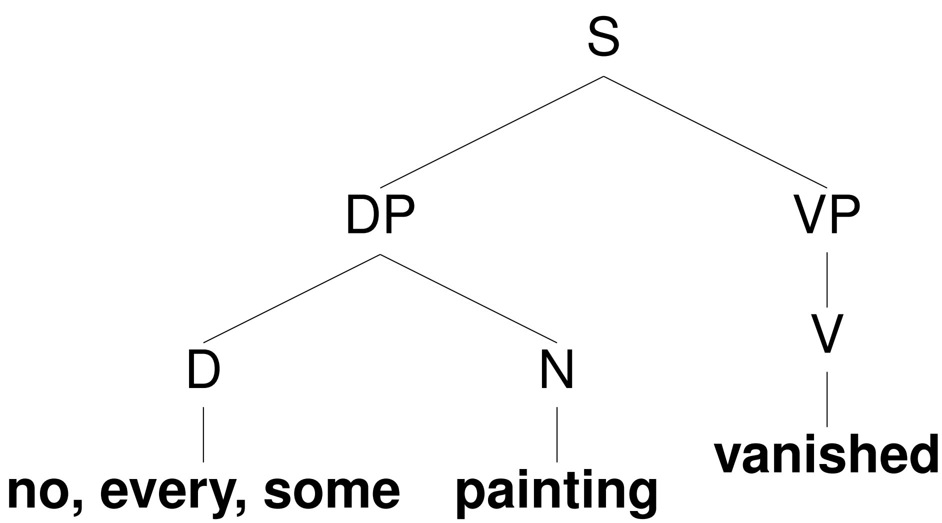
Quantifiers: Their Semantic Type (Part 2)

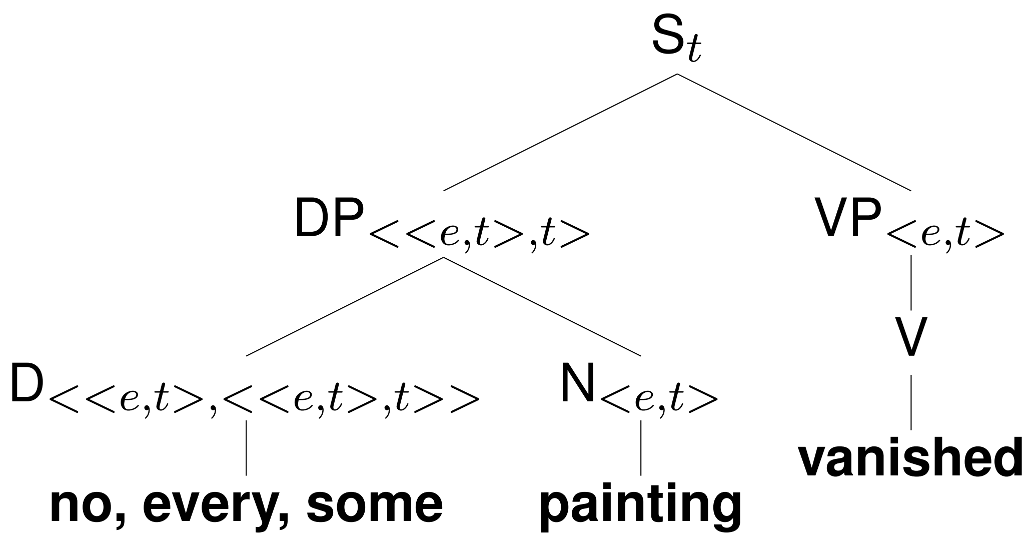
Heim and Kratzer
Chapter 6

6.4 Quantifying determiners

- Now that we have defined denotations for quantifying DPs like “nothing”, “everything” and “something”, we can find denotations for quantifying determiners like “no”, “every”, and “‘some”.



6.4 Quantifying determiners (cont.)



⟦**every**⟧ = λf $\in$ $D_{<e,t>}$. λg $\in$ $D_{<e,t>}$. for all x $\in$ D_e such that f(x) = 1, g(x) = 1]

⟦**no**⟧ = λf $\in$ $D_{<e,t>}$. λg $\in$ $D_{<e,t>}$. there is no x $\in$ D_e such that f(x) = 1 and g(x) = 1]

⟦**some**⟧ = λf $\in$ $D_{<e,t>}$. λg $\in$ $D_{<e,t>}$. there is some x $\in$ D_e such that f(x) = 1 and g(x) = 1]

6.4 Quantifying determiners (cont.)

- This $<< e, t >, << e, t >, t >>$ treatment of quantifying determiners was first proposed by David Lewis in “General Semantics”, 1972.

