Tutorial 1. *Monopoly and Price Discrimination.*

Question 1. Suppose there are 10 students in a class and teacher brings a bag with 10 candies. Assume all students have identical preferences and have positive marginal value of candy.

(a) Is allocation in which each student gets a candy Pareto efficient?

(b) Is allocation in which one student gets all the candy Pareto efficient?

Problem 1. Monopoly sells its product to $N = 10$ identical consumers each of whom has individual demand $P = 30 - 2q$, where q is quantity demanded by a single consumer at price P. The firm has constant marginal cost $MC = 5$ and no fixed cost.

(a) Suppose the firm cannot price discriminate. Derive aggregate market demand $P(Q)$, where Q is the quantity demanded by all consumers at price P. Set up firm's profit function $\pi(Q)$, find profit-maximizing quantity Q_M and price P_M. Calculate firm's profits, consumer surplus and welfare loss. Demonstrate on a diagram.

(b) Suppose the firm decides to use two-part pricing. Describe optimal two-part pricing scheme, calculate total quantity produced by the firm, firm's profits, consumer surplus, and deadweight loss.

(c) Give definition of perfect price discrimination. Calculate quantity produced and profits if firm employs this pricing strategy. Compare your results to part (b).

Problem 2. A firm is selling its product in two markets. In market A the demand is given by $Q_A = 100 - 2P$ and in market B the demand is $Q_B = 80 - 4P$. The firm's total cost is $C = \frac{Q^2}{12}$ where $Q = Q_A + Q_B$ is the total output.

(a) Find the profit-maximizing prices and quantities, calculate the profits if firm can successfully price discriminate.

(b) Calculate the price elasticities of demand at the equilibrium prices and quantities.

(c) What is the relationship between the price elasticities and the prices charged in each market?

Problem 3. A monopoly is facing two groups of consumers. Consumers of type 1 have relatively low demand given by $P_L = 5 - q_L$ and consumers of type 2 have relatively high demand $P_H = 10 - q_H$, where q_i is quantity demanded by one consumer per month, $i \in \{L, H\}$. The firm's marginal cost is $MC = 0$.

(a) Suppose the types are observable and the firm offers two packages with quantity q_i at fixed monthly fee F_i, what quantities and fees maximize the firm's profits? *(Hint: use the strategy similar to two-part tariff)*

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(b) Now suppose that the firm is unable to distinguish the types, would the pricing scheme from part (a) work if the firm kept offering the same two packages?

(c) If the firm is unable to distinguish between the types, but wants to offer two different packages with quantities you found in part (a), what monthly fees should it charge to the different types of consumers?

Problem 4. A monopoly is facing a non-linear inverse market demand given by $P = \frac{100}{\sqrt{Q}}$ and has cost function $C = 20 + 10Q$. Derive firm's MR and MC curves, calculate profit-maximizing price and quantity if the firm is charging single price. Show on a diagram.

Additional Questions

1. Go back to problem 1, part (a). Calculate price elasticity of demand at Q_M. Calculate Lerner index, give a verbal interpretation of the value you obtain.
2. Prove formally (using mathematical equations) that a monopoly with a non-zero marginal cost operates on the elastic part of the demand curve.
3. YYoga offers several membership options: drop in at $17 dollars per class, package of 10 tickets for $145, monthly memberships at $105 and yearly membership at approximately $90/mo. Is this an example of price discrimination? Argue why yes or no. Explain how the firm can increase profits this way compared to charging single price.
4. If a market is monopolistic, how would you determine whether it is a natural monopoly? Is Canada Post a natural monopoly?
5. Suppose that there are two outcomes one is efficient and the other is inefficient. In the efficient outcome everyone is better off compared to the inefficient one. Is this true or false?
6. Exercises 6.5.5.1 and 6.5.5.2 on page 6-240 of MAF.

Technical Notes.

Deriving Aggregate Demand.

Recall that individual demand curve is the result of maximizing utility subject to budget constraint $q_1 = q_1(P_1, P_2, M)$ - quantity demanded of good 1 is a function of consumer's income M and prices P_1 and P_2. When we draw a demand curve for good 1 on a diagram we keep M and P_2 constant, and, to be precise, we plot inverse demand curve $P(q)$ - which tells us the marginal value of the good (sometimes referred to as reservation price). To derive market demand we need to find total quantity demanded by all consumers at each price (horizontal summation of individual demand curves):

$$Q(P) = \Sigma q_i(P) \tag{1}$$

where Q is the total quantity demanded and q_i is quantity demanded by a consumer i.

When there are N identical consumers, like in problem 1, then to find market demand you need to

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- Single out $q(P)$ from individual demands
- Write market quantity as $Q = N \cdot q$, sub $q(P)$
- Single out P, which now will be function of Q

For **example:** there are $N = 100$ consumers with individual inverse demands $P = 50 - 10q$. Follow the steps above: $q(P) = \frac{50-P}{10}$;
$Q = 100(\frac{50-P}{10}) = 500 - 10P$, $P = 50 - 0.1Q$ - is the inverse market demand. Notice that is has the same vertical intercept and flatter slope compared to individual demands.

For exercises 6.5.5.1 and 6.5.5.2 you need to find aggregate demand when you are given two market demand curves. The idea is the same: single out Q_i demanded in each market segment, to find total Q add them up, solve for P.

Elasticity.

$\varepsilon = \frac{\Delta\%Q}{\Delta\%P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} = \frac{dQ(P)}{dP} \cdot \frac{P}{Q}$. If you have equation of the inverse demand $P(Q)$ you can use $\varepsilon = \frac{1}{\frac{dP(Q)}{dQ}} \cdot \frac{P}{Q}$ where $\frac{dP(Q)}{dQ}$ is the slope of the inverse demand.

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