Chapter 5

The nature of turbulence and velocity distributions in rivers

Viscosity and turbulence effects in flowing water
Viscosity and laminar flow
Turbulence and turbulent flow
Velocity distributions in open channels

Viscosity and turbulence effects in flowing water

Viscosity and laminar flow

We noted earlier that, depending on the nature of the streamlines with respect to space and time, flow can be classified generally as uniform or nonuniform and as steady or unsteady. Flow also can be classified according to the factor which is most important in limiting its rate of deformation: viscosity or turbulence.

In viscosity-dominated flow (viscous flow) in open channels, water particles move in response to gravity but only as rapidly as viscosity will allow. Such flows are visualized as moving in finely divided layers or laminae which slide one over the other as shown in Figure 5.1. Any tendency for a water particle to move vertically as a result of inertial forces and mix with those of adjacent layers is overcome or dampened by the viscous forces. Thus we refer to such viscosity-dominated motion as laminar flow. The downstream displacement of an individual layer per unit time is the velocity of that layer and the rate of change in velocity in successive layers above the bed is the velocity gradient.

The relation between viscosity and the motion of laminar flow is summarized in the definitional equality (see Figure 5.1):
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\[ \text{viscosity} = \frac{\text{shear stress}}{\text{strain rate}} \] .................................(5.1)

A fluid in which the ratio of fluid deformation (strain rate) is directly proportional to the applied stress \( \tau \), is known as a Newtonian fluid; thus laminar flow describes the behaviour of a Newtonian fluid. Here the strain rate is the relative displacement of a given layer of water per unit time with respect to the underlying layer, or the local velocity gradient, \( v/y \), where \( y \) is height above the bed. So, for the particular case of laminar flow, equation (5.1) takes the differential form:

\[ \mu = \frac{\tau}{dv/dy} \] ...............................................................(5.2)

from which it follows that \( \tau = \mu \frac{dv}{dy} \)

5.1: The laminar flow model. Displacement of a fluid particle is visualized as occurring within (but not between) discrete layers so that the gradient of the velocity profile at any given point is a direct measure of the strain rate in the flow at that point.

Laminar flow only prevails where the viscous forces dominate the inertial forces. A guide to the relative importance of these two forces is given by the dimensionless Reynolds Number, \( R_e \):

\[ R_e = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{vL}{v} \]
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The term L represents a characteristic length specifying the scale of the flow and in open channel flows can be taken as the mean depth of flow (in pipes, \( L = \) pipe diameter). Note also that, in order for \( Re \) to be dimensionless, the viscosity term here must be kinematic viscosity (\( \nu = \mu/\rho \)).

It turns out, however, that laminar flow is a rather rare occurrence in natural channels. Experimentally it has been shown that laminar flow will only prevail if \( Re < 1000 \) (very approximately) but it is far higher than this in even the smallest of streams. A quick calculation for a 10 cm-deep stream flowing at a slow 10 cms\(^{-1} \) (at 10\(^\circ\)C water temperature, for example; see Figure 2.1 for the appropriate viscosity) yields

\[
Re = \frac{0.1 \times 0.1}{1.306 \times 10^{-6}} = 7657, \text{ well beyond the domain of laminar flow.}
\]

Nevertheless, even though rivers may not exhibit laminar flow in terms of their mean flow properties, it is argued that very close to the boundary where velocities are lowest, the viscous forces may be relatively important and cannot be ignored when considering the flow behaviour there. This thin near-boundary zone of flow is sometimes referred to as the *viscous sublayer*, the implication being that the flow above is not at all viscosity-dominated (laminar).

**Turbulence and turbulent flow**

In flows in which \( Re > 1000 \), the viscous forces are so small relative to the inertial forces that they can be ignored as a significant factor controlling the rate of fluid deformation (velocity gradient). In these turbulent flows viscosity is no longer capable of damping the tendency for water particles to veer off the laminar streamline directions and as a result mixing between the layers takes place to the extent that the layering analogy is no longer a useful model of the flow. Instead we envisage turbulent flow to be an entirely
chaotic and dominantly random three-dimensional motion which is superimposed on the mean flow.

