Tutorial 7. *Choice under Uncertainty (cont'd).* *Solutions*

Problem 1. Ana's utility function is $U = \sqrt{w}$, where w is her wealth. She owns a bakery that will be worth 69 or 0 dollars next year with equal probability.

(a) Suppose her firm is the only asset she has. What is the lowest price P at which she will agree to sell her bakery? Demonstrate the solution on diagrams.

In the 'good' state her firm is successful and her wealth is $w_1 = 69$; if things turn out badly, the firm is worth nothing and she has $w_2 = 0$. The expected utility of this lottery for her is $EU = 0.5\sqrt{69} + 0.5\sqrt{0} = 0.5\sqrt{69}$. The lowest price P at which she'll sell her firm must give her utility equal to the expected utility of operating the firm by herself. $U(P) = 0.5\sqrt{69}$, $P = 17.25$.

(b) Redo part (a) assuming that she has 100 dollars safely stored under her mattress. Demonstrate on diagrams.

In the 'good' state her firm is successful and her wealth is $w_1 = 100 + 69$; in the unfortunate situation the firm dies on her, but she still has the \$100 under her mattress, therefore $w_2 = 100$. The expected utility of this lottery for her is $EU = 0.5\sqrt{169} + 0.5\sqrt{100} = 11.5$. The lowest price P at which she'll sell her firm must give her utility equal to the expected utility of operating the firm by herself. $\sqrt{100 + P} = 11.5$, $100 + P = 132.25$, $P = 32.25$.

(c) Compare and discuss your results in parts (a) and (b). What is the relationship between Ana's income and her risk aversion?

When we look at the numbers we notice the following. In part (a) P=17.25 is much lower than $E(w) = 0.5 \cdot 0 + 0.5 \cdot 69 = 34.5$. In part (b) the expected profits generated by the firm are still the same, however, she would only sell the firm at $P \geq 32.25$, which is much closer to the expected value of the business. This suggests that when she had nothing to fall back on in terms of assets (in part (a)), having some certainty was quite important for her, and she would participate in the risky project only if the risk premium was sufficiently high. In part (b) the wealth she had regardless of the success of the business was higher and at that point a small risk premium would be enough for her to take the risk. ***Conclusion****: as her wealth increased, she became less risk-averse.*

Recall from the lecture that the degree of individual's risk-aversion is reflected in the curvature or the U function. Let's see what is Arrow-Pratt measure of absolute risk aversion for this utility function:

$$\rho = -\frac{U''(w)}{U'(w)} = -\frac{-0.25w^{-1.5}}{0.5w^{0.5}} = \frac{1}{2w}$$

Which is diminishing in w.

Problem 2. Albina owns a car worth 50,000 which can get stolen with probability 1%. She can purchase coverage of the amount $q \in [0; \quad 50,000]$ at premium $\pi = 0.05$ dollars for each dollar covered. Her utility function is $U = \ln(w)$. Assume she has no other assets.

(a) Set up her maximization problem.

In the 'good' state her wealth is $w_1 = 50,000 - 0.05q$ *= the value of the car net of the total cost of the insurance she chooses to buy. In case the car gets stolen, her income will be equal to the amount of coverage she purchased net of the cost:* $w_2 = q - 0.05q$. *Her objective is to choose q that will maximize her expected utility:*

$$\max EU \quad = \quad 0.99 \ln(50,000 - 0.05q) + 0.01ln(q - 0.05q)$$

(b) How much insurance will she choose to buy?

$$\textit{Find the F.O.C.: } \frac{dEU}{dq} = \frac{0.99 \cdot (-0.05)}{50{,}000 - 0.05q} + \frac{0.01(0.95)}{0.95q} = 0,$$

$$\frac{0.0495}{50{,}000 - 0.05q} = \frac{0.01}{q}$$

$$0.0495q + (0.01)(0.05)q = 500$$

$$0.05q = 500$$

And $q = 10,000$. $10,000 \cdot 0.05 = 500$ *dollars. Given the premium that the insurance company is charging Albina will choose to only purchase partial insurance, meaning that she 'redistributed' some of her wealth from the good state to the bad state, but she is still bearing some risk by herself.*

(c) How much profits does the insurance company earn on insuring Albina?

The expected cost of the insurance is $0.01 \cdot 10,000 = 100$, *so the expected profits are exp. revenue - exp. cost = 400.*

(d) Does the fact that the insurance company earn profits mean that Albina is worse off? Explain what is happening.

Albina's MU of wealth is $1/w$, *which is diminishing, which means she is risk averse, so her CE is lower than the expected value of the lottery and there are potential gains for both Albina and the risk-neutral insurance company that they share.*

(e) How much insurance will she buy if insurance companies charge an actuarially fair insurance rate?

Redo the EU maximization problem to verify that when $\pi = 0.01$ *she will buy full insurance. Her F.O.C becomes 50,000-0.01q=.99q,* $q^* = 50,000$.

Problem 3. A person with $U(w) = w^2$ is facing the following lottery: there are four tickets sold, of which only one wins the prize of \$100.

(a) Calculate the expected value of the lottery. $E(w) = 25$