Tutorial 5. *Repeated Games. Hotelling Model.*

Problem 1. Firms A and B sell a product that consumers view as perfect substitute. Firms have an option to advertise, which is costly. If none of the firms advertises or both advertise, they share the market equally. If only one firm advertises it will capture the entire market. Denote strategies $A = advertise$ and $N = not$. Payoffs are summarized below

		Firm B	
		A	N
Firm A	A	20; 20;	70, 0
	N	0; 70	50, 50

(a) Discuss the NE of a one-shot game.

(b) Discuss how the equilibrium may change if game is played repeatedly.

(c) If firms adopt grim trigger strategy, at what discount factor can the efficient outcome be supported as a subgame perfect NE?

(d) How does your answer to part (c) change if cheating on the cooperative agreement is detected with a lag? For example, if firm A cheats in period 1 firm B will only notice it in period 2 and thus the punishment is delayed to period 3.

(e) Use this game to discuss how payoffs affect the likelihood of cooperation (when is it likely and when it is unlikely that the firms will cooperate?).

Problem 2 We did not cover this model formally in class, but you still may find it instructive to go over this problem as it is an application of game theory.
Consider the basic Hotelling model: consumers's preferences are distributed uniformly along a [0; 1] line. There are $N = 4$ firms. Each consumer will purchase one unit of good from the firm that is closest to him. If two or more firms are located at equal distance, consumer will buy from each firm with equal probability. For example, if one firm is located at 0 and three other firms at 1, then the first firm captures zero to 0.5 share of the market, $\pi_1 = 0.5$; the other three firms share segment from 0.5 to 1 equally, for $i = 2, 3, 4$ $\pi_i = \frac{1-0.5}{3}$

(a) Is it a NE for all firm to locate at 0.5?

(b) Is combination of locations (0, 1/3, 2/3, 1) a NE?

(c) Is (0.25, 0.25, 0.75, 0.75) a NE?

Problem 3. Go back to problem 2 from the tutorial 4. Discuss how cooperation is possible if the game is repeated infinite number of times. Calculate δ for which cooperation can be supported as SPNE.

Additional questions: *Bargaining games.*
In this question we will discuss possible ways of dividing economics surplus.

(a) Consider the following simultaneous game. Two players have to divide a dollar. Each of them chooses the share he demands: 0, 0.01, ... 0.99, 1.00. If the shares add up to 1 or less each player gets the share he demanded. If the shares add up to more than one both get zero. What are the possible NE divisions of the dollar in this case?

(b) This is another famous game, called ultimatum game or take-it-or-leave-it offer. Now suppose that player one makes a division offer $(x,\ 1-x)$, were x is the share of the dollar that player 1 wants to keep. If player 2 accepts the offer the dollar is divided accordingly. If player 2 rejects, both get zero. What is SPNE of this game?

(c) In most of human subject experiments of the ultimatum game the divisions were around (0.6; 0.4), meaning that first player demanded slightly higher share, but very rarely the offers were very unequal. Discuss why in this case the offers were different from what the theory predicts they should be.