The distinction between laminar and turbulent flow is readily apparent in a simple experiment first conducted by the English engineer Osborne Reynolds (from whom $Re$ takes its name) in 1883. Reynolds injected a thread of dye into a laminar flow passing through a glass tube and noted that the thread remained intact, displaying no tendency to mix with the surrounding fluid (Figure 5.2a). The sustained integrity of the dye thread implied both steady and uniform flow and demonstrates the qualitative validity of the laminar flow model. In contrast, when he injected the same dye into the tube carrying turbulent flow, the dye thread broke up and rapidly diffused throughout the flow, thus demonstrating that turbulent mixing must have transported water particles vertically (and horizontally) as well as in the downflow direction (Figure 5.2c).
A current meter measuring the velocity in such a laminar flow would yield a quite flat velocity trace indicating a perfectly constant (steady) velocity over time (Figure 5.2b). The velocity record in the turbulent flow, however, would be characterized by unsteadiness, with excursions above and below the mean reflecting local acceleration and retardation of the flow as parcels of chaotically swirling water pass by the current meter.

It may be useful to think of turbulent flow as having a primary downstream velocity of advection on which are imposed all manner of secondary motions. That is, the mean flow transports the eddies and other flow structures which constitute the secondary motion and at a point adds to, or subtracts from, the mean velocity.

In a somewhat more formal sense we can note that, if we resolve the complex threedimensional motion of turbulent flow into the orthogonal component vectors defined in Figure 5.3, we can state that,

\[
\begin{align*}
\vec{v}_x &= \bar{v}_x + v'_x \quad \text{(5.3a)} \\
\vec{v}_y &= \bar{v}_y + v'_y \quad \text{(5.3b)} \\
\vec{v}_z &= \bar{v}_z + v'_z \quad \text{(5.3c)}
\end{align*}
\]

where \(v_x\), \(v_y\) and \(v_z\) are the instantaneous point velocities in the three orthogonal directions (x, y, and z), \(\bar{v}_x\), \(\bar{v}_y\), and \(\bar{v}_z\) are the respective turbulent fluctuating velocity components.

The mean values of \(\bar{v}_x\), \(\bar{v}_y\), and \(\bar{v}_z\) are necessarily zero but their standard deviations are non-zero and an intensity of turbulence, \(t_i\), can be defined as the root mean square (rms):

\[
t_i = \sqrt{\frac{v'_x^2 + v'_y^2 + v'_z^2}{3\bar{v}_x}} \quad \text{(5.4)}
\]
Thus, the *turbulence intensity* is the mean standard deviation of the velocity fluctuations for the three orthogonal flow directions relative to the mean downstream velocity - a coefficient of velocity variation. It is a matter of common experimental observation that $t_i$ is greatest near the channel boundary where the turbulence is being generated, and declines away from the bed towards the water surface.

*Turbulence scale* refers to the wavelength or period of the fluctuations and this is much more difficult to measure in a meaningful way because most velocity records include several compounded scales of velocity fluctuation and so far it is not possible to decouple the various signals. All we can say with certainty, is that velocity records in natural streams are characterized by (a) strong *periodicity* over a wide range of time scales and (b) strong *intermittency* over a wide range of time scales.

Part of the problem of analyzing turbulent velocity fluctuations in rivers is that they appear to be caused by several distinctly different processes. In this sense, equations (5.3) and (5.4) are deceptively simple because they perhaps imply a full understanding of the flow processes involved. On the contrary, however, the causes of the fluctuating velocity components are very poorly understood indeed. At this stage in our enquiry we should at least recognize that the term *turbulence* means different things to different people.

*True turbulence* is defined by hydrodynamicists as the very fine scale of velocity fluctuation present uniformly (*isotropically*) throughout a turbulent flow and described generally by properties predicted by Gaussian statistical theory. It is characterized by a random and normally distributed series of velocity pulsations that are largely beyond the measurement resolution of most current meters used to measure velocity in rivers. True turbulence typically has been measured in flumes using hot-film anemometers and involves velocity pulsation frequencies greater than about 10 hertz (10 cycles per second or a period of 0.1 second). It is a persistent, non-periodic, and isotropic pattern of velocity disturbance that is generated at the boundary but rapidly diffuses throughout
the entire flow. This type of 'true' turbulence has been studied by engineers and physicists in laboratory experiments for many years.

