Tutorial 2. *Game Theory: Basic Consepts. Simultaneous Games.*

Warm-up questions: which of the following situations involve strategic thinking and can be modeled as a game?

1. OPEC members choosing their annual output.
2. Two manufacturers: one of nuts and one of bolts decide whether to use metric or American standards.
3. An electric company deciding whether to order a new power plant given its estimate of demand for electricity in ten years.

Problem 1. Consider a "Boxed Pigs" Game[1]: There is a Big Pig (player 1) and a Small Pig (player 2). For experimental purposes they are put in a box. In the box there is a panel in one end of the box and a food dispenser at the other end of the box. When a pig presses the panel, food is dispensed at the other end of the box. It costs 2 utility units to press the panel and total utility worth of the food dispensed is 10 units. Big Pig is strong and eats really fast - if he gets to the dispenser first - he eats 9 units of food and only 1 unit of food is left for the Small Pig. If Small Pig gets to the dispenser first - he can eat 4 units. If they arrive to the food dispenser at the same time Small Pig gets 3 units of food. The actions available to the pigs are to Press and Wait (by the food dispenser). If both choose W nothing happens. If one presses and the other waits, the one who presses will get to the dispenser second.

(a) Represent this game in normal form.

(b) For each player determine whether there is a dominant strategy.

(c) Find pure strategy NE in this game.

Problem 2. When I was teaching ECON 383 in 2007 one of the graded components on the course outline was 'surprise quizzes', however, the number of quizzes was not specified (obviously so weren't the dates). Let's model this situation as a game. The resulting simultaneous game we played each week was as follows. Students have two pure strategies: prepare (P) and relax (R). Iryna also has two pure strategies: quiz (Q) and no quiz (N). Assume students do not care about learning and just want to maximize "payoff = grade - disutility from studying". It is costly for Students to study, so they want to study only if there is a quiz; if there is no quiz they prefer to relax. It is costly for Iryna to make and grade a quiz, therefore, if Students are prepared Iryna prefers not to give quiz, but if students are not prepared, Iryna prefers quiz. The payoffs that satisfy these preferences over the outcomes can be as follows:

		Iryna	
		Q	N
Students	P	15; -10;	-20, 0
	R	-20; 15	15, -15

[1] Baldwin and Meese, 1979, you do not have to read the article

(a) If Iryna chooses Q, what is Students' best response (BR)? If Iryna chooses N, what is Students' BR?

(b) For each player determine whether there is a dominant strategy.

(c) Find pure strategy NE in this game.

(d) Find the NE is mixed strategies. Denote p =probability that students study; q =probability with which Iryna gives the quiz.

(e) Suppose that Iryna has to ensure that students study every week, does this mean that she must give a quiz every single week? In other words, is it possible that $q < 1$ and students still study?

Problem 3. 'Matching Pennies' is an example of a ***zero-sum game***: *a player wins at the expense of the other player, therefore, in each outcome the sum of the payoffs is zero.* Each round of the game Player 1 and Player 2 choose heads H or tail T. If they chose the same, as in outcomes (H, H) and (T, T) Player 1 gets one dollar from Player 2. If they choose different, Player 1 pays one dollar to Player 2. Represent this game in normal form and find all NE (in pure and mixed strategies).

Problem 4. This is another famous game, called 'Battle of Sexes'. On a night a Man and a Woman can either go to a boxing match or to listen to opera. Man likes boxing more than opera and the Woman prefers the opera to boxing, at the same time the like each other, so they would prefer to be together to doing their favorite thing on their own (for example, Woman would rather attend boxing match with the Man than to be at opera by herself). Construct a payoff matrix that would represent this game. Find all NE.

Problem 5. Consider a two-player Rock-Paper-Scissors game: rock beats scissors, scissors beat paper, paper beats rock (this is another zero-sum game). This game has no pure strategy NE and you can prove that in the mixed strategies NE each strategy is played with probability $\frac{1}{3}$. Suppose winning gives player payoff of 1 and losing results in -1 payoff. Lets add one more strategy called Dynamite. Dynamite beats everything, but if both players choose it, they end up in the hospital, payoff -10,000 for both. Discuss how adding this new strategy affects the equilibrium outcomes of the game.

More questions: answer true/false and explain.

1. If all players have a dominant strategy, there is a unique NE.
2. Since players seek to maximize their payoffs, NE of a game must be efficient.
3. If a player has a dominant strategy, he is guaranteed to get the highest payoff in the NE.