Modeling and Analysis of Multipath Video Transport over Lossy Networks Using Packet-Level FEC

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Abstract

The use of forward error correction (FEC) coding is often proposed to combat the effects of network packet losses for error-resilient video transmission on packet-switched networks. On the other hand, path diversity has recently been proposed to improve network transport for both single-description (SD) and multiple-description (MD) coded video. In this work we model and analyze an SD coded video transmission system employing packet-level FEC in combination with path diversity. In particular, we provide a precise analytical approach to evaluating the efficacy of path diversity in reducing the burstiness of network packet-loss processes. We use this approach to quantitatively demonstrate the advantages of path diversity in improving end-to-end video transport performance using packet-level FEC.

1. Introduction

To transmit packet video over lossy packet-switched networks, packet-level forward error correction (FEC) is often proposed to combat packet losses typically due to network congestion, link failures, and timeouts. The efficacy of packet-level FEC is often limited by the bursty nature of typical network packet-loss processes. The use of path diversity, where packets are routed over multiple paths, has recently been proposed to improve network transport for both single-description (SD) and multiple-description (MD) coded video [3, 4]. Path diversity can reduce the effects of extended packet bursts as seen at the FEC decoder, thereby improving the FEC performance. In this work, we model and analyze a SD-coded video transmission system using a combination of both packet-level FEC and path diversity, and demonstrate the efficacy of path diversity in improving joint source-channel coding (JSCC) performance for packet video transmission.

We consider two specific multipath transport scenarios: In the first scenario, we simply assume the paths share no joint links and are totally independent of each other. We provide a precise quantitative analysis of the resulting effective loss-burst-length distribution and residual decoded packet-loss rate using path diversity. Using these results, we demonstrate the efficacy of path diversity in improving end-to-end video transmission performance using packet-level FEC. In the second scenario, we assume different paths may share some joint links and, therefore, the packet-loss processes on different paths may be correlated. We investigate the effect of the resulting path correlation on FEC performance.

The paper is organized as follows: In Section 2 we describe a general system model for packet-video transmission over networks using packet-level FEC and path diversity. In Section 3 we investigate the multipath video transport system for the case of disjoint paths using packet-level FEC. In Section 4 we consider the multipath video transport system for the case of joint paths. Finally, in Section 5 we provide a summary and conclusions.

2. Multipath Video Transport System

Figure 1 illustrates the general SD video multipath transmission system model considered in this paper. Several important system model parameters are also indicated. As shown in this figure, a video transmission system has the following components: a video encoder and decoder, a packet-level FEC encoder and decoder, and a multipath transport network.

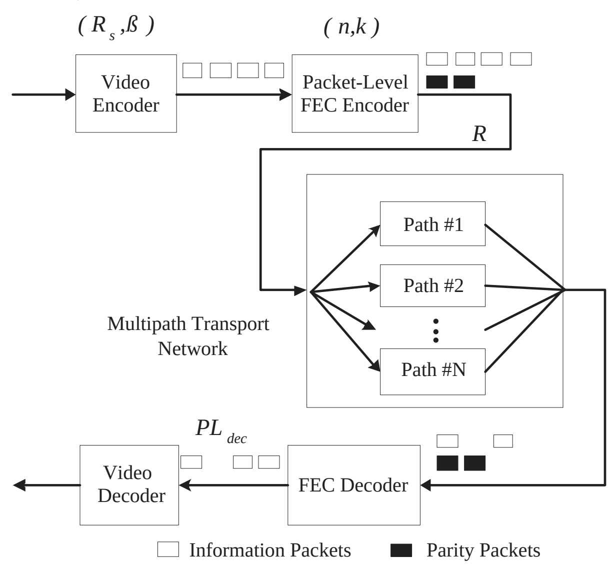

Figure 1. Video transmission system model.

2.1. Video Encoder/Decoder

Assume the source generates a space-time video signal which is used as input to the video encoder. We assume the encoder is a typical block-based hybrid motion-compensated video encoder[1], which encodes the video signal at rate R_s bits/sec with INTRA refresh rate β (as a fraction of macroblocks coded in the intra mode). The latter parameter is indicative of the error resilience capability of the encoded video. We assume the compressed video data are packetized with M macroblocks (MBs) per transport packet.

