Tutorial 1. *Monopoly and Price Discrimination.*

Question 1. Suppose there are 10 students in a class and teacher brings a bag with 10 candies. Assume all students have identical preferences and have positive marginal value of candy.

(a) Is allocation in which each student gets a candy Pareto efficient?

An allocation is ***Pareto efficient*** *when there is no other allocation such that at least one person is better-off and no one is worse-off. The allocation in which each student gets a candy satisfies the definition in this case: since all candy is already distributed, the only way to make one of the students better off (give him more candy) is to make some other student worse-off (take candy away).*

(b) Is allocation in which one student gets all the candy Pareto efficient?

In this situation whether the allocation is efficient or not depends on whether utility is transferrable. If transfers are not possible, then this allocation is efficient (provided that for that one lucky student the value of candy is does not fall to zero). If transfers are possible, then the efficient allocation must maximize the total well-being, in which case each candy should be given to a student who places the highest value on it. For example, if the marginal value of candy is diminishing and $MV = 25$ *cents for the first candy and falls to* $MV = 5$ *cents for the tenth candy, a situation when the student who got all ten candies 'sells' his last candy at any price* $5 \leq P \leq 25$ *is a Pareto improvement.*

Problem 1. Monopoly sells its product to $N = 10$ identical consumers each of whom has individual demand $P = 30 - 2q$, where q is quantity demanded by a single consumer at price P. The firm has constant marginal cost $MC = 5$ and no fixed cost.

(a) Suppose the firm cannot price discriminate. Derive aggregate market demand $P(Q)$, where Q is the quantity demanded by all consumers at price P. Set up firm's profit function $\pi(Q)$, find profit-maximizing quantity Q_M and price P_M. Calculate firm's profits, consumer surplus and welfare loss. Demonstrate on a diagram.

At price P each consumer demands $q = \frac{30-P}{2}$ *units, therefore, market quantity demanded is* $Q = 10q = 10\frac{30-P}{2} = 150-5P$ *which solves for inverse market demand* $P = 30-0.2Q$. $\pi = TR - TC = P \cdot Q + AC \cdot Q$. *Since* $FC = 0$ *and MC is constant, AC=MC=5; P depends on Q, sub inverse demand for P, obtain* $\pi = (30 - 0.2Q)Q - 5Q$. *To maximize profits F.O.C. is* $\frac{d\pi}{dQ} = 30 - 0.4Q - 5 = 0$*; notice that this is same as 'choosing output so that marginal revenue equals marginal cost':* $MR = 30 - 0.4Q = 5 = MC$. *The profits are maximized at* $Q_M = 62.5$. *Sub into demand to find* $P_M = 17.5$. *Calculate profits* $\pi = 781.25$, *CS=390.625. To find welfare loss you need the efficient output, which satisfies* $MV = MC$, *that is MC curve intersects demand curve:* $P = 30 - 0.2Q = 5 = MC$, *denote* $Q^* = 125$. *DWL=390.625*

(b) Suppose the firm decides to use two-part pricing. Describe optimal two-part pricing scheme, calculate total quantity produced by the firm, firm's profits, consumer surplus, and deadweight loss.

When consumers have identical demands optimal two-part pricing is to set per unit price $P = MC$, which is 5 in this case, and then charge the fixed fee equal to CS at that price. Use individual demands to find that at P=5 each consumer demands q=12.5 and for each consumer CS=156.25, which is the highest fee he is willing to pay to be able to purchase the good at P=5. Notice that in this case the firm does not make any profits on the unit sales as P=AC, so the firm's profits come from the fixed fees collected, which totals 1,562 for the 10 consumers. Firm produces efficient output, however, in this specific instance efficiency does not benefit consumers since CS is captured as firm's profits.

(c) Give definition of perfect price discrimination. Calculate quantity produced and profits if firm employs this pricing strategy. Compare your results to part (b).

Perfect price discrimination is a rather unrealistic strategy when for each unit sold firm is able to extract highest possible price, the unit's MV that is. Notice that in this case to sell an additional unit the firm does not lower the price on the units it is selling otherwise, which means MR=MV=Demand curve. This results in the profit-maximizing output (MR=MC) being same as the efficient output (MV=MC), therefore $Q_M = Q^* = 125$. Firm collects $TR = \sum MV's$, which is the total value of the good to the consumers (area under demand curve for Q^*). In conclusion, same as in part (b) the firm produces efficient output, however it does not mean that consumers get any benefit out of it: the entire CS is captured as firm's profits.

