Tutorial 10. *Externalities.* *Solutions*

Problem 1. Alfred and Ben own cabins at a lake. Alfred has a Jet Ski that he loves to ride on the lake, which understandably disturbs Ben. Alfred's utility is $U_A = 20\sqrt{x} + C_A$, where x is the number of hours he spends riding the Ski and C_A is the amount of money he spends on all other goods (formally called composite commodity). Ben's utility is $U_B = C_B - x$, where C_B is the amount of money Ben has for his private consumption. Both Alfred and Ben have income of 120 dollars. The cost of one hour of Jet Skiing is 2 dollars.

(a) If there is no bylaw regarding the noise created by the Jet Skis, how many hours will Alfred spend having fun on his watercraft? (for now assume they do not communicate).

Maximize Alfred's utility subject to the budget constraint $P_1q_1 + P_2q_2 = M$, which in this case is $2x + C_A = 120$; notice that since C_A is measured in dollars, the price of one unit of this good is one dollar. You can substitute $C_A = 120 - 2x$ from the budget constraint into the utility to obtain $U_A = 20\sqrt{x} + 120 - 2x$, find first-order conditions $\frac{dU_A}{dx} = \frac{20 \cdot 0.5}{\sqrt{x}} - 2 = 0$, so in his selfish utility-maximizing equilibrium Alfred will spend $x^* = 25$ hours riding the Jet Ski.
Alternative way to solve this, which is more in line with the lecture, is as follows. Alfred will choose his equilibrium x^* such that his MPB=MPC. MPB is his marginal rate of substitution $MRS = \frac{MU_x}{MU_C}$ which is defined as the max amount of C_A he's willing to sacrifice for one more hour of fun with the Ski. MPC is the relative price of the time spent on the Jet = opportunity cost of x. The utility function is quasilinear meaning that MRS only depends on x, so optimal x must satisfy $\frac{10}{\sqrt{x}} = 2$ in the interior solution. Since $x = 25$ is affordable, we don't have to worry about the corner solution.
The corresponding utilities are $U_A = 20\sqrt{25} + 120 - 50 = 170$ and $U_B = 120 - 25 = 95$; $U_A + U_B = 265$.

(b) What is the efficient number of hours that Alfred should spend on the Jet Ski?

You can find it by maximizing $U_A + U_B = 20\sqrt{x} + 120 - 2x + 120 - x$ or using the F.O.C. for efficiency: $MSB = MSC$. In this case the only person who benefits from the activity is Alfred, so social marginal benefit is the same as private marginal benefit (MRS from part (a)) and social marginal cost is $MPC + MEC = 2 + 1 = 3$. Which solves for $x_E = 11.1$
The resulting utilities are $U_A = 20\sqrt{11.1} + 120 - 22.2 = 164.4$, $U_B = 120 - 11.1 = 108.9$, $U_A + U_B = 273.3$. In equilibrium Alfred skis too much, in the sense that the net value of the units above the efficient quantity to Alfred is smaller than the disutility to Ben. As we reduce amount of skiing to the efficient level, A gets smaller U, but since at x^* his MPB was smaller than MSC, when he does reduce x the overall wealth between the two increases.

(c) In this particular situation can we expect Alfred and Ben to reach efficient outcome or is government intervention absolutely necessary to guarantee efficiency?

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In this case, if they are diplomatic and honest enough, achieving efficiency should not be a problem and as long as they agree who has the property rights over the externality the outcome should be efficient. If there are no regulations regarding the amount of noise it may logically follow that Alfred is free to do whatever he wants, so Ben will compensate him for reducing the amount of skiing to the efficient level (notice that there is no unique transfer, in this case Ben will be willing to pay at most the amount what will leave him indifferent with his utility from part (a), in that case all gains from the noise reduction are captured by Alfred, or Ben will pay Alfred minimum compensation that will be enough to make Alfred reduce the hours (in this case all gains will accrue to Ben), obviously any transfer in between will make both strictly better-off.

Problem 2. *Externalities, Efficiency, and Property Rights.* Alfred and Ben share an apartment. Alfred likes loud music and Ben does not. Suppose that music imposes an increasing marginal cost on Ben $MEC = Q$ where MEC is dollars and Q is hours of loud music. Alfred's marginal value of each hour of loud music measured in dollars is $MPB = 10 - Q$.

(a) Represent this situation on diagram. Show efficient consumption of loud music and situation when Alfred listens to loud music as much as he wants.

