Chapter 5: Working with contours

Introduction

Contoured topographic maps contain a vast amount of information about the three-dimensional geometry of the land surface and the purpose of this chapter is to consider some of the ways in which these data can be extracted and presented.

Natural three-dimensional surfaces such as topography are extremely difficult to summarize in any quantitative way and for this reason we invariably look at landscape in terms of a series of two-dimensional slices such as those presented in Figure 4.2 in Chapter 4. These slices, or topographic cross-sections (or terrain profiles) of the surface, although only providing a partial view of landform, are extremely useful in visualizing the surface. They also lead us directly to the notion of slope.

Slope of the land

The slope or gradient of the land, as we all intuitively appreciate, expresses the relationship between the change in height of the surface with respect to a horizontal shift in position. The familiar expression 'rise over run' captures this notion of slope. In a formal sense slope is the first derivative of the elevation (E)/distance (D) function, $E = f(D)$. Some of you may not fully appreciate, however, the variety of ways in which slope can be expressed.

Commonly, slope is specified as (a) a ratio, (b) a fraction, (c) an angle in degrees, (d) a percentage, and less commonly as (e) an angle in radians. Figure 5.1 illustrates how these various measures of slope are derived and how they relate, one to the other. The first three measures will be familiar to those who remember their high school trigonometry. Perhaps one of the most familiar expressions of slope is the percentage, because road gradient conventionally is represented in this way on road signs. Conversion of percentage gradient to a slope in degrees is straightforward. For example, a steep slope of 30% means that the \[
\frac{\text{rise}}{\text{run}} \times 100 = 30, \quad \text{so that} \quad \frac{30}{100} = 0.3 \]

is the tangent of the slope; thus the angle of slope (from tangent tables or the inverse tangent function on your calculator) is $16.70^\circ$. Remembering that there are $60'$ in $1^\circ$ of angle, this decimal angle can be expressed in terms of degrees and minutes as $16^\circ + (0.70 \times 60)' = 16^\circ 42'$. Note that a 50% gradient is tangent $0.5 = 26.57^\circ$ and that a 100% gradient is tangent $1.0 = 45^\circ$. These calculations serve to remind us of course that there is not a linear relationship between an angle in degrees and its tangent.

It is likely that the only unfamiliar notion in Figure 5.1 is the radian. A radian is the angle at the centre of a circle subtended by an arc whose length is equal to the radius, $R$. Since the circumference, $C$, of a circle is $2\pi R$ in length, $360^\circ$ corresponds to $2\pi$ radians and $1^\circ = 2\pi/360 = 0.0175$ radians. This type of angular measurement sometimes is used by geomorphologists interested in defining the forces
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5.1: Various measures of slope.

controlling the stability of slopes but it is not a slope measure in common everyday use.

Topographic cross-sections

A terrain profile or topographic cross-section is a diagram showing the rise and fall of the surface of the ground along a line or section between two points. The line of section is often a straight line but it may also be non-linear, such that drawn along a sinuous river course or along a winding road. We have seen several examples of linear profiles already in the contour planforms and profiles of Chapter 4.

The slope at any point on one of these profile is the same as the corresponding slope on the ground only if the horizontal and vertical scales are the same. In most cases, however, it is both
convenient and instructive to exaggerate the vertical scale to make the relief fully apparent. The reason for this exaggeration becomes clear if we consider an example. If we were interested in the relief along a section as it appears on a 1:50 000 topographic sheet, relief of 100 metres (five times the contour interval) would show up on a vertical scale of 1:50 000 as a mere blip of 1/5 of a centimetre (2 mm). Relief detail can be seen much more clearly if the vertical scale (VS) is exaggerated with respect to the horizontal scale (HS). The degree of scale distortion in terrain profiles is specified by the vertical exaggeration, $VE$, where $VE = \frac{VS}{HS}$. For example, if $VS = 1/5,000$ and $HS = 1/50,000$, $VE = \frac{1/5,000}{1/50,000} = \frac{50,000}{5,000} = 10$. When constructing a cross-section the vertical exaggeration should be set as low as possible while remaining consistent with the meaningful display of relief information.