To most river scientists, however, turbulence includes not only the fine-scale isotropic velocity fluctuations but also a variety of larger scale anisotropic flow structures collectively termed macroturbulence.

Macroturbulence includes, for example, eddies or vortices shed from protuberances on the bed and banks. Although they might be quite localized disturbances to the flow, collectively they may be far more important in some river processes (such as contributing to the resistance to flow or initiating the movement of bed material or the suspension of sediment) than is the finer-scale turbulence.

Unlike the hydrodynamicist's turbulence, however, macroturbulence does not obey the same Gaussian statistical laws. In fact, very little is known about the nature of macroturbulence in rivers; it is a prime area for future study which is only now starting to attract the attention of researchers.

Matthes (1947) has provided perhaps the only classification of these macroturbulent flow structures to date. He recognizes three major types:

(a) Rythmic and cyclic surges: the entire flow surges in response to waves in the flow and causes a shift in the mean velocity. These may be seasonal in nature, or related to storm events or to diurnal-scale processes. Included here might be surges related to choking or to the forming and reforming of hydraulic jumps (at relatively short timescales on the order of hours to minutes). These hourly to minute-long surges are very common in rivers although their cause often is very difficult to isolate.

(b) Continuous rotary motions, include separation eddies and von Karman vortex trails (or streets). Conventionally eddies have vertical axes of rotation while rollers have horizontal axes.

(c) Discontinuous or intermittent vortex action refers to the boils or 'kolks' commonly seen in the water surface of rivers. Vortices shed from the boundary reach the surface as
eruptions (local water-surface elevation and outward spreading of flow). The origin of boils is not known although some (but not all) appear to be linked with the presence of dunes on the bed.

We should note that it is not entirely clear to many other river scientists whether structures in (b) and (c) are mutually exclusive. In particular, very little is known about the character of kolks in spite of their apparent importance in suspending material in the flow. It is an area of enquiry which deserves much additional study.

In general, measurements of macroturbulent velocity records indicate that turbulence intensity is greatest at the boundary and weakens as the structures diffuse during transport away from the source of disturbance. This process sees large eddies driving smaller ones until the turbulent energy is diffused at ever-decreasing scales until viscosity dampens it completely. Thus energy for driving the macroturbulence is abstracted from the main flow and it cascades through flow structures of ever decreasing size and finally is lost as heat.

In short, the notion of ‘turbulence’ covers a whole host of exceedingly complex processes!

**Velocity distributions in open channels**

Throughout most of the preceding discussion we have considered mean velocity as though it applied everywhere in the flow. We must now relax this assumption by recognizing what we all know to be true: velocity varies in a river channel from relatively low values near the boundary to the highest values where the flow is least retarded by the effects of skin resistance.

This more realistic pattern of velocity is often represented by *isovels* or isolines joining points of equal velocity across the channel cross-section (Figure 5.4). Here we can see that the highest velocity filament of the flow occurs in the centre of the channel near the surface of the flow and that velocity declines away from this zone.
5.9 A channel cross-section with isovels showing a typical pattern of velocity distribution in the flow.

towards the boundary. In many channels, such as the one depicted in Figure 5.4, the point of highest velocity is not at the surface of flow but rather it is depressed to some extent beneath the free surface. The reason for this vertical depression of the highest velocity is that the free-water surface often is deformed slightly by the complex forces operating at the interface of flowing water and air, locally increasing the flow resistance there. The effect can be quite exaggerated if the water surface is very rough, particularly if the water surface is shearing against an upstream wind.

The velocity distribution in laminar flow follows from the relations among shear stress, viscosity, and velocity gradient specified by equation (5.5):

\[
\tau = \mu \frac{dv}{dy} = \nu \rho \frac{dv}{dy} \tag{5.5}
\]

Integrating, we obtain

\[
\int \frac{dv}{dy} dy = v = \int \frac{g \rho}{V} (d-y) \ dy = \frac{g \rho}{V} (yd - \frac{y^2}{2}) \tag{5.6}
\]