- A similar account was proposed by Richard Montague 1973, and developed further by Cresswell 1973, Barwise and Cooper 1981, and Keenan and Stavi 1986.

6.5.1 A little history

- Aristotle proposed that quantified sentences have the form: QXY.
- X,Y are universal terms (1-place predicates) and Q is chosen from “all”, “some”, “no”, “not all”.
- Such sentences can be combined into 256 three-sentence arguments called “syllogisms”, which come in one of four “figures”:

Q_1ZY	Q_1YZ	Q_1ZY	Q_1YZ
Q_2XZ	Q_2XZ	Q_2ZX	Q_2ZX
———	———	———	———
Q_3XY	Q_3XY	Q_3XY	Q_3XY

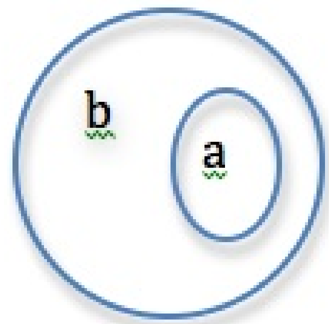
- Aristotle showed exactly which such syllogisms are valid.
- BARBARA: All animals are mortal, all cats are animals, all cats are mortal.
- CELARENT: No reptiles have fur, all snakes are reptiles, no snakes have fur.

6.5.1 A little history (cont.)

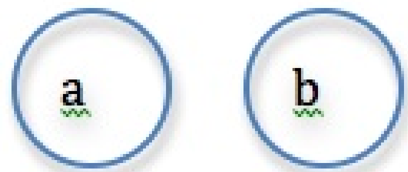
- Euler popularized Leibniz's device of illustrating logical relations by geometrical analogues.

- Euler illustrated the four Aristotelian forms of sentences by three relations of closed figures:

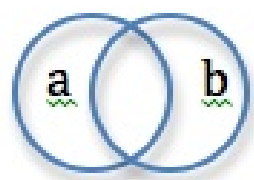
Every a is b



No a is b



Some a is b



6.5.1 A little history (cont.)

- Frege is famous for inventing predicate logic, with 1-place quantifiers.

- But in various places, he also endorsed the relational view of quantifiers.

- ”The words **all**, **every**, **no**, **some** combine with concept words. In universal and particular affirmative and negative statements, we express relations between concepts that indicate the specific nature of these relations by means of those words.”

6.5.2 Relational and Schönfinkeled denotations for determiners

- On the relational theory of quantification, quantifiers denote relations between sets: set inclusion ("every"), set disjointness ("no"), and set non-disjointness ("some").

- Our semantics for quantifying determiners in the last section is not strictly an instance of the relational theory.

- But there is a very straightforward connection between our determiner denotations and relations between sets.

6.5.2 Relational and Schönfinkeled denotations for determiners (cont.)

- Here is a sample of the relations that quantifying determiners would express on a relational theory:

(1) For any A $\subseteq$ D and any B $\subseteq$ D:

(a) $<$A, B$>$ $\in$ R_{every} iff A $\subseteq$ B.

(b) $<$A, B$>$ $\in$ R_{some} iff A $\cap$ B $\neq \emptyset$.

(c) $<$A, B$>$ $\in$ R_{no} iff A $\cap$ B = $\emptyset$.

(d) $<$A, B$>$ $\in$ $R_{at\ least\ two}$ iff $|$ A $\cap$ B $|$ $\geq$ 2.

(e) $<$A, B$>$ $\in$ R_{most} iff $|$ A $\cap$ B $|$ $>$ $|$ A $-$ B $|$.

- Since we are using function application, we'll need to transform these relational denotations into functional ones. But note that it is sometimes useful to talk in terms of sets.

6.5.2 Relational and Schönfinkeled denotations for determiners (cont.)

- Quantificational determiners of type $<< e, t >, << e, t >, t >>$ are analogous to transitive verbs, of type $< e, < e, t >>$

- Both can be viewed as expressing 2-place relations, the only difference being that transitive verbs are *first-order relations* (they express relations between *individuals*); whereas quantifying determiners are *second-order relations* (they relate *sets of individuals* or characteristic functions thereof).

- Thus, we will use Schönfinkelization to arrive at their denotations.