At the video decoder, the received video packets are depacketized and the video signal is reconstructed. For lost video MBs, passive error concealment will be used to mitigate the distortion due to unrecovered packets.

2.2. Packet-Level FEC Encoder/Decoder

In this paper we use an interlaced Reed-Solomon $RS(n,k)$ coding scheme [6] to provide FEC. For every block of k information packets an additional $p = n - k$ redundant packets are transmitted. The channel-coding rate is then given by

$$R_c = k/n\,; \qquad \textit{bits/channel use} \qquad (1)$$

Assume the total available network bandwidth is fixed at R bps. As a result of the overhead introduced by channel coding, the source coding rate has to be throttled to

$$R_s = R * R_c \text{ bps.} \qquad (2)$$

At the FEC decoder, packets received from different paths will be reordered as necessary. Some packets may be lost due to network congestion, link failures, and timeouts. Let $P(j,n)$ denote the block error distribution seen by the FEC decoder after packet reordering, $i.e.$, the probability that j packet losses occur within a block of n consecutive packets, $n \geq 1, 0 \leq j \leq n$. With N_p denoting the number of lost packets within this block, if $N_p > n - k$ we assume the lost packets within this block cannot be recovered by the FEC decoder. Then the residual packet-loss rate of the original video packets after channel decoding can be shown to be given by

$$PL_{dec} = \Big(\sum_{j=n-k+1}^{n} j * P(j,n)\Big)/n\,. \qquad (3)$$

2.3. Multipath Transport Network

As illustrated in Fig. 1, the encoded video packets are transported over the multipath transport network composed of N paths. To simplify the analysis, we assume a simple cyclic or round-robin multipath transport scheme: the 1-st packet is transported on path #1, the 2-nd packet is transported on path #2, etc., until the $(N+1)$-th packet which will then be transported on path #1.

[1]In Section 3.3 we will make specific use of the ITU-JVT JM 6.1 codec for the newly developed H.264 video coding standard to provide some numerical examples.

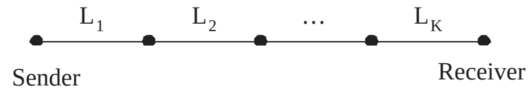

Figure 2. A transport path consisting of K links.

As illustrated in Fig. 2, each path consists of several transmission links connected by routers. Due to temporary buffer overflow and link outages, the packet losses typically occur in bursts with possibly varying burst lengths. We use a two-state discrete-time Markov-chain model, called the Gilbert model [7], to capture the bursty nature of each link. The Gilbert model for link i has two states, "Reception" and "Loss", and two independent parameters: P_{01}^i and P_{10}^i, representing the associated state transition probabilities.

In the literature, two alternative parameters are often used to characterize the Gilbert model: the steady-state packet-loss rate PL^i and the average loss burst length LB^i. The relationships between these parameters are given by:

$$PL^i = \pi^i(1) = \frac{P_{01}^i}{P_{10}^i + P_{01}^i}\,;\; LB^i = \frac{1}{P_{10}^i}\,, \qquad (4)$$

where $\pi^i(1)$ is the steady-state probability of being in the loss state.

We assume the packet-loss processes of the links along a given path are independent. Then, the end-to-end packet loss process of the entire path consisting of K links can be modeled as an aggregate Gilbert channel, with average packet loss rate (PL) and average burst length (LB) given by

$$PL = 1 - \prod_{i=1}^{K} \pi^i(0)\,, \qquad (5)$$

and

$$LB = \frac{1 - \prod_{i=1}^{K} \pi^i(0)}{(\prod_{i=1}^{K} \pi^i(0))(1 - \prod_{i=1}^{K}(1 - P_{01}^i))}\,, \qquad (6)$$

where $\pi^i(0) = 1 - \pi^i(1)$ is the steady-state probability of being in the reception state for intermediate link i and P_{01}^i is the corresponding transition probability from the reception state to the loss state [7].