Notice that in parts (b) and (c) the firm captures all potential gains from trade as profits, that is why in those cases π equals to the sum of CS, DWL, and π from part (a).

Problem 2. A firm is selling its product in two markets. In market A the demand is given by $Q_A = 100 - 2P$ and in market B the demand is $Q_B = 80 - 4P$. The firm's total cost is $C = \frac{Q^2}{12}$ where $Q = Q_A + Q_B$ is the total output.

(a) Find the profit-maximizing prices and quantities, calculate the profits if firm can successfully price discriminate.

This would be a case of ordinary price discrimination: segregate consumers into market segments and charge different prices in each segment[1]*. Find inverse demands in both markets $P_A = 50 - 0.5Q_A$ and $P_B = 20 - 0.25Q_B$. Profits are the difference between the sum of the revenues generated in both market segments and the total cost of producing the output:*

$$\pi(Q_A, Q_B) = TR_A(Q_A) + TR_B(Q_B) - TC(Q_A + Q_B) = P_A \cdot Q_A + P_B \cdot Q_B - TC(Q_A + Q_B)$$

Sub demands and the cost function to obtain

$$\pi = (50 - 0.5Q_A)Q_A + (20 - 0.25Q_B)Q_B - \frac{(Q_A + Q_B)^2}{12}$$

[1] Recall that this is not always possible: the firm must be able to prevent re-sales and also identify consumers' types.

Find the first-order conditions:

$$\frac{\partial \pi}{\partial Q_A} = 50 - Q_A - \frac{Q_A + Q_B}{6} = 0 \quad (1)$$

$$\frac{\partial \pi}{\partial Q_B} = 20 - 0.5Q_A - \frac{Q_A + Q_B}{6} = 0 \quad (2)$$

Notice that from the FOCs it follows that the profit-maximizing combination of Qs must satisfy $MR_A = MR_B = MC(Q_B + Q_B)$. *Think about why it must be true (prove why if it is not satisfied the profits are not maximized). The quantities that satisfy both FOCs are* $Q_A = 40$, $Q_B = 20$ *and the respective prices are* $P_A = 30$ *and* $P_B = 15$.

(b) Calculate the price elasticities of demand at the equilibrium prices and quantities.

$\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q}$. *In Segment A* $\frac{dQ_A}{dP_A} = -2$, $\varepsilon_A = -2 \cdot \frac{30}{40} = -1.5$. *In Segment B* $\frac{dQ_B}{dP_B} = -4$, $\varepsilon_B = -4 \cdot \frac{15}{20} = -3$.
You can play around with the numbers: recall that $MR = P(1 + \frac{1}{\varepsilon})$, *use* $MR = MC$ *to find that* $P = \frac{MC}{1+\frac{1}{\varepsilon}}$. *In total firm produces* $Q = 60$, *at which point MC=10, so* $P_A = \frac{10}{1+\frac{1}{-1.5}} = 30$ *and* $P_B = \frac{10}{1+\frac{1}{-3}} = 15$.

(c) What is the relationship between the price elasticities and the prices charged in each market?

Problem 3. A monopoly is facing two groups of consumers. Consumers of type 1 have relatively low demand given by $P_L = 5 - q_L$ and consumers of type 2 have relatively high demand $P_H = 10 - q_H$, where q_i is quantity demanded by one consumer per month, $i \in \{L, H\}$. The firm's marginal cost is $MC = 0$.

(a) Suppose the types are observable and the firm offers two packages with quantity q_i at fixed monthly fee F_i, what quantities and fees maximize the firm's profits?

For consumers of type 1 the package is $q_L = 5$, $F_L = 12.5$; *for consumers of type 2 the package is* $q_H = 10$, $F_H = 50$. *Similar to the two-part pricing, the firm will choose* q_i *s.t.* $MV_i = MC = 0$ *and charge the fee* F_i *equal to the total value (area under the D curve) for the package.*

(b) Now suppose that the firm is unable to distinguish the types, would the pricing scheme from part (a) work if the firm kept offering the same two packages?

Now consumers of type 1 keep buying their package (for them the other package does not look attractive at all), but consumers of type 2 prefer the package designed for type 1: If they consumer 5 units they get total value $TV = 37.5$ *(area under their D-curve for q=5), but only pay 12.5 dollars, which leaves them with a positive consumer surplus, whereas their own package results in* $CS = 0$.

(c) If the firm is unable to distinguish between the types, but wants to offer two different packages with quantities you found in part (a), what monthly fees should it charge to the different types of consumers?