Thus we conclude that the vertical velocity distribution in laminar flow must have a parabolic form. But we also know that laminar flow is not very common in real open channels so we must modify equation (5.5) to accommodate the important additional influence of turbulence.
The velocity distribution in fully turbulent flow can be derived by integration of the conceptual equivalent of equation (5.5):

\[ \tau = (\nu + \varepsilon) \rho \frac{dv}{dy} \]  ............................................................(5.7)

where \( \varepsilon = \) a coefficient of eddy viscosity. In other words, \( \varepsilon \) is an index of the additional resistance to fluid deformation resulting from the internal chaotic fluid motions that characterize turbulent flow. Of course, in order to operationalize equation (5.7) for use in solving real problems, the coefficient \( \varepsilon \) must be evaluated in terms of properties that we can measure in the flow. It turns out, however, that this task is easier recognized than achieved; countless fluid dynamicists have spent more than a century wrestling with the problem and their solutions remain essentially empirical and only approximate. Given the very complex nature of turbulence we noted earlier, this outcome is hardly surprising. We do know, however, that when velocity is measured in natural (turbulent) flows, typically it is semi-logarithmic in vertical distribution so that \( v \propto \log y \). This observation suggests that \( \varepsilon \) is much more important than \( \nu \) in determining the velocity gradient in turbulent flow. Indeed, we might safely ignore \( \nu \) in equation (5.7) and restate the relationship as

\[ \tau = \varepsilon \rho \frac{dv}{dy} \]  ............................................................(5.8)

Before developing equation (5.8) further, it might be useful at this point in our discussion to consider the way in which laminar and turbulent flow interact in a developing 'boundary layer' in the flow. In order to do this we need to imagine an infinite fluid moving as a laminar flow towards a fixed wedge-shaped body suspended in the flow as depicted in Figure 5.5. Upstream of the wedge the velocity field is uniform \( \frac{dv}{dy} = 0 \) because here it is not influenced by the presence of a boundary. As the flow impinges on the leading edge of the wedge the resulting drag causes a boundary layer to develop as the flow decelerates over the boundary below.
5.11: The developing boundary layer over the surface of a fixed body suspended in an infinite moving fluid.

The strain rate here is completely controlled by viscosity so that the resulting velocity distribution is parabolic in form. In other words, although the boundary layer continues to increase in thickness along the wedge in the direction of flow, the rate of deformation is limited by viscous stresses. These changes occur in the zone of laminar flow and boundary layer development shown in Figure 5.5 and correspond with the flow resistance domain on the Stanton diagram described by equation (4.19).

As the boundary layer develops, the inertial forces become much more important and turbulence begins to develop at the boundary; in other words the flow near the boundary reaches critical Reynolds Number. Thereafter a thickening layer of turbulent flow overlies a thinning layer of laminar flow in the laminar or viscous sublayer. Laboratory studies indicate that the thickness of the viscous sublayer, $\delta$, is described well by

$$\delta = \frac{11.6v}{\sqrt{\tau_0/\rho}} = \frac{11.6v}{v^*} \quad \text{.................................................................}(5.9)$$
You may recall that $v^* = \sqrt{\frac{\tau_o}{\rho}}$ is an important quantity known as the shear velocity; it does not represent a real physical velocity but it has the dimensions of a velocity.

Equation (5.9) shows that $\delta$ thins under conditions of high shear stress. Between the viscous sublayer and the fully developed turbulent flow there is a transitional or buffer zone in which both viscosity and turbulence play a part in determining the velocity and momentum gradient.

In a river the turbulent boundary layer may never be fully developed because the depth of flow simply is not great enough. Nevertheless, the boundary layer thickness is often taken as the flow depth. A quick operation with a calculator will also show that, even in very low shear stress conditions (see Sample Problem 5.1), the viscous sublayer in natural channels will have negligible thickness and can be ignored as a factor influencing near-boundary velocity.

**Sample Problem 5.1**

**Problem**: Determine the thickness of the viscous sublayer in a small stream in which $y = 0.2$ m, $\nu = 1.139 \times 10^{-6}$ m$^2$/s$^{-1}$, $s = 0.0001$, and $\rho = 999.1$ kg/m$^3$.