The steps for constructing a topographic cross-section follow:

1. Select the horizontal and vertical scales of the diagram. Typically, the horizontal scale is set by the scale of the map from which the section is derived. The vertical scale should be chosen to include the full range of elevations along the line of section. These scales should be shown on the axes of the graph together with a clear statement of the vertical exaggeration.

2. Draw a line on the map between the points for which the profile is required and determine the heights of the start and end points (by interpolation, if necessary).

3. Place the edge of a piece of paper along the cross-section marked on the contour map and, without moving the paper, mark the intersections of each contour and such features as rivers and roads on the paper in the manner depicted in Figure 5.2.

4. Transfer the contour-intersection marks from the edge of the piece of paper to the horizontal axis of the graph. Extend vertical lines from each intersection mark to its appropriate height indicated on the vertical axis; mark each projected intersection with a dot at the appropriate height.

5. The points plotted on the graph are then joined together by a smooth line (not a series of straight segments).

6. The cross-section is completed by the addition of a title, the horizontal and vertical scales, and a clear statement of the vertical exaggeration if any is involved.

Although the topographic cross-section shown in Figure 5.2 is based on a straight line of section, exactly the same principles of profile construction are applied to a non-linear line of section. The only difference is that the strip of paper in this latter case must be rotated so that its edge follows the curve of the winding line of section. Care must be taken here to ensure that each centre of rotation of the paper strip remains fixed with respect to the map during rotation otherwise the plotted section length will be shorter than the actual length. Although this may be a tedious procedure it is necessary in order to transfer the true horizontal distance to the cross-section.

There are a number of practical uses of terrain profiles, including the routing of roads (where
gradient changes are important) and in testing for intervisibility. In the case of the latter, for example, the siting of microwave towers must be achieved so that a line-of-sight exists between adjacent towers. Another recently more important application of the principle of intervisibility is in mapping forested slopes to identify areas which can and cannot be seen from highways in order to minimize the negative aesthetic impacts of logging scars. This condition of intervisibility is tested easily with an appropriate cross-section or series of cross-sections. For example, if two points, A and C, lie on a proposed line of sight, we can see from their relative positions plotted in Figure 5.3 that a third point, B, represents an obstruction because it falls above the line of sight. In other words, this graphical test of intervisibility dictates that C will be visible from A (and *vice versa*) if B is below the line AC, and obstructed if it is above it.

**Mathematical determination of intervisibility**

In this simple case of determining intervisibility it may actually be easier to solve the problem mathematically rather than drawing a full cross-section. The mathematical equivalence of this graphical
5.3: Determination of intervisibility (positive slope); see text for explanation.

The test of intervisibility is to find the equation of the line AC and solve it for the x coordinate of the point B. If this calculated point is greater than the elevation of B then A can be seen from C; if it is less, B obstructs the view from A to C and vice versa.

In our example we can simplify this task by setting A as the origin; the equation describing AC is given by $E = mD$ where $E$ is elevation above A, $D$ is distance from A, and $m$ is the slope of the line. From Figure 5.3, for the line passing through A and C, $m = \frac{-220 - 80}{3450 - 500} = 0.0475$, so that $E = 0.0475D$. Location B has an elevation of 155 m above sea level (or 155 - 80 = 75 metres above A) and is 1 000 metres from A (i.e., $D = 1 000$). Thus, at location B the line-of-sight elevation with respect to A is $E = 0.0475D = 0.0475(1 000) = 47.5$ metres. Since B is located 75 m above A, it must be above the line of sight for its location and therefore obstructs the view from A to C.