2.4. Video Distortion Models

The end-to-end distortion of a reconstructed video sequence, denoted by D, results from two components: the distortion induced by source compression, denoted by D_s, and the channel distortion due to packet losses, denoted by D_c. We make use of the additivity assumption in [1], which states that the end-to-end distortion is the sum of D_s and D_c, $i.e.$,

$$D = D_s + D_c\,. \qquad (7)$$

As shown in [1], the compression distortion D_s can be expressed as:

$$D_s = \frac{\theta}{R_s - R_0} + D_0\,, \qquad (8)$$

where R_s is the source coding rate and θ, R_0 and D_0 depend on the INTRA rate β and other model parameters, which are

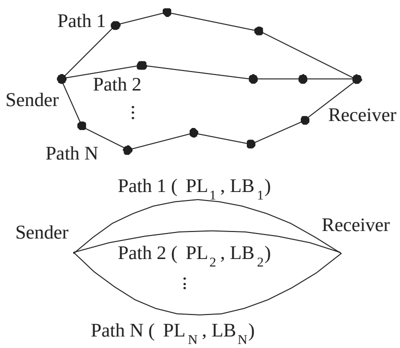

Figure 3. Multipath transport network with disjoint paths.

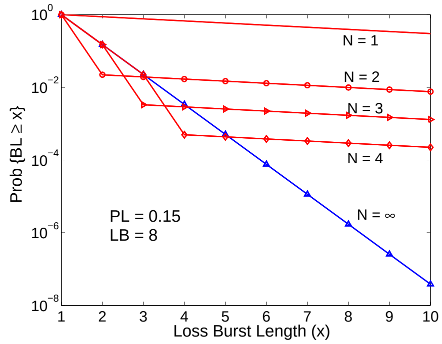

Figure 4. Complementary cdf of loss-burst-length for different N.

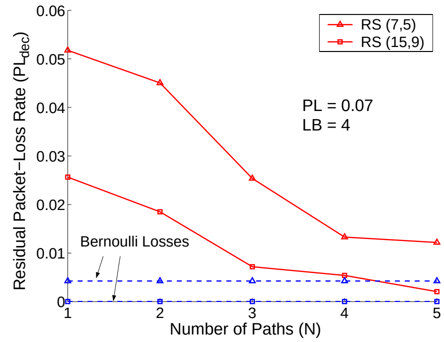

Figure 5. Residual packet-loss rates PL_{dec} vs. number of paths N.

specific to the encoded video sequence and can be obtained by fitting the model to experimental data.

The channel distortion (due to packet losses) D_c can be expressed as a function of the decoded packet-loss rate PL_{dec} and the INTRA coding rate β as [1]

$$D_c = \alpha PL_{dec} \sum_{t=0}^{T-1} \frac{1-\beta t}{1+\gamma t}\,, \tag{9}$$

where $T = 1/\beta$ and the parameter γ describes the efficiency of loop filtering to remove the effects of errors due to packet losses. Likewise, the model parameters γ and α can be obtained by fitting the model to experimental data.

3. Disjoint Paths

First, consider the case where the different paths share no joint links so that packet-loss processes on different paths are independent. Therefore, as illustrated in Fig. 3, each end-to-end path can be modeled as an independent Gilbert channel. The channel parameters associated with the aggregate Gilbert channel for the i-th path, PL_i and LB_i, can be obtained from (5) and (6), respectively[2].

3.1. Analysis of Loss-Burst-Length Statistics

The FEC performance is dependent on the burstiness of the underlying packet-loss processes. Generally, the less bursty the packet losses are, the better performance FEC can achieve. In this subsection, we quantitatively investigate the effectiveness of multipath transport in reducing the burstiness of packet losses. The results are used to explain the FEC performance improvement using path diversity described in the next subsection.

Suppose the random sequence $\{Y_n\}$ represents the packet loss process perceived by an end node, with 1 denoting loss and 0 denoting reception. The effective average loss burst length perceived by an end node can be expressed as

$$ALB_{eff} = \sum_{k=1}^{\infty} k * Pr\{LB_{eff} = k\} = \sum_{k=0}^{\infty} (k+1) * P\{1^k 0|01\}\,, \tag{10}$$

where the random variable LB_{eff} denotes the loss-burst-length seen by the receiver after packet reordering.

