right denotations for many DPs.

6.1.1 Predictions about truth-conditions and entailment patterns

- Naive intuition might tell us that “only John” should denote John, but this isn’t right: If ⟦**only John**⟧ = John, then ⟦**only John left**⟧ = 1 iff John left. But if John and Sam left, this sentence is false, not true as predicted.

- Naive intuition might tell us that “no man” denotes nothing, but we’ll see that this is wrong too. We need semantic tests to help us decide what types of denotations various DPs have.

- Some tests will tell us that a *particular* assignment of a denotation won’t work, and sometimes a test will tell us that *no denotation of a certain type* will do.

DPs that fail to validate subset-superset inferences

- The following inference is intuitively valid.

(1) **John came yesterday morning.**
∴ **John came yesterday.**

- It is *predicted* to be valid by any semantics that implies these three assumptions:

(i) $[\![\textbf{John}]\!] \in D_e$.
(ii) $[\![\textbf{came yesterday morning}]\!] \subseteq [\![\textbf{came yesterday}]\!]$
(iii) A sentence whose subject denotes an individual is true iff that individual is a member of the set denoted by the VP.

- How can we prove this? Note that no concrete assumption about *which* element of D, **John** denotes is needed.

DPs that fail to validate subset-superset inferences

- But the following inference is invalid:

(2) **At most one letter came yesterday morning.**
∴ **At most one letter came yesterday.**

- If we want to maintain (ii) and (iii), it follows that ⟦**at most one letter**⟧ $\notin \mathrm{D}_e$.

- Other DPs that fail this inference scheme are DPs with the determiners “no”, “few”, “less than n”, “at most n”, “exactly n”.

- For all such DPs, we thus have a strong reason to assume that their denotations are not of type e.

DPs that fail the Law of Contradiction

- If you choose two VPs with disjoint extensions and combine first one, then other, with a given proper name, you get two sentences that contradict each other. The following sentence is contradictory:

(4) **Mount Rainier is on this side of the border, and Mount Rainier is on the other side of the border.**

- This can be proven from the following assumptions:

(i) $[\![$**Mount Rainier**$]\!] \in \mathrm{D}_e$.
(ii) $[\![$**be on this side of the border**$]\!] \cap [\![$**be on the other side of the border**$]\!] = \emptyset$.
(iii) (composition rule for subject + VP, same as above)
(iv) standard analysis of **and**.

DPs that fail the Law of Contradiction

- But many such sentences are not contradictory, e.g.:

(5) **More than two mountains are on this side of the border, and more than two mountains are on the other side of the border.**

- We conclude that **more than two mountains** does not denote anything in D_e.

- This argument extends to “a mountain”, “n mountains”, “no mountain”, and lots of others.

DPs that fail the Law of of Excluded Middle

- Again we choose two sentences whose subjects are identical but whose VPs differ. This time we choose two VPs the union of whose extensions exhausts everything there is, and we coordinate the two sentences by "or". The result is a tautology.

(6) **I am over 30 years old, or I am under 40 years old.**

- The proof requires the following assumptions:

(i) $[\![\textbf{I}]\!] \in \mathrm{D}_e$.
(ii) $[\![\textbf{be over 30 years old}]\!] \cup [\![\textbf{be under 40 years old}]\!] = \mathrm{D}$
(iii) (as above)
(iv) standard analysis of **or**.

DPs that fail the Law of of Excluded Middle

- The following sentence is not a tautology, so the subject is not of type e.

(7) **Every woman in this room is over 30 years old, or every woman in this room is under 40 years old**

- There are other such systematic differences in the entailments, tautologies,and contradictions that we get for proper names on the one hand and for a lot of nondefinite DPs on the other hand.

- We now turn to an argument of a somewhat different kind.

