Tutorial 4. *Imperfect Competition. Solutions.*

From EEA Ch. 16 do problems 4, 8 13, 14 and 15.

Additional Problems

Problem 1. There are $N = 5$ firms selling homogeneous product in a market with demand $P = 180 - Q$, where Q is the output produced by all firms. Firms have identical costs $C_i = 150 + 30q_i$.

(a) Suppose all firms except firm i produce identical output q_j. Derive $q_i^{BR}(q_j)$.

Firm i maximizes its profits taking output of all the other firms, which is $(N-1)q_j = 4q_j$ *as given.*

$$\begin{aligned}\pi_i &= (180 - q_i - 4q_j)q_i - 30q_i - 150 \\ \frac{\partial \pi_i}{\partial q_i} &= 180 - 2q_i - 4q_j - 30 = 0 \\ q_i^{BR} &= 75 - 2q_i\end{aligned}$$

(b) Find Cournot equilibrium.

To answer this question you could either figure that each of the other firms will produce according to its BR function, which would be symmetric, and substitute that into firm i's BR or simply recognize that since firms are identical, in Cournot equilibrium they should produce exactly the same output: $q_i = q_j = q^*$. *Substitute into* q_i^{BR} *for* q_i *and* q_j: $q^* = 75 - 2q^*$, $q^* = 25$. *Market output* $Q = 125$ *and price is* $P = 55$. *Each firm will earn profits* $\pi_i = 475$

(c) Is this industry in long-run equilibrium?

No, positive profits attract new firms who will enter the market (that is as long as profits do not become negative after entry).

(d) How many firms will there be in the long-run in this market?

Recall the formula for Cournot profits with N firms that we derived in the lecture: $\pi_i = \frac{(a-c)^2}{b(N+1)^2}$. *Notice that we derived this formula assuming FC=0, so technically it is not the profits but TR-VC. So the firms keep entering the market as long as* $\frac{(a-c)^2}{b(N+1)^2} > FC$, *that is provided that the marginal entrant does not turn profits to be negative. Once you substitute the values you obtain:* $\frac{(180-30)^2}{1(N+1)^2} > 150$. *Which gives us* $N < 11.25$. *There will be* $N = 11$ *firms in the market each making profits* $\frac{(150)^2}{(12)^2} - 150 = 6.25$, *which is still positive, but you can verify that at N=12 all firms will end up making a loss and the* 12^{th} *firm has no incentive to enter the market.*

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Problem 2. There are two identical firms in a market, each with costs $C_i = 30q_i$, where q_i is output produced by each firm. Market demand is $P = 210 - 1.5Q$, where $Q = q_1 + q_2$

(a) Find Cournot equilibrium.
$q_i^{BR} = 60 - \frac{1}{2}q_j$, $q^* = 40$, $Q = 80$, $P = 90$ $\pi = (90 - 30)40 = 2,400$

(b) What will be the outcome if the firms decide to collude?

Firms will max joint profits when they jointly act as a monopoly and produce total Q such that market MR=MC: 210-3Q=30, $Q_M = 60$ *and charge price* $P = 120$*. Since firms have identical costs, each will produce half of the monopoly output* $q_M = 30$ *and earn profits* $\pi_i = (120 - 30)30 = 2,700$

(c) Suppose firm one believes that firm 2 honors collusion agreement. Does it have an incentive to honor it as well if this is a one-shot game?

No, if it is a one-shot game then firm has an incentive to defect on the agreement and produce according to its BR: $q_1^D = 60 - \frac{1}{2}30 = 45$ *units of output.* $Q = 75$, $P = 97.5$, $\pi_1 = (97.5 - 30)45 = 3,037.5$.

Problem 3. There are two firms producing a differentiated product. For each firm the quantity demanded depends not only the price it is charging, but also on the price charged by the other firm. Firms' demands are:

$$\begin{aligned} q_1 &= 24 - 5p_1 + 2p_2 \\ q_2 &= 24 - 5p_2 + 2p_1 \end{aligned} \tag{1}$$

Assume production costs are zero.

(a) For each firm find the best response function.
You could to it separately for each firm, or use symmetry: since costs and demands are similar, their BRs should be similar. $\pi_i = p_i q_i = (24 - 5p_i + 2p_j)$

$$\begin{aligned} \pi_i &= p_i q_i = p_i(24 - 5p_i + 2p_j) \\ \frac{\partial \pi_i}{\partial p_i} &= 24 - 10p_i + 2p_j \\ p_i^{BR} &= 2.4 + 0.2p_j \end{aligned}$$

(b) Find equilibrium prices and quantities when firms choose prices simultaneously.

Sub one BR into the other to find $p_i = 3$*;* $\pi_i = 45$

(c) Suppose the firms decide to collude. What prices should they charge? What are the profits if they collude?
When firms collude they will maximize joint profits subject to the constraint that $p_1 = p_2$

$$\pi_{joint} = (24 - 5p_1 + 2p_2)p_1 + (24 - 5p_2 + 2p_1)p_2$$

subject to the constraint that $p_1 = p_2$*. This will give you* $p_1 = p_2 = 4$ *and* $\pi_1 = \pi_2 = 48$

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(d) Using a diagram with p_1 on the horizontal axis and p_2 on the vertical, plot each firm's BR function; demonstrate the results from parts (b) and (c), draw the appropriate iso-profit curves.

See the last page. Notice one interesting thing. If you look at the isoprofit lines through the NE you can see the 'lens of the missed opportunities' - combinations of prices above isoprofit1 and to the right of isoprofit2 - in those points both firms are better off. When you think about efficiency as 'impossible to benefit one firm without hurting the other' in this context, any combination of prices such that the isoprofit curves are above the NE level and are tangent is efficient and both firms are better off compared to NE (that is why if you try to max joint profits without constraint you would not get a unique solution.)

(d) Suppose one of the firms decides to defect on the agreement they formed in part (c). If firm 1 believes that firm 2 will honor the agreement, what price should it charge? How much profits will it make? How is firm 2 doing in this scenario?

$p_1 = 3.2$, $\pi_1 = 51.2$, $\pi_2 = 46.1$.

(e) Find subgame perfect equilibrium if firm 1 chooses its price first.

Use same approach as in Stackelberg model. Firm 1 expects that firm 2 will be maximizing its profits by producing according to its BR: maximize π_1 s.t. $p_2 = p_2^{BR}$.

$$\begin{aligned} \pi_1 &= p_i(24 - 5p_1 + 2(2.4 + 0.2p_1)) \\ \frac{\partial \pi_1}{\partial p_1} &= 28.8 - 9.2p_1 \end{aligned}$$

$p_1 = 3.13$; $p_2 = 3.026$

Do parts (a) and (b) from problem 11 in EEA Ch. 16.

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