Here we model each of the N independent paths as an aggregate Gilbert model. To simplify the analysis, we assume these Gilbert models are homogeneous, each with average packet-loss probability and average burst length PL and LB, respectively. Furthermore, we assume the packets are transmitted over the network using the cyclic multipath transport scheme described in Section 2.3. Then the loss-burst-length distribution, $Pr\{LB_{eff} = k+1\}$, $k \geq 0$, can be expressed as

$$P\{1^k 0|01\} = \begin{cases} PL^k(1-PL) & ; \quad 0 \leq k \leq N-3 \\ PL^{N-2} P_{00} & ; \quad k = N-2 \\ PL^{N-2} P_{01} P_{11}^{k-N+1} P_{10} & ; \quad k \geq N-1\,, \end{cases} \tag{11}$$

where P_{01}, P_{10} are the state transition probabilities of each of the aggregate Gilbert models and can be computed from PL and LB according to (4) with $P_{00} = 1-P_{01}$, $P_{11} = 1-P_{10}$.

Therefore, from (10) and (11), the effective average loss burst length is

$$\begin{aligned} ALB_{eff} &= \textstyle\sum_{k=0}^{\infty} (k+1) P\{1^k 0|01\} \\ &= \textstyle\sum_{k=0}^{N-3} (k+1) PL^k (1-PL) \\ &+ \textstyle\sum_{k=N-2}^{N-2} (k+1) PL^{N-2} P_{00} \\ &+ \textstyle\sum_{N-1}^{\infty} PL^{N-2} P_{01} P_{11}^{k-N+1} P_{10} \\ &= 1/(1-PL)\,. \end{aligned} \tag{12}$$

This last expression indicates that, somewhat surprisingly, when the number of paths $N \geq 2$, the effective average loss-burst-length ALB_{eff} is independent of N and LB, but only depends on PL. Figure 4 provides a numerical example of the complementary cdf of the loss-burst-length, $Pr\{LB_{eff} \geq x\}$, with different orders of path diversity N. For comparison, we indicate the corresponding complementary cdf for the Bernoulli channel ($N = \infty$), where the losses are totally independent[3]. It indicates that, although the effective average loss-burst-length ALB_{eff} is the same for $N \geq 2$, the loss-burst-length distributions are quite different. More specifically, it shows that, with an increase of path

[2]For simplicity, we assume each of the disjoint paths are of the same length.

[3]When $N \rightarrow \infty$, each packet will be transmitted on a different path. In this case, the channel reduces to a Bernoulli channel, where the packet losses are totally independent.

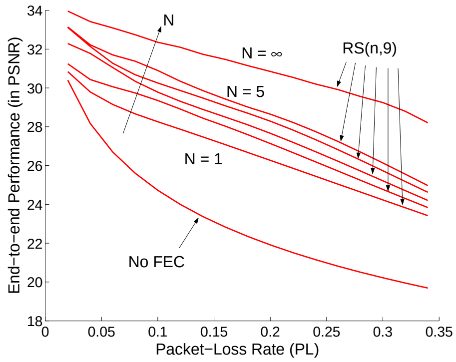

Figure 6. The end-to-end PSNR performance vs. packet-loss rate PL for the Susie sequence with $RS(n, 9)$ codes and path diversity N, where the FEC coding rates R_c are optimally selected; Other model parameters are set as follows: $R = 80$ Kbps, $\beta = 0.02$, $LB = 4$.

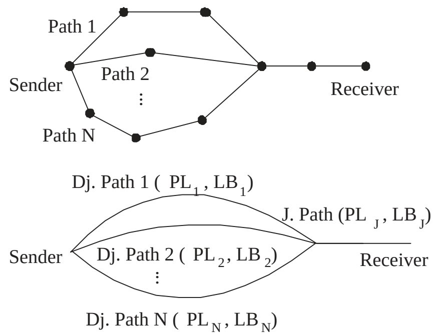

Figure 7. Multipath transport network with joint paths.