**Solution**: The shear stress at the bed, $\tau_o$, can be determined from the depth-slope product [equation (4.5)] as $0.0001 \times 999.1 \times 0.2 \times 0.0001 = 0.1959$ Nm$^{-2}$, so equation (5.9) gives

$$\delta = \frac{11.6\nu}{\sqrt{\frac{\tau_o}{\rho}}} = \frac{11.6 \times 1.139 \times 10^{-6}}{\sqrt{0.1959/999.1}} = 0.0009 \text{ m or about 1 mm.}$$

In the fully turbulent boundary layer the velocity gradient is given by equation (5.8). The generally accepted approach to the evaluation of $\varepsilon$ is that known as the Prandtl-von Karman theory in which eddy viscosity is considered to be given by

$$\varepsilon = \frac{2\nu^2}{dy} ..............................................(5.10)$$

where $l$ measures the depth of eddy penetration (the 'mixing length') and the velocity gradient controls the frequency of penetration.

5.12
Turbulent flow is envisaged as motion in which momentum exchange between 'layers' is achieved by eddies which act over a distance (l) far beyond the molecular scale. High-velocity (and high-momentum) fluid is brought close to the boundary while low-velocity (and low-momentum) parcels of water move from the bed into the flow above. The overall effect is to average out velocity and momentum differences in the flow far more significantly than can be achieved by viscous forces alone.

The character of Prandtl's 'mixing length' has been the subject of considerable experimental investigation. It is thought to be dependent on proximity to the boundary so that

\[ l = Ky \] .................................................................(5.11)

where \( K \) is known as the von Karman constant. Substituting \( l = Ky \) in equation (5.10) yields

\[ \varepsilon = K^2 y^2 \left( \frac{dv}{dy} \right) \] ............................................................................(5.12)

and substituting this version of \( \varepsilon \) in equation (5.8) yields

\[ \tau = K^2 y^2 \rho \left( \frac{dv}{dy} \right)^2 \] ............................................................................(5.13)

Noting that, near the bed \( \tau = \tau_o \), rearranging equation (5.13) gives:

\[ \sqrt{\frac{\tau_o}{\rho}} = V^* = Ky \frac{dv}{dy} \]

and

\[ \frac{dv}{dy} = \frac{V^*}{Ky} \] ............................................................................(5.14)

The relationship between velocity and height above the bed is obtained by integrating equation (5.14):

\[ \int \frac{dv}{dy} dy = v = \int \frac{V^*}{Ky} dy = \frac{V^*}{K} \int \frac{1}{y} dy = \frac{V^*}{K} \ln y + c \] ............................................................................(5.15)

Thus the velocity distribution in turbulent flow is logarithmic in form.
The intercept $y_o$ has been experimentally determined to depend on the height, $k$, of the roughness elements on the boundary relative to the thickness of the viscous sublayer, $\delta$. When $k<\delta$, the boundary is said to be 'hydrodynamically smooth' because the roughness elements are contained within the viscous sublayer, corresponding to the minimum relative roughness and lowest flow resistance on the Stanton diagram. Although this case has little relevance in most river channels (but may be important in some estuaries or lake inflows), we should note for the sake of completeness that $y_o$ simply decreases as shear stress increases according to

$$y_o = \frac{v}{9v^*} \quad \text{(5.16)}$$

Hydrodynamically rough bound-aries are defined as those for which $k>5\delta$, in which case the intercept $y_o$ is given by

$$y_o = \frac{k}{30} = \frac{D_{65}}{30} \quad \text{(5.17)}$$

where $D_{65}$ is the 65th percentile of the size distribution of bed particles expressed in the same units as $y_o$ (metres).

Since $v = 0$ at $y = y_o$, the constant of integration in equation (5.15) can be evaluated as $c = -\frac{v^*}{K} \ln y_o$ so that

$$v = \frac{v^*}{K} \ln y - \frac{v^*}{K} \ln y_o$$

and

$$\frac{v}{v^*} = \frac{1}{K} \ln y - \frac{1}{K} \ln y_o$$

5.6: The logarithmic velocity profile in turbulent flow and a definitional diagram for roughness length ($k$).
and \( \frac{v}{v^*} = \frac{1}{K} \ln \left( \frac{y}{y_o} \right) \) ...................................................................................................................(5.18)

Experiments to determine the magnitude of the von Karman constant suggest that it is a variable quantity rather than a constant. Nevertheless, it is claimed that for many flows, as a first approximation, \( K=0.4 \).