6.1.2 Predictions about ambiguity and the effects of syntactic reorganization

- Sometimes we can rearrange the words in a sentence and get the same truth conditions, as in topicalization and the *such that* construction:

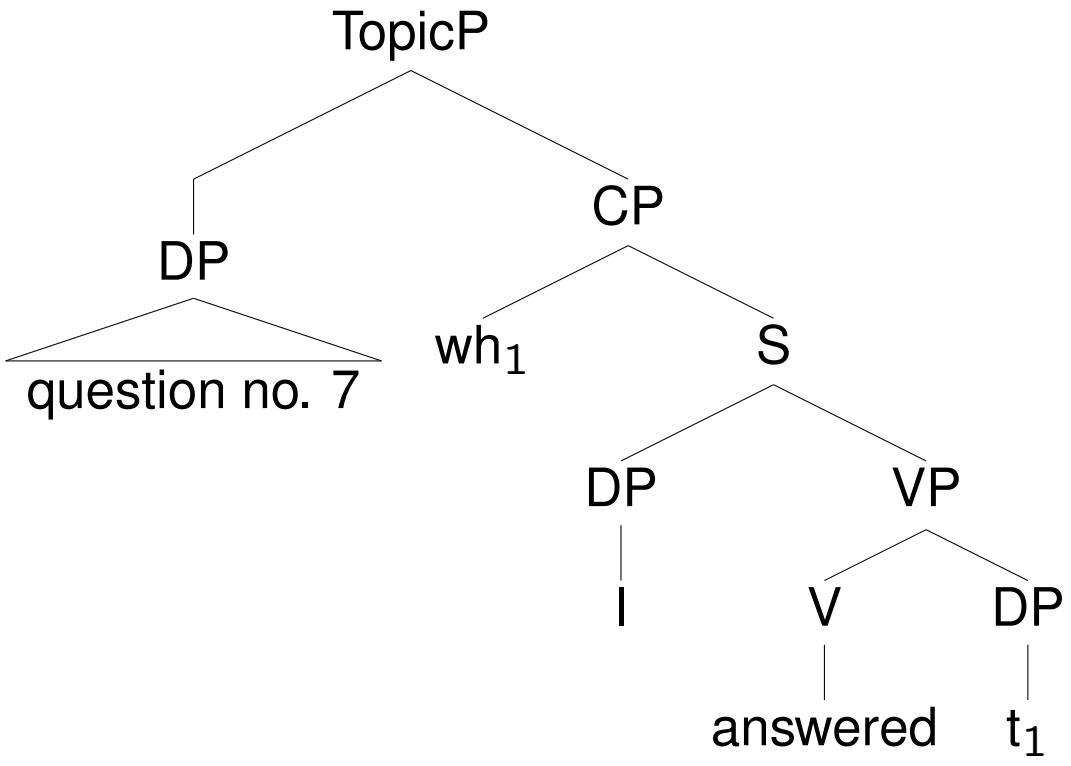
(8a) **I answered question no. 7.**
(8b) **Question no. 7, I answered.**

(9a) **John saw Mary.**
(9b) **Mary is such that John saw her.**
(9c) **John is such that he saw Mary.**

- If we take the pronouns in (9b) and (9c) to be variables, and "such that" as signaling predicate abstraction, then [[**such that John saw her**]] = (the characteristic function of) the set $\{x : \text{John saw } x\}$. This is also the value of the whole VP, (9b) is thus predicted to true iff Mary is in the set – which means nothing more and nothing less than that John saw her. So we have proved the equivalence of (9b) to (9a). The same goes for (9c).

6.1.2 Predictions about ambiguity and the effects of syntactic reorganization (cont.)

- The topicalization in (8b) may also involve predicate abstraction, if we derive it as suggested in Chomsky's "On *Wh*-Movement". The CP here looks like another predicate abstract, and that gives us the right meaning and the equivalence with (8a).



6.1.2 Predictions about ambiguity and the effects of syntactic reorganization (cont.)

- But when we replace the names and first person pronoun in (8) and (9) with other kinds of DPs, the equivalences don't hold.