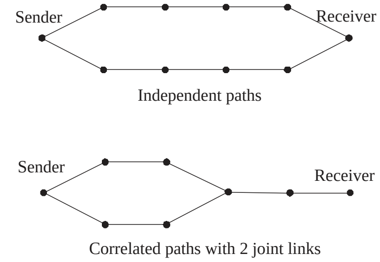

Figure 8. Path correlation model.

diversity order N, the probability $Pr\{LB_{eff} \geq x\}$ will decrease for relatively large x. This means that with increasing path diversity N, the residual packet-loss probability with FEC, PL_{dec}, will be reduced. We will further demonstrate this in the next subsection.

3.2. Analysis of Residual Packet-Loss Rates

We need to obtain the block error distribution $P(j, n)$ to evaluate PL_{dec} according to (3). Consider an arbitrary block of n packets. Without loss of generality, we assume the 1-st packet within this block is transmitted on path #1. Assume there are N paths in total. Then, out of n packets, the number of packets transmitted on the i-th path is

$$l_i = \left\lfloor \frac{n}{N} \right\rfloor + h_i, \qquad 1 \leq i \leq N \,, \tag{13}$$

where

$$h_i = \begin{cases} 1, & \text{if } i \leq n \bmod N \,, \\ 0, & \text{otherwise} \,. \end{cases} \tag{14}$$

Out of these n packets, let b_i denote the number of lost packets transmitted on the i-th path. The total number of lost packets is then

$$\sum_{i=1}^{N} b_i = j \,. \tag{15}$$

Since the packet-loss processes of these N path are independent of each other, we have

$$P(j, n) = \sum_{S} \left(\prod_{i=1}^{N} P_i(b_i, l_i) \right) \,, \tag{16}$$

where the sum is over the set

$$S = \left\{ (b_1, b_2, \ldots, b_N) : \sum_{i=1}^{N} b_i = j \right\} \,, \tag{17}$$

and $P_i(b_i, l_i)$ denotes the block error distribution on the i-th path. If we model the packet-loss process over each path as a Gilbert model, as expressed by (5) and (6), then the block error distribution $P_i(b_i, l_i)$ on each path can be obtained by a recursive algorithm first proposed in [8].

Figure 5 provides a numerical example of the efficacy of path diversity in reducing the residual packet-loss rates for the $RS(7, 5)$ and $RS(15, 9)$ codes, where each path is modeled as a homogeneous and independent Gilbert model with $PL = 0.07$ and $LB = 4$. It demonstrates that, as expected, packet transport with path diversity can improve the efficacy of FEC coding significantly. We have also indicated in Fig. 5 the limiting residual packet loss performance if the losses are independent ($N \to \infty$) [4], $i.e.$, a Bernoulli channel. Observe the rapid approach of the residual packet-loss rates to their limiting values with increasing N. Also note that for use of a fixed code there is little advantage to path diversity orders $N > 4$. Additional results described in [9] also demonstrate the effectiveness of multipath transport in reducing the probability of large bursts and their effect on end-to-end performance.

3.3. Video Performance Using Multipath Transport

Figure 6 demonstrates a comparison of the end-to-end video performance with different network packet-loss rates (PL) for the following cases: 1) without coding ($R_c = 1$) or path diversity ($N = 1$); 2) with an $RS(n, 9)$ code, but no path diversity ($N = 1$), where R_c is optimally selected[5]; 3) combined $RS(n, 9)$ code and path diversity ($N \geq 2$), where R_c is again optimally selected, for the QCIF Susie test sequence. For comparison, we have also indicated the performance achieved on a Bernoulli channel ($N = \infty$). The QCIF Susie sequence (176×144) consists of 150 frames at $f_r = 30$

[4]The limiting value for the $RS(15, 9)$ code is extremely small and indistinguishable from zero in Fig. 5.

[5]Optimally selecting R_c to maximize end-to-end performance for a given overall transmission rate R represents a joint source-channel coding (JSCC) approach.

frames/sec. We assume every $M = 11$ MBs are packetized into one packet. Therefore, each QCIF frame is packetized into $99/M = 9$ packets. Other system parameters are indicated in the figure caption. This figure indicates that, compared to the case of JSCC without path diversity, the combination of JSCC and path diversity can provide significantly improved end-to-end video performance. For example, for $PL = 15\%$, there is a performance advantage of 2 dB in going from $N = 1$ to $N = 4$. However, unlike the fixed code case illustrated in Fig. 5, observe that when the code rate is optimally chosen as part of a JSCC approach there is a considerable performance advantage to path diversity orders $N > 4$.