For hydrodynamically rough boundaries, setting \( K=0.4 \) and introducing \( y_o \) from equation (5.17) yields:

\[
\frac{v}{v^*} = \frac{1}{0.4} \ln \left( \frac{y}{D_{65}/30} \right) \...........................................................(5.19)
\]

Alternatively, we can express equation (5.19) in base 10 logarithms by noting that \( \ln x = 2.303 \log_{10} x \),

\[
\frac{v}{v^*} = \frac{2.303}{0.4} \log \left( \frac{30y}{D_{65}} \right) = 5.75 \log \left( \frac{30y}{D_{65}} \right) = 5.75 \log \left( \frac{y}{D_{65}} \right) + 5.75 \log 30
\]

Multiplying by \( v^* \) and simplifying leads to

\[
v = 5.75 v^* \log \left( \frac{y}{D_{65}} \right) + 8.5v^* \ ..................................................(5.20)
\]

Again, for the sake of completeness we might note that, for hydrodynamically smooth boundaries, equation (5.16) applies so that, substituting for \( y_o \) and \( K=0.4 \) in equation (5.18) yields:

\[
\frac{v}{v^*} = \frac{1}{0.4} \ln \left( \frac{y}{\nu/9v^*} \right) \...........................................................(5.21)
\]

or the base 10 log alternative,

\[
v = 5.75 v^* \log \left( \frac{v}{v} \right) + 5.5 v^* \ .........................(5.22)
\]

Equations (5.20) and (5.22) are known as the universal velocity equations for respectively hydrodynamically rough and smooth boundaries. Velocity distributions conforming to
this 'law' of the wall' as equation (5.18) sometimes is known, plot on semi-logarithmic paper as a straight line. An example of the use of equation (5.20) is shown in Sample Problem 5.2.

Clearly the accuracy of the universal velocity equations depends on the underlying assumptions that velocity is distributed logarithmically with depth, that \( K = 0.4 \), and that equations (5.16) and (5.17) accurately specify the zero-velocity depth. This is quite a list of assumptions! As we might expect, the equations give quite variably accurate estimates but in the absence of any other data are nevertheless quite useful.

\textit{Shear stress determination from the vertical velocity distribution.}

Implicit in any of the above discussions of fluid mechanics to this point is that shear stress is the \textit{average} shear stress for the entire channel cross-section as expressed by equation (5.20). We now are in a position, however, to use the 'law of the wall' to compute the intensity of shear stress \textit{at a point} on the bed and thus consider the distribution of shear stress over the boundary. Indeed, equation (5.18) is commonly used for this purpose by river scientists. Shear stress determined from velocity distributions at regularly spaced points across the channel should average out to the mean shear stress for the channel determined by equation (5.20).

The minimum data necessary to compute the point shear stress is a pair of velocity measurements at two different depths on the same vertical, preferably as close as possible to the bed. A more reliable estimate of \( \tau_o \) is obtained from the average velocity gradient determined from the velocity profile. The procedure involves solving equation (5.18) for the shear velocity and is illustrated in Sample Problem 5.3.
Sample Problem 5.2

Problem: Estimate the vertical velocity distribution in a wide rectangular open channel with the following flow and channel characteristics: flow depth = 2.0 m, \( \nu = 1.139 \times 10^{-6} \text{ m}^2\text{s}^{-1} \), \( s = 0.0001 \), \( \rho = 999.1 \text{ kgm}^{-3} \) and bed material \( D_{65} = 0.5 \text{ cm} \) (0.005 m).

Solution: We use equation (5.18), noting that the shear stress at the bed, \( \tau_0 \), can be determined from the depth-slope product [equation (4.5)] as \( 9.806 \times 999.1 \times 2.0 \times 0.0001 = 1.959 \text{ Nm}^{-2} \).