If the firm wants to sell $q_L = 5$ *and* $q_H = 10$ *respectively, it should lower the* F_H *t $25 so that the customers of this type get at least as high CS as when they buy the other package.*
Notice that if there is equal number of consumers of both types, this does not really make sense: it would be better to just offer the package with 10 units at fee 50 dollars and only serve high demand customers. However, if the proportion of the low-demand clients is relatively high, the firm will get higher profits by serving both types. Finally, if firm is facing heterogeneous consumers, it can also charge $P > MC$ *and FEE=CS of low-demand clients at that price. That way firm will be making profits from both fee and marking up the price.*

Problem 4. A monopoly is facing a non-linear inverse market demand given by $P = \frac{100}{\sqrt{Q}}$ and has cost function $C = 20 + 10Q$. Derive firm's MR and MC curves, calculate profit-maximizing price and quantity if the firm is charging single price. Show on a diagram.

$MR = \frac{50}{\sqrt{Q} = MC = 10}$. $Q_M = 25$ *and* $P_M = 20$.

Additional Questions

1. Go back to problem 1, part (a). Calculate price elasticity of demand at Q_M. Calculate Lerner index, give a verbal interpretation of the value you obtain. $\varepsilon = -1.4$
2. Prove formally (using mathematical equations) that a monopoly with a non-zero marginal cost operates on the elastic part of the demand curve. *Use* $MR = P(1 + \frac{1}{\varepsilon})$ and $MR = MC$ *to prove that when* $MC > 0$ *it must be that* $|\varepsilon| > 1$.
3. YYoga offers several membership options: drop in at $17 dollars per class, package of 10 tickets for $145, monthly memberships at $105 and yearly membership at approximately $90/mo. Is this an example of price discrimination? Argue why yes or no. Explain how the firm can increase profits this way compared to charging single price. *This can be an indirect price discrimination: consumers with inelastic demand go for the drop in or the 10 ticket option while consumers with more elastic demand choose to buy memberships (in this case marginal cost of attending a class becomes zero, which results in higher CS, part of which is captured by the firm).*
4. If a market is monopolistic, how would you determine whether it is a natural monopoly? Is Canada Post a natural monopoly?
5. Suppose that there are two outcomes one is efficient and the other is inefficient. In the efficient outcome everyone is better off compared to the inefficient one. Is this true or false? *False. You can use your calculations for problem 1 as an example.*
6. Exercises 6.5.5.1 and 6.5.5.2 on page 6-240 of MAF.

[idudnyk@sfu.ca]

Technical Notes.

Deriving Aggregate Demand.

Recall that individual demand curve is the result of maximizing utility subject to budget constraint $q_1 = q_1(P_1, P_2, M)$ - quantity demanded of good 1 is a function of consumer's income M and prices P_1 and P_2. When we draw a demand curve for good 1 on a diagram we keep M and P_2 constant, and, to be precise, we plot inverse demand curve $P(q)$ - which tells us the marginal value of the good (sometimes referred to as reservation price). To derive market demand we need to find total quantity demanded by all consumers at each price (horizontal summation of individual demand curves):

$$Q(P) = \Sigma q_i(P) \tag{3}$$

where Q is the total quantity demanded and q_i is quantity demanded by a consumer i.

When there are N identical consumers, like in problem 1, then to find market demand you need to

- Single out $q(P)$ from individual demands
- Write market quantity as $Q = N \cdot q$, sub $q(P)$
- Single out P, which now will be function of Q

For **example:** there are $N = 100$ consumers with individual inverse demands $P = 50 - 10q$. Follow the steps above: $q(P) = \frac{50-P}{10}$;
$Q = 100(\frac{50-P}{10}) = 500 - 10P$, $P = 50 - 0.1Q$ - is the inverse market demand. Notice that is has the same vertical intercept and flatter slope compared to individual demands.

For exercises 6.5.5.1 and 6.5.5.2 you need to find aggregate demand when you are given two market demand curves. The idea is the same: single out Q_i demanded in each market segment, to find total Q add them up, solve for P.

Elasticity.

$\varepsilon = \frac{\Delta\%Q}{\Delta\%P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} = \frac{dQ(P)}{dP} \cdot \frac{P}{Q}$. If you have equation of the inverse demand $P(Q)$ you can use $\varepsilon = \frac{1}{\frac{dP(Q)}{dQ}} \cdot \frac{P}{Q}$ where $\frac{dP(Q)}{dQ}$ is the slope of the inverse demand.

[idudnyk@sfu.ca]