(10a) **Almost everybody answered at least one question.**
(10b) **At least one question, almost everybody answered.**

(11a) **Nobody saw more than one policeman.**
(11b) **More than one policeman is such that nobody saw him.**
(11c) **Nobody is such that he saw more than one policeman.**

- Suddenly, the "transformations" affect truth-conditions. How?

- These truth-conditional effects are completely unexpected if the DPs in (10) and (11) are treated as individuals, like those in (9) and (10).

6.1.2 Predictions about ambiguity and the effects of syntactic reorganization (cont.)

- A final problem with trying to assimilate the semantics of all DPs to that of proper names is that such an approach does not anticipate certain judgments of *ambiguity*.

- Thus, (13) is ambiguous in a way that (12) is not. How?

(12) **It didn't snow on Christmas Day.**

(13) **It didn't snow on more than two of these days.**

- There may be differences in the way the two readings are pronounced, but there is no ambiguity in (12) and can't be because there is only one way of putting the three semantic parts of (12) together.

6.2 Problems with having DPs denote sets of individuals

- See the quote from Geach about ”quantificatious thinking”, i.e. thinking that words like “all”, “most”, “some”, “none” tell us how large a part of a set is being considered, so “all men” would refer to the entire set of men, “most men” would refer to the greater part of the set of men, “no men” would refer to the empty set, etc.

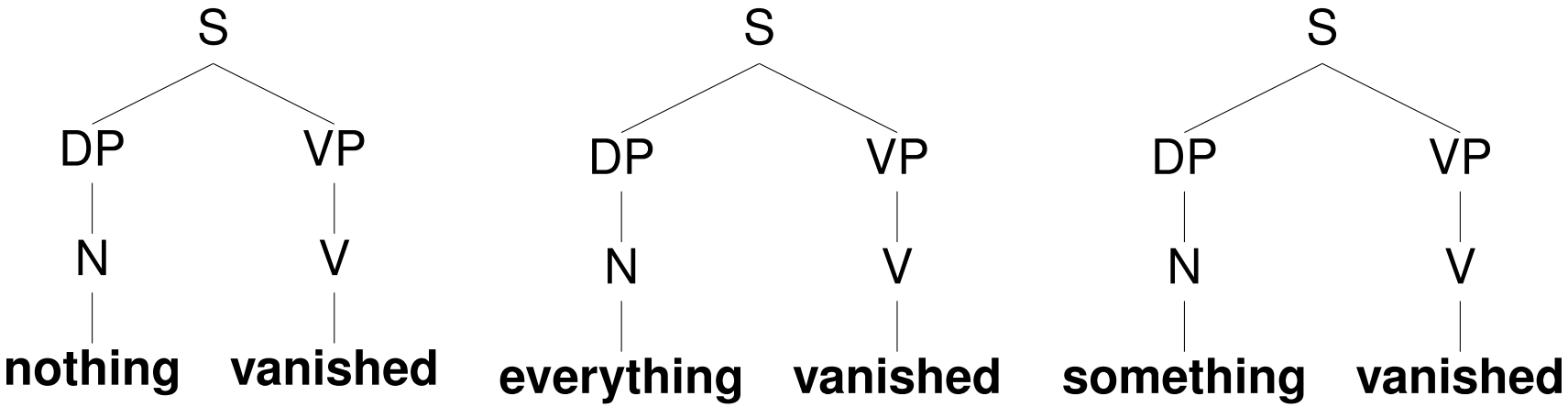
- Work through the extended exercise to see what is wrong with this approach.

6.3 The solution: generalized quantifiers

6.3.1 "*Something*", "*nothing*", "*everything*"

- If quantifier DPs like "something", "nothing" and "everything" are not of type e, then what type are they?

- Since they combine with a VP of type $< e, t >$ to form a sentence of type t, and we want to follow our Fregean strategy of using function application whenever possible, they must be of type $<< e, t >, t >$



6.3.1 "*Something*", "*nothing*", "*everything*" (cont.)