So \( v^* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{1.959}{999.1}} = 0.044 \text{ ms}^{-1} \) and equation (5.22) takes the particular form:

\[
v = 5.75 \left( 0.044 \right) \log \left( \frac{y}{0.005} \right) + 8.5 \left( 0.044 \right) \quad \text{or} \quad v = 0.253 \log \left( \frac{y}{0.005} \right) + 0.374
\]

Solving for the following arbitrary depths:

<table>
<thead>
<tr>
<th>depth, ( y )</th>
<th>log depth</th>
<th>velocity, ( v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>-2.000</td>
<td>0.450</td>
</tr>
<tr>
<td>0.1</td>
<td>-1.000</td>
<td>0.703</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.301</td>
<td>0.880</td>
</tr>
<tr>
<td>1.0</td>
<td>0.000</td>
<td>0.956</td>
</tr>
<tr>
<td>1.5</td>
<td>0.176</td>
<td>1.001</td>
</tr>
<tr>
<td>2.0</td>
<td>0.301</td>
<td>1.032</td>
</tr>
</tbody>
</table>

We can now graph the velocity distribution as a linear or straight-line semi-logarithmic plot:

![Graph of velocity distribution](image-url)
Sample Problem 5.3

**Problem:** Determine the shear stress on the bed of a river at the vertical for which the following velocity data are available:

<table>
<thead>
<tr>
<th>height above bed, m</th>
<th>velocity, m/s</th>
<th>ln height above bed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.55</td>
<td>-2.303</td>
</tr>
<tr>
<td>0.30</td>
<td>0.75</td>
<td>-1.204</td>
</tr>
<tr>
<td>0.60</td>
<td>1.10</td>
<td>-0.511</td>
</tr>
<tr>
<td>1.20</td>
<td>1.40</td>
<td>0.182</td>
</tr>
<tr>
<td>1.50</td>
<td>1.30</td>
<td>0.405</td>
</tr>
<tr>
<td>1.80</td>
<td>1.45</td>
<td>0.588</td>
</tr>
</tbody>
</table>

**Solution:** We can employ equation (5.18) to compute the shear velocity but first we must determine \( y_0 \), the height above the bed at which the logarithmic velocity profile reaches zero. Note that a graph of \( \ln y \) versus \( v \) based on real data will not conform exactly to a logarithmic distribution so we use a best-fit regression curve to average the data as shown below:

![Graph of ln y vs velocity](image)

The intercept at \( v = 0 \) is \( \ln y_0 = -3.707 \), so taking the antilog, \( y_0 = 0.02455 \) m. Setting \( K = 0.4 \) and \( y_0 = 0.02455 \) in equation (5.18) gives

\[
\frac{v}{v^*} = 2.5 \ln \left( \frac{y}{0.02455} \right).
\]

We can solve for any \( v/y \) combination from the regression equation but \( y = 1.0 \) m (\( \ln y = 0 \)) easily yields \( v = 1.252 \) ms\(^{-1}\).

Therefore, equation (5.18) becomes

\[
\frac{1.252}{v^*} = 2.5 \ln \left( \frac{1.0}{0.02455} \right) \quad \text{or} \quad \frac{1.252}{v^*} = 9.268,
\]

and the shear velocity \( v^* = \sqrt{\tau_0/\rho} = 0.135 \) ms\(^{-1}\). Assuming a fluid density of 1000 kgm\(^{-3}\), \( \sqrt{\tau_0/1000} = 0.135 \) and the boundary shear stress, \( \tau_0 = 18.23 \) Nm\(^{-2}\).

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Some concluding remarks

In this Chapter we have been concerned with the flow of water through channels with rigid boundaries. In all alluvial river channels, however, the boundary is to some extent moulded by the flows they convey. Even if a channel boundary is completely rigid at low flow, there will be some higher discharge which will deform the channel through the processes of erosion and deposition. Thus the flow and the channel boundary are part of a dynamic and mutually adjusting system which is far more complex than allowed by
the theory presented in the previous pages. That body of theory remains useful only as an ideal model against which we can measure the character of real river geomorphology. Nevertheless, although the theory of rigid channel flow yields a rather limited set of practically useful quantitative predictions of flow behaviour, it does provide us with the conceptual tools to deal qualitatively with the complex process-form relationships which constitute the behaviour of real rivers.

In the pages to follow we will consider the problems of accounting for the adjustable or 'alluvial' channel boundary of natural channels. Our first task, taken up in Chapter 6, is to consider the nature of sediment entrainment, transport, and deposition, within the channel.

References

Matthes, G.H., 1947, Macroturbulence in natural stream flow: Transactions of the American Geophysical Union, 28 (2) 255-262.