Tutorial 11. *Public Goods.*
Solutions.

Problem 1. And now suppose that Alfred and Ben still live in the cabins on a lake and this time they are married and have kids, so they are contemplating building a common playground for their kids. Their utilities are $U_i = \sqrt{x} + c_i$ where x is the total amount spent on the playground $x = x_A + x_B$ and c_i is the composite commodity, which is the amount of money each has left for private consumption. Suppose each has income of 5 dollars.

(a) How much can we expect Alfred and Ben to spend at most on the playground in total?

Each maximizes $U_i = \sqrt{x_i + x_j} + 5 - x_i$, taking the other person's contribution as given; each chooses contribution so that MPB=MPC (MRS equals relative price), F.O.C give us $\frac{1}{2\sqrt{x_i + x_j}} = 1$. Or any combination of x_A and x_B such that $x = .25$. In the non-cooperative outcome we cannot expect them to provide more than 0.25 units of the public good in total.

(b) Is the outcome in part (a) efficient? Explain.

Efficient spending should satisfy MSB=MSC, or $MRS_A + MRS_B = 1$ (or, can max $U_A + U_B$. $\frac{1}{\sqrt{x_i + x_j}} = 1$, efficient spending is x=1.) Notice that the public goods are a special case of externalities: when Alfred or Ben choose their private contributions, they only consider their own benefits, ignoring the positive impact on the other person.
Remark: that if they share the cost of the public good (each pays the share equal to their MV at the efficient Q) both are better off in the efficient outcome, however, the free-rider problem may still present. For extra readings on efficient provision of public goods you can look up Lindahl equilibrium.

Problem 2. Now Alfred and Ben are two students who share an apartment. For each of them the disutility from cleaning the apartment is 12 dollars and the value of having the apartment clean is 10. If they decide to clean up together, naturally they will share the cost equally. Represent this strategic situation in a normal form and find the Nash equilibrium outcome of this game. Discuss what is happening here.

Ben

		Clean	Ignore
Alfred	Clean	4; 4;	-2, 10
	Ignore	10; -2	0, 0

Since the public good is non-excludible, once the apartment is clean, both enjoy the benefits, so each prefers the other person to do the job. If the cost of cleaning was smaller than 10, there would be 2 NE in which only one person cleans, you may want to discuss with the students that again each person would prefer the other one to clean, so in reality we may observe an outcome that is not a NE.

If time left do problem 11 from Ch. 18 of EEA, p.637