Tutorial 5. *Repeated Games. Hotelling Model.* *Solutions.*

Problem 1. Firms A and B sell homogeneous product. Firms have an option to advertise, which is costly. If none of the firms advertises or both advertise, they share the market equally. If only one firm advertises it will capture the entire market. Denote strategies $A = advertise$ and $N = not$. Payoffs are summarized below

Firm B

		A	N
Firm A	A	20; 20;	70, 0
	N	0; 70	50, 50

(a) Discuss the NE of a one-shot game.

This is another example of Prisoners' Dilemma: NE in DS is inefficient. In the outcome (N, N) both firms are better off, but it cannot be supported as an equilibrium in one-shot game: each firm has an incentive to advertise and there is nothing that will stop it from doing so.

(b) Discuss how the equilibrium may change if game is played repeatedly.

When a game is played repeatedly, the benefits from cooperation become larger. However, if the game is repeated finite number of times, SPNE is still (A; A): in the last period players choose NE strategies of the stage game, which means in the period before that the idea of 'future punishment for non-cooperation' does not work, so players choose their NE strategies in that stage game, etc.
Cooperation can only be supported in SPNE if the game is repeated infinite number of times.

(c) If firms adopt grim trigger strategy, at what discount factor δ can the efficient outcome be supported as a subgame perfect NE in an infinitely repeated game?

Recall that NE is a combination of BR's. Find δ s.t. the present value of the stream of payoffs in cooperative outcome is at least as high as the PV of deviating and receiving NE payoffs forever: $\pi(cooperate) \geq \pi(deviate)$. Let's calculate these.

$$\pi(cooperate) = 50 + \delta 50 + \delta^2 50 + \delta^3 50 + ... = \frac{50}{1-\delta} \tag{1}$$

If firm decided to defect on the agreement it will receive current payoff of 70, but starting from the next period the other firm will start advertising and will keep doing that for the rest of the game:

$$\pi(deviate) = 70+\delta 20+\delta^2 20+\delta^3 20 = 70+\delta(20+\delta 20+\delta^2 20+\delta^3 20)+... = 70+\frac{\delta 20}{1-\delta} \tag{2}$$

$$\frac{50}{1-\delta} \geq 70 + \frac{\delta 20}{1-\delta} \Rightarrow \delta \geq 0.4 \tag{3}$$

(d) How does your answer to part (c) change if cheating on the cooperative agreement can only be punished with a lag? For example, if firm A cheats in period 1 firm B cannot start advertising next period, and thus the punishment is delayed to period 3.

The PV of the cooperative payoff is still the same. If a firm decides to cheat on the cooperative agreement it can enjoy the higher profits for two periods:

$$\pi(deviate) = 70 + \delta 70 + \delta^2 20 + \delta^3 20 + ... = 70(1+\delta) + \frac{\delta^2 20}{1-\delta} \tag{4}$$

$$\pi(deviate) = 70 + \delta 70 + \delta^2 20 + \delta^3 20 + ... = 70(1+\delta) + \frac{\delta^2 20}{1-\delta} \tag{5}$$

To support cooperation in SPNE need $\frac{50}{1-\delta} \geq 70(1+\delta) + \frac{\delta^2 20}{1-\delta}$*; multiply both sides by* $1-\delta$*:* $50 \geq 70(1-\delta^2) + \delta^2 20$*. Now* $\delta \geq 0.63$*.*

(e) Use this game to discuss how payoffs affect the likelihood of cooperation (when is it likely and when it is unlikely that the firms will cooperate?).

As you could see from the parts (c) and (d), when payoff from defecting on the agreement is higher, higher δ *is required to maintain cooperation. Other things equal cooperation is easier to maintain when the difference between NE and cooperative payoffs is large.*

Problem 2. Consider the basic Hotelling model: consumers's preferences are distributed uniformly along a [0; 1] line. There are $N = 4$ firms. Each consumer will purchase one unit of good from the firm that is closest to him. If two or more firms are located at equal distance, consumer will buy from each firm with equal probability. For example, if one firm is located at 0 and three other firms at 1, then the first firm captures zero to 0.5 share of the market, $\pi_1 = 0.5$; the other three firms share segment from 0.5 to 1 equally, for $i = 2, 3, 4$ $\pi_i = \frac{1-0.5}{3}$

(a) Is it a NE for all firm to locate at 0.5?

