Tutorial 3. *More Games. NE in Continuous Strategies. Sequential Games.*

Problem 1. There are 2 students taking a game theory class, Zulu and Yva. Their teacher suggests they play the following simultaneous-move game in tutorial: each student writes a nonnegative number on a piece of paper. Denote Zulu's number by Z and Yva's number by Y. Teacher says that the reward is the bonus points they can earn toward their grade, denote U_Z and U_Y respectively. The payoffs are calculated according to the following formulas:

$$U_Z = (4 - Z - Y)Z; \qquad U_Y = (4 - Z - Y)Y$$

This means for example, that if Zulu writes 1 and Yva writes 5, Zulu gets $(4 - 1 - 5)1 = -2$ points and Yva gets $(4 - 1 - 5)5 = -10$ points.

a) What are players' strategies and how many strategies does each player have? Is teacher a player?

b) Find Zulu's best response function. *(max U_Z with respect to Z, solve $\frac{\partial U_Z}{\partial Z} = 0$ for $Z(Y)$)*

c) Find Yva's best response function.

d) Find Nash equilibrium of this game. Draw BR's on a diagram with Z on horizontal and Y on the vertical axis. How many bonus marks do they earn in equilibrium? *(To find NE find Z and Y that satisfy both BRs at the same time.)*

e) Is Nash equilibrium efficient? Find the efficient outcome in which they receive the same payoffs[1].

f) Show that $Z = 1$ and $Y = 1$ is not a NE.

Problem 2. Consider the following game. 'Incumbent' is currently a monopoly that makes profits in its market. Another firm, Entrant, contemplates entering the market. If the new firm enters, the market will be shared and the incumbent firm will earn lower profits if it accommodates the entrant. Another possibility is that when the new firm enters the market, the incumbent will engage in price war which will result in negative profits for both firms. The payoffs are summarized in the matrix below (obviously if Entrant says out it does not matter what Incumbent's strategy would be in case of entry).

Entrant

		Enter	*Stay Out*
Incumbent	Accommodate	50; *50*	100; *0*
	Fight	-50; *-100*	100; *0*

a) Find all Nash equilibria in this game.

b) Represent this game in extensive form and find SPNE.

[1] Any combination of Z and Y s.t. $Z + Y = 2$ will maximize joint payoff and will be efficient. If you try to max joint payoff $\pi = U_Z + U_Y$, both FOC's will be $\frac{\partial \pi}{\partial Z} = \frac{\partial \pi}{\partial Y} = 4 - 2Y - 2Z = 0$, meaning the efficient outcome is not unique. That is why let's just look at the 'fair' one - in which both get the same payoff.

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Do **Problem 9** from Ch. 15 p. 542 in Eaton, Eaton and Allen textbook.

Additional Questions

1. Stag Hunt is another famous game. Two hunters decide whether to go hunting a stag or a hare. Stag is much larger than hare, and joint effort is required to kill it. The payoffs are summarized in matrix below.

		Hunter 2	
		Stag	*Hare*
Hunter 1	Stag	5; *5*	0; *3*
	Hare	3; *0*	2; *2*

   a) Find all NE if the game is played simultaneously.

   b) Draw the extensive form and find SPNE if the game is played sequentially. Does it matter which Hunter moves first?

2. If a game is played sequentially it is always better to be the first to make the decision. Is this true or false?

3. If each of the players has a DS, then if the game is played sequentially the SPNE will be the same as NE of a simultaneous game. True or false?

4. Before each class teacher gives students a question to work on. In the beginning of the class the teacher asks whether any one of the students wants to answer the question. If one of the students volunteers to answer, the teacher is happy and everyone enjoys the class - all students get value of $V = 10$. If no one is prepared to answer the question the teacher gets upset and gives students a quiz, all students get payoff $V = -3$. The time cost of preparing the answer for the question is $C = 5$. Being prepared does not affect the payoff in case there is a quiz - the teacher will ask a different question. There are $N = 5$ students in class. How many pure strategy NE are in this game?

5. By now you all know that your grades in ECON classes are curved, so the letter grade you will get in the end of this course depends not only how much you study, but also on how much your classmates study. Do you think that the amount of studying students do in NE is efficient? To answer the question assume an average student cares more about grades than learning[2], so *Payoff = Utility(grade) - Cost(studying).* Which game does this look like?

6. Problem 7 from Ch. 15 on p.542 in Eaton, Eaton, Allen textbook.

[2]Adding a term that captures utility from studying would complicate the efficiency analysis, that's why I left it out. I do know that many students enjoy learning, and I admit that the assumption is wrong.