Since you are not given any info about private costs, assume MPC=0; On his own Alfred will consume where his MPB=MPC, Q=10.
For efficiency we need MSC=MSB. $MSC = MEC$ and $MSB = MPB$, efficient consumption of loud music is 5 hours, at Q=5 MSB=10-5=5=MSC.
Welfare in unregulated outcome: *DWL=25 Alfred's total benefits from the music=total value of 10 hours=50, denote $U^A = 50$, Ben's total costs are summation of MEC from each hour=area under MEC, which is 50, denote $U^B = -50$.*

(b) Suppose that there are no legal restrictions on volume of music and Alfred has a right to listen to music as loud as he wants and as much as he wants. Ben approaches Alfred with an offer to pay money for reducing the number of hours of the loud music. Will negotiations achieve efficient outcome?

For simplicity suppose both guys know that if there is no agreement Alfred will listen to music for 10 hours. Negotiations will work as follows. For each hour when Alfred ***does not*** *turn up the volume Ben will be willing to offer payment up to the amount of the marginal cost imposed on Ben. For example 10^{th} hour causes Ben MEC=10, meaning that it makes Ben so sick he will pay up to 10 dollars to avoid it. Alfred's marginal value of 10^{th} hour is $MPB = 10 - 10 = 0$, meaning that he will accept any payment in order to give up the last hour of music. Clearly they can agree on some payment that will make both better off. They will continue negotiations until price that Ben is willing to pay is equal to price that Alfred is willing to accept, which happens at 5 hours of music, which is the efficient level of consumption.*
Suppose that Ben paid 5 dollars for each hour of music reduction. Let's calculate guys' welfare in this case: $U^A = 37.5 + 25 = 62.5$, which is Alfred's total value of 5 hours consumed[1] *+ payment from Ben. Ben's utility $U^B = -12.5 - 25 = -37.5$, which is total damage from 5 hours of music and the payment. Notice what happened: both are better*

[1] Total value = area under D curve for the quantity consumed.

off compared to situation when Alfred listened to music for 10 hours, and sum of each guys' increase in welfare = DWL, this means that ***during negotiations they managed to capture the efficiency loss brought about by the externality****. Actual price for which the externality is traded does not have to be \$5/hour and is indeterminate - there are many prices that will be acceptable for both guys, but regardless of the price* ***at least one of them will benefit from reducing level of music while the other one will not be hurt****.*
Internalizing the externality in this case works as follows. although Alfred does not have to pay any price in order to listen to music, possibility of getting paid for not listening creates the ***opportunity cost****, which as you remember is a part of economic costs: every time Alfred listens to 1 hour of music he loses an opportunity to get some money from Ben.*

(c) Suppose Ben has a right for quiet environment. This means that loud music is legal, but Ben has a right to prohibit Alfred from playing music loudly any time. Suppose Alfred approaches Ben with an offer to compensate Ben for each hour of loud music. Will negotiations reach efficient outcome?

In order for Ben to accept an offer the compensation per hour should be at least as high as MEC, for example the minimum he will accept for the first hour is 1 dollar etc. Alfred will only pay up to his marginal value of the music, for the first hour he will pay about 9 dollars. Clearly it is possible for both to benefit from 'trading' on the first hour. Alfred will purchase 5 hours of music in total: all gains are exploited when price that Alfred is willing to pay for one more hour is equal to price that Ben is willing to accept . Let's again calculate welfare assuming that Alfred paid 5 dollars per each hour. $U^A = 12.5$*, which is his consumer surplus = TV-payment, Ben's welfare* $U^B = 12.5$*, which is the payment he received minus the cost of the music.*

(d) Compare parts (b) and (c) and make a conclusion.

If you did everything correctly you already know that in both cases the level of music is the same and it is efficient. The difference is apparently in who pays. Notice that whoever has property rights is in advantageous position and will be able to gain from selling the rights.

(e) **Discussion.** This problem is a demonstration of ***Coase theorem***. Explain how does Coase theorem apply to the situation described in this problem. Can you think of any reasons why negotiations may fail between Ben and Alfred?

Coase theorem*: when there is small number of parties involved, transaction costs are zero, there is no hidden information and property rights are clearly defined, private parties will achieve efficient solution even if externality is present. More or less obvious reason for unsuccessful negotiations is asymmetric information: if guys do not know each other's costs and benefits they might not reach an agreement. Transaction costs/contract enforcement - what if Alfred takes money from Ben and then cheats and listens to music anyways? Notice that if guys lived in a residence building where half of people loved loud music and the other half did not, transaction costs would be extremely high and even if property rights were defined, it would be very hard to reach efficient solution.*