4. Correlated Paths

In the previous section, we assumed that the different paths share no joint links so that the packet-loss processes on different paths are independent. However, in actual multipath transport networks there may be some shared or joint links between different transport paths. In this case, the packet-loss processes on different paths may be correlated. In actual networks the connection topologies (joint/disjoint links) between the sender and the receiver may be quite varied so that precise modeling of video transport using path diversity can be fairly complex. However, it has been shown in [4, 5] that the important end-to-end properties of a 2-path network can be captured using a simplified three-subpath topology, where subpaths 1 and 2 are formed by the disjoint links along the two paths and subpath 3 is formed by the joint links along both paths. In this section we consider a similar approach as in [4, 5], except that we will consider a more general N-path network. We will specifically concentrate on the effect of path correlation on the end-to-end video performance using packet-level FEC. To model the effect of path correlation on FEC performance, we consider a simplified scenario: different paths share common joint links, as shown in the upper subfigure of Fig. 7, *i.e.*, a number of disjoint links followed by a series of joint links.

4.1. Analysis of Residual Packet-loss Rates

We assume each subpath can be described by a Gilbert model, as illustrated in the bottom subfigure of Fig. 7, where the associated model parameters can be derived from the corresponding portion of the original path using (5) and (6). To evaluate the residual packet-loss rates PL_{dec} after FEC decoding, again we need to determine the block error distribution $P(j, n)$ as seen by the FEC decoder after packet reordering. Assume there are j packets lost out of n consecutive packets. If the total number of packets lost over disjoint links is $b^D = d$ then the number of packets lost over the joint links is $b^J = j - d$. Therefore, we have

$$P(j,n) = \sum_{d=0}^{j} \Big(Pr\{b^D = d\} * Pr\{b^J = j-d\} \Big) . \quad (18)$$

Since out of these n packets d packets have already been lost over the disjoint links, only the remaining $n - d$ packets will be transmitted over the joint links. Therefore, we have

$$Pr\{b^J = j-d\} = P^J(j-d, n-d) , \quad (19)$$

where $P^J(j - d, n - d)$ denotes the block error distribution over the joint path.

Let b_i^D denote the number of packets lost over disjoint path i. Therefore, we have

$$b^D = \sum_{i=0}^{N} b_i^D . \quad (20)$$

Following a similar approach as in going from (15) to (17), it can be shown that [9]

$$P(j,n) = \sum_{d=0}^{j} \bigg(\Big(\sum_{S_d} \Big(\prod_{i=1}^{N} P_i^D(b_i^D, l_i) \Big) \Big) * P^J(j-d, n-d) \bigg) , \quad (21)$$

where l_i is given by (13) and

$$S_d = \left\{ \left(b_1^D, b_2^D, \ldots, b_N^D \right) : \sum_{i=1}^{N} b_i^D = d \right\} . \quad (22)$$

If we model the packet-loss process over each joint/disjoint subpath as a Gilbert model, then the block error distributions $P_i^D(b_i^D, l_i)$ and $P^J(j - d, n - d)$ on each of the joint/disjoint paths can be obtained by the recursive algorithm originally proposed in [8].

4.2. Effect of Path Correlation on Packet-Level FEC Performance

In order to investigate the effect of path correlation on FEC performance, we consider a simplified 2-path transport scheme and assume there are 5 intermediate links on each of the 2 paths, as illustrated in Fig. 8. We further assume that each intermediate link is independently modeled by a Gilbert channel with parameters PL^i and LB^i. Therefore, the corresponding end-to-end PL and LB can be computed from (5) and (6), respectively, for each path. However, the two paths may share some joint links. Let J denote the number of joint links. The larger J is, the more correlated the two paths are. Figure 8 shows the case of $J = 0$ and $J = 2$. The block packet-loss distribution $P(j, n)$ and the residual packet-loss rate after channel decoding PL_{dec} can be obtained from (21) and (3), respectively.