- Rather than denoting an individual or a set of individuals, "nothing" in "nothing vanished" says something about the denotation of the predicate "vanished".

- It states that there is no individual of which the predicate is true; that is, there is no individual that vanished.

- If we replace "nothing" with "everything", the claim is that the predicate is true of all individuals; and with "something", that there is at least one individual for whom the predicate is true.

- Quantificational DPs denote functions whose arguments are characteristic functions of sets, and whose values are truth values.

- Such functions are sometimes called "second-order properties", first-order properties being functions of type $< e, t >$.

6.3.1 "*Something*", "*nothing*", "*everything*" (cont.)

- More recently, second-order properties are referred to as "generalized quantifiers".

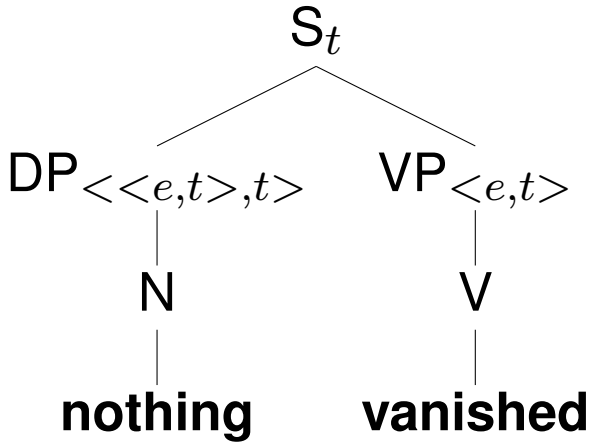
- Thus, we have the following lexical entries:

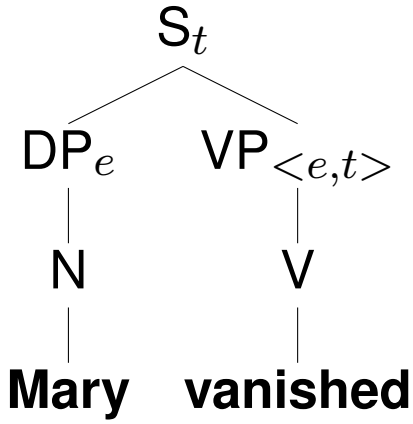
[[**nothing**]] = λf $\in$ $D_{<e,t>}$. there is no x $\in$ D_e such that f(x) = 1.
[[**everything**]] = λf $\in$ $D_{<e,t>}$. for all x $\in$ D_e, f(x) = 1.
[[**something**]] = λf $\in$ $D_{<e,t>}$. there is some x $\in$ D_e such that f(x) = 1.

6.3.1 “*Something*”, “*nothing*”, “*everything*” (cont.)

- Our semantic composition rules guarantee the right modes of combinations for subject DPs that are proper names or generalized quantifiers, without any further machinery.

- The difference between the denotations of the lexical items “nothing” and “Mary” brings about a difference in the way the respective DP-nodes are semantically combined with their VPs.





6.3.2 Problems avoided

- Before we continue, we want to make sure that this approach avoids the problems we detected for the simpler types e and $< e, t >$.

Subset to superset

- **At most one letter came yesterday morning** does not entail **At most letter came yesterday**.

- We must show that there exists a function f $\in$ $D_{<<e,t>,t>}$ such that it is possible that f(⟦**came yesterday morning**⟧) = 1, but f(⟦**came yesterday**⟧) = 0.

- This is quite evident.

- We only need to point out that it is possible for the actual facts to be such that ⟦**came yesterday morning**⟧ ≠ ⟦**came yesterday**⟧

- There are as many different functions in $D_{<<e,t>,t>}$ as there are ways of mapping the elements of $D_{<e,t>}$ to {0,1}.

- So for each given pair of distinct elements of $D_{<e,t>}$, there are lots of functions in $D_{<<e,t>,t>}$ that map the first to 1 and the second to 0.