6.5.2 Relational and Schönfinkeled denotations for determiners (cont.)

- To get from R_{every} to [[**every**]], we start with R_{every} in (1a) and create its characteristic function F_{every}, in (2). We then Schönfinkel from left to right and call the result f_{every} as shown in (3).

(1) $R_{every} = \{<A, B> \in Pow(D) \text{ x } Pow(D) : A \subseteq B\}$.
(2) $F_{every} = \lambda<A, B> \in Pow(D) \text{ x } Pow(D) . A \subseteq B$.
(3) $f_{every} = \lambda A \in Pow(D). [\lambda B \in Pow(D) . A \subseteq B]$.

- Now we must replace the sets A and B with functions for NP and VP, as in (4):

(4) [[**every**]] = $\lambda f \in D_{<e,t>} . [\lambda g \in D_{<e,t>} . \{x \in D : f(x) = 1\} \subseteq \{x \in D : g(x) = 1\}]$.

6.6 Formal properties of relational determiner meanings

- If quantifiers correspond to binary relations, we may investigate whether they do or do not have standard properties of relations.
- Below is a list of some potentially interesting mathematical properties of relational determiner denotations.
- δ stands for a determiner, and A, B, C range over subsets of the domain D.

6.6 Formal properties of relational determiner meanings (cont.)

Definiendum	*Definiens*
δ is reflexive	for all A : $<$A, A$>$ $\in$ R_δ
δ is irreflexive	for all A : $<$A, A$>$ $\notin$ R_δ

- "every child is a child" TRUE
  "no child is a child" FALSE

δ is symmetric	for all A, B : if $<$A, B$>$ $\in$ R_δ, then $<$B, A$>$ $\in$ R_δ
δ is antisymmetric	for all A, B : if $<$A, B$>$ $\in$ R_δ and $<$B, A$>$ $\in$ R_δ, then A = B

- "some child cries" $\Rightarrow$ "some crier is a child"
  "every child cries" $\not\Rightarrow$ "every crier is a child"

δ is transitive	for all A, B : if $<$A, B$>$ $\in$ R_δ and $<$B, C$>$ $\in$ R_δ, then $<$A, C$>$ $\in$ R_δ

6.6 Formal properties of relational determiner meanings (cont.)

δ is conservative for all A, B : $<$A, B$>$ $\in$ R_δ iff $<$A, A $\cap$ B$>$ $\in$ R_δ

- Modern research within the generalized quantifier tradition has shown that some of these properties may help formulate constraints for possible determiner meanings.

- Keenan and Stavi have proposed that all determiners are conservative.

- Hence: “every child cries” $\Leftrightarrow$ “every child is a child that cries”

- But: “only children cry” $\not\Leftrightarrow$ “only children are children that cry”

6.6 Formal properties of relational determiner meanings (cont.)

δ is left upward monotone:

for all A, B, C : if $A \subseteq B$ and $<A, C> \in R_\delta$, then $<B, C> \in R_\delta$

δ is left downward monotone:

for all A, B, C : if $A \subseteq B$ and $<B, C> \in R_\delta$, then $<A, C> \in R_\delta$

δ is right upward montotone:

for all A, B, C : if $A \subseteq B$ and $<C, A> \in R_\delta$, then $<C, B> \in R_\delta$

δ is right downward monotone:

for all A, B, C : if $A \subseteq B$ and $<C, B> \in R_\delta$, then $<C, A> \in R_\delta$

- “every animal walks” $\Rightarrow$ “every cat walks” (left downward monotone)
  “every cat walks” $\Rightarrow$ “every cat moves” (right upward monotone)

- “some cats walk” $\Rightarrow$ “some animals walk” (left upward monotone)
  “some cats walk” $\Rightarrow$ ”some cats move” (right upward monotone)

- “no animal walks” $\Rightarrow$ “no cat walks” (left downward monotone)
  “no cat moves” $\Rightarrow$ ”no cat walks” (right downward monotone)

6.6 Formal properties of relational determiner meanings (cont.)

- Do the exercises on pp. 152-153. Try determiners like “every”, “most”, “many”, “a few”, “few”, “exactly two”, “at least two”, “some” or “a”, among others if you wish.

- Determiners that can appear in a *there*-insertion context are termed “weak” determiners, while those that can’t appear there are called “strong” determiners, following Milsark 1974. Try the definite article and demonstrative determiners too.

- For the exercise on negative polarity, note that (iii) has “ever” in the VP, and (iv) has “ever” in the subject DP.