No. In this case each firm earns profits 0.25. If a firm locates at .49, it will increase it payoff to 0.495. So each of the firms has an incentive to move left/right from that point.

(b) Is combination of locations (0, 1/3, 2/3, 1) a NE?

This is also not a NE. Firms 1 and 2 have an incentive to locate closer to center.

(c) Is (0.25, 0.25, 0.75, 0.75) a NE?

In this case each firm is getting exactly 25% of the market. This combination of locations is a NE because no firm can increase profits by moving. If firm 1 moves closer to zero, for example 0.24, its profits will become 0.245. If this firm moves to 0.26 it *will get* $\pi = 0.5(.26 - .25) + 0.5(0.75 - 0.26) = .25$*, so it cannot improve its profits and therefore is currently choosing its BR to the location of all the other firms. Since the locations are symmetric the same is true for all other firms.*

Problem 3. Go back to problem 2 from the tutorial 4. Discuss how cooperation is possible if the game is repeated infinite number of times. Calculate δ for which cooperation can be supported as SPNE.

Additional questions: *Bargaining games.*
In this question we will discuss possible ways of dividing economics surplus.

(a) Consider the following simultaneous game. Two players have to divide a dollar. Each of them chooses the share he demands: 0, 0.01, ... 0.99, 1.00. If the shares add up to 1 or less each player gets the share he demanded. If the shares add up to more than one both get zero. What are the possible NE divisions of the dollar in this case?

All divisions in which shares add up to exactly one are NE. Check: when shares add up to exactly one, increasing the demand will result in zero payoff, which is strictly worse if the player gets something, and as good as current strategy if the player is currently getting zero $\Rightarrow$ *no incentive to do that; lowering a demand results in lower payoff, clearly no incentive to do that. Notice that any combination of demands that adds up to less than one is not a NE because in this case BR is to increase the demand. Any combination that adds up to more than one dollar is not a NE, each player has incentive to lower demand which can increase the payoff. Obviously 101x101 payoff matrix is unnecessary in this case; if you like to demonstrate this using the normal form just use strategies (0, 1/3, 2/3, 1). Conclusion: multiple NE in simultaneous game, all are efficient.*

(b) This is another famous game, called ultimatum game or take-it-or-leave-it offer. Now suppose that player one makes a division offer $(x, 1 - x)$, were x is the share of the dollar that player 1 wants to keep. If player 2 accepts the offer the dollar is divided accordingly. If player 2 rejects, both get zero. What is SPNE of this game?

Use backward induction: player two accepts anything greater than zero: Accept if $1-x \geq$ *o. Given this strategy Player 1's BR is to offer division (.99, .01). Notice that if the offer is continuous the optimal x=1. To be very precise, but strangely enough player 2's BR to offer (1, 0) is accept as well, because he is indifferent between accepting and rejecting, so division (1, 0) is the 'official' SPNE of this game: first player captures the entire gains (it is equilibrium because player 2 has no incentive to reject).*

(c) In most of human subject experiments of the ultimatum game the divisions were around (0.6; 0.4), meaning that first player demanded slightly higher share, but very rarely the offers were very unequal. Discuss why in this case the offers were different from what the theory predicts they should be.

In experiments most offers in which player 1 demanded more than 75% were rejected meaning that somehow player 2 prefers to have zero. This could be expected, recall the focal points: if there are many NE people are more likely to choose the ones that are more appealing for one reason or another. Peoples' sentiments toward fairness are common knowledge and since players who made the offers could expect that unequal divisions will be rejected, they chose their BR which is to offer more or less equal division. Notice that (0.6, 0.4) division is still a NE outcome if player 2's strategy is to reject all offers in which he gets less than 0.4.