Using a slightly different line of reasoning, the intervisibility problem can also be solved for these simple cases by comparing line-of-sight slopes. In Figure 5.3 it is evident that C will only be visible from A if the slope from A to B is less than that from B to C. Since slope A to B = $\frac{155 - 80}{1500 - 500} = 0.075$, and that from B to C = $\frac{220 - 155}{3450 - 1500} = 0.033$, it follows that C is not visible from A.

5.4: Determination of intervisibility (negative slope); see text for explanation.

In these calculations slopes are positive because our reference direction is 'uphill' to the right. In other examples, negative slopes may be involved and signs must be strictly observed in order to give consistent and comparable measurements among slope segments.

For example, consider the case of the negatively sloping surface depicted in Figure 5.4. Once again, to determine if C is visible from A, we let A be the origin as before and now the equation of the line AC is $E = \frac{80 - 220}{3450 - 500}D = -0.0475D$. 

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Location B has an elevation of 155 metres above sea level (or 155-220 = -65 metres with respect to an origin at A) and is 1 000 metres from A (i.e., D = 1 000). The line-of-site elevation at B is \( E = -0.0475D = -0.0475(1 000) = -47.5 \) metres. Since B has an elevation of -65 m, it is below the line-of-sight for its location and therefore does not obstruct the view from A to C.

Again, using the relative slope criterion as before, A will be visible from C if the slope of line segment AB is greater than that of BC, and vice versa. Since slope \( AB = \frac{220-155}{500-1500} = -0.065 \) and slope \( BC = \frac{155-80}{1500-3500} = -0.038 \), it follows this time that A is visible from C.

You might note that we could have solved this intervisibility problem by comparing slope AB to that for AC; if slope AB>slope AC, C can be seen from A and if slope AB<slope AC, point B must obstruct the line of sight.

**Slope maps**

You should also note that the slope along a line of section is usually not the maximum slope for any point on that line. Only if the line of section fortuitously crosses the contours at right-angles will the section slope and the maximum slope coincide. Thus slopes along a line of section usually are less than the local slope maximum but they can never exceed it.

Slope maps such as that shown in Figure 5.5 can be drawn for an area by constructing isolines (lines joining equal quantities) through a field of spot slope-values derived from slopes measured at right-angles to the contours. These maps have been used for such purposes as identifying zones of slope instability and areas too steep for conventional logging practices. The task of preparing such maps is rather tedious if done by hand but computers have considerably streamlined the process in cases where the surface configuration has been digitized.

Slope maps represent a class of thematic (special purpose) map known as an isometric map because they use isolines joining equal-slope values to depict the spatial pattern of this surface property. In Figure 5.5 the pattern is generalized by using various graded shadings to group a number of slope ranges. Other maps of this type are discussed in Chapter 9.

Slope maps are a step removed from the depiction of the spatial pattern of topographic elevation and they are not interpreted as readily as contour maps. Like most other kinds of graphics in cartography, however, the more these maps are used the more useful they become; practice makes practical, if not perfect!

Whereas topographic contour maps depict the variation in elevation with distance \([E = f(D)]\), slope isoline maps represent the differential form of that surface \([dE/dD = S = f'(D)]\). The slope can also vary spatially, however, and we refer to this property of a surface as its convexity or concavity. We have encountered the notions of convexity and concavity in a qualitative way already (see Figures 4.2C and D) but we now should refine these concepts a little by treating them as statements of the rate of change in

This type of mathematical description of river profiles quite readily can be generalized to three-dimensional surfaces. Mapping of these higher-order surfaces rarely is undertaken, however, because the maps involve too many transformations and simply are too difficult to interpret. Here mapping gives way completely to mathematics. The description of topography in these quantitative terms is the concern of certain sub-fields of geomorphology such as morphometry and topology.

But now it is time, once again, to leave these theoretical notions and return to 'hands on' work with maps in order to consolidate these ideas. Soon we will turn our attention to the problems of surveying in the field.
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