Law of Contradiction

- **More than two cats are indoors and more than two cats are outdoors** is not a contradiction.

- We must show that there exists a function f ∈ $D_{<<e,t>,t>}$ such that it is possible that f(⟦**indoors**⟧) = 1 and f(⟦**outdoors**⟧) = 1.

- Obviously there are plenty such functions.

Law of Excluded Middle

- **Everybody here is over 30 or everybody here is under 40** is not a tautology.

- Again, the proof is trivial.

- Thus, it is easy to show that our approach avoids the first type of problem we noted with assuming that quantifier DPs are of type e.

- It is more challenging to reflect on our second argument against the uniform type e analysis of DPs.

6.3.2 Problems avoided (cont.)

Truth-conditional effects of syntactic reorganization

- We won't be able to handle examples like (3a) and (3b) until the next chapter.

(3) (a) **Everybody answered many questions correctly.**
(b) **Many questions, everybody answered correctly.**

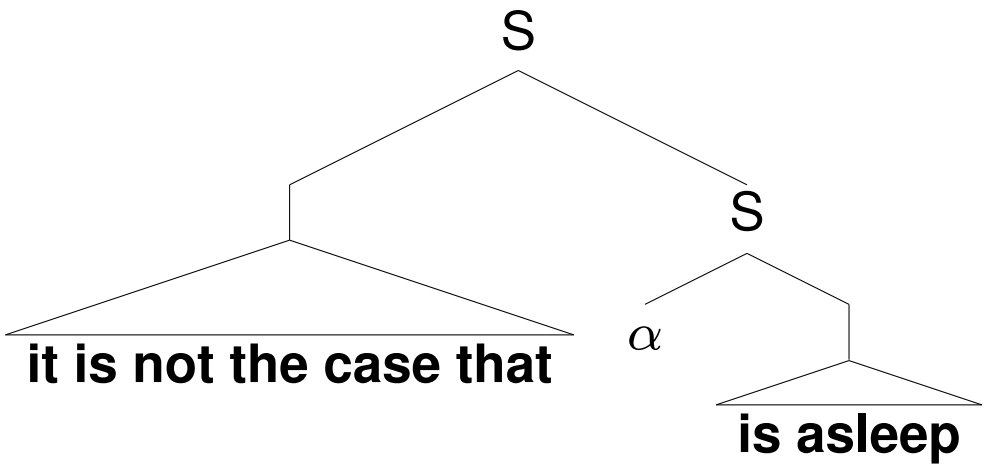
- But we can consider a hypothetical case that shows how it is possible in principle to alter truth-conditions by the topicalization of a phrase of type $<< e, t >, t >$.

- Suppose we topicalize the embedded subject α in a structure of the form (4a), yielding (4b), creating the following trees. (Never mind that (4b) is ungrammatical.)