Table 1 shows the effect of path correlation on the FEC performance with two different RS codes and for two channel conditions. More specifically, it shows the residual packet-loss rates PL_{dec} using a relatively weak $RS(22, 18)$ code and a stronger $RS(31, 18)$ code under two different channel conditions: a relatively bad channel with $PL = 10\%$ and $LB = 8$ and a relatively good channel with $PL = 5\%$ and $LB = 4$. Generally, the results indicate that increased correlation among paths results in a higher residual packet-loss rate PL_{dec} after channel decoding. However, it should be noted that, when the channel conditions are relatively good, or when a weak code is used, the path correlation level (J) does not make much difference in the FEC performance. But when the channel conditions are relatively bad ($PL = 10\%$ and $LB = 8$) and a stronger code is used, the path correlation level J can have a significant effect on the FEC performance. This means that path correlation has a significant impact on FEC performance only when channel conditions are severe and strong FEC codes are used. These analytical results are consistent with the conclusions in [2], where the results there were obtained by simulations.

J	$PL = 10\%, LB = 8$		$PL = 5\%, LB = 4$	
	$RS(22, 18)$	$RS(31, 18)$	$RS(22, 18)$	$RS(31, 18)$
0	8.05 %	3.03 %	2.84 %	0.32 %
1	8.12 %	3.09 %	2.90 %	0.34 %
2	8.20 %	3.34 %	2.95 %	0.39 %
3	8.28 %	3.58 %	3.00 %	0.43 %
4	8.37 %	3.80 %	3.05 %	0.46 %
5	8.46 %	4.01 %	3.11 %	0.49 %

Table 1. Decoded packet-loss rates (PL_{dec}) of a 2-path transport network with different path correlations and FEC code schemes.

4.3. Effect of Path Correlation on Video Performance

Figure 9 demonstrates the effect of path correlation on video performance transmitted over different multipath transport networks. Specifically, it demonstrates a comparison of the end-to-end video performance achieved over multipath transport networks with different numbers of joint links (J) for different path diversity orders (N) and coding strategies (with or without coding) as shown in Fig. 6. The path correlation model is the same as described in Section 4.2, but with different path diversity N. For the coded systems, the code rate R_c is chosen optimally. Other model parameters are indicated in the caption. This figure indicates that, for all path correlation levels (J), increased path diversity combined with JSCC generally can provide improved end-to-end video performance. However, with increased path correlation, the advantage achieved with the use of path diversity will decrease, and with $J \geq 4$ there is little difference in performance independent of the value of N.

5. Summary and Conclusions

We have modeled and analyzed an SD coded video transmission system employing packet-level FEC in combination with path diversity. We provided a precise analytical approach to evaluating the efficacy of path diversity in reducing the burstiness of packet-loss processes. Using this approach we have quantitatively demonstrated the advantages of path diversity in improving end-to-end video transport using packet-level FEC. Finally, we have quantitatively demonstrated the effect of path correlation on the packet-level FEC performance.

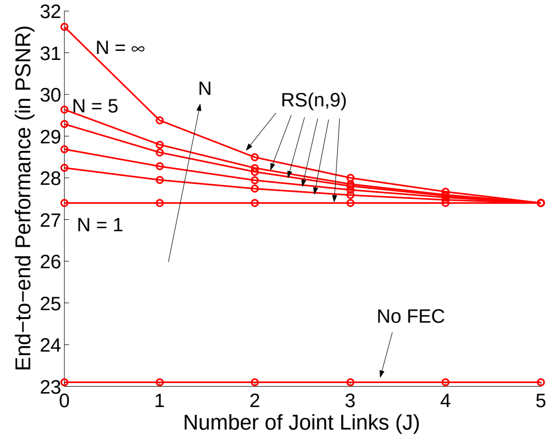

Figure 9. The end-to-end PSNR performance vs. the number of joint links J for the Susie sequence with $RS(n, 9)$ codes and path diversity N, where the FEC coding rates R_c are optimally selected; Other model parameters are set as follows: $R = 80$ Kbps, $\beta = 0.02$, $PL = 15\%$, $LB = 4$.

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