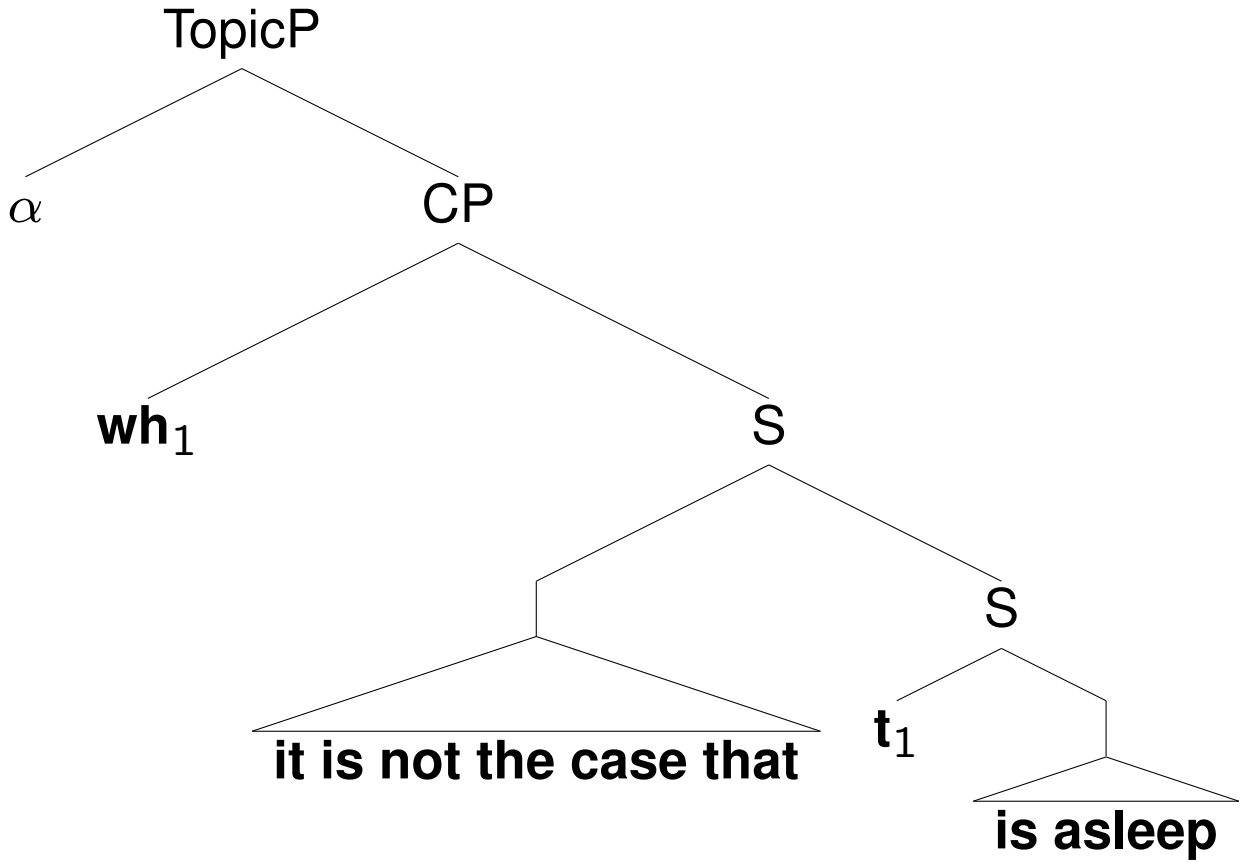
(4) (a) **It is not the case that α is asleep.**

(b) α, **it is not the case that t is asleep.**

(4a')



(4b')



- if $[\![\alpha]\!] \in D_e$, the two structures are provably equivalent. (Give the proof.)

- But if $[\![\alpha]\!] \in D_{<<e,t>,t>}$, their truth-conditions may differ. Suppose that α = **everything**. Then we calculate as follows.

(5) ⟦(4a)⟧ = 1

iff

⟦**everything is asleep**⟧ = 0

iff

⟦**everything**⟧(⟦**asleep**⟧) = 0

iff

⟦**asleep**⟧(x) = 0 for some x ∈ D.

(6) ⟦(4b)⟧ = 1

iff

⟦**everything**⟧(⟦**wh$_1$ it is not the case that t$_1$ is asleep**⟧) = 1

iff

⟦**wh$_1$ it is not the case that t$_1$ is asleep**⟧(x) = 1 for all x ∈ D

iff

⟦**it is not the case that t$_1$ is asleep**⟧$^{[1 \to x]}$ = 1 for all x ∈ D

iff

⟦ **t$_1$ is asleep**⟧$^{[1 \to x]}$ = 0 for all x ∈ D

iff

⟦**asleep**⟧(x) = 0 for all x ∈ D.

- The last lines of (5) and (6) express clearly distinct conditions.

- So we have shown that topicalization of a phrase with type $<< e, t >, t >$ can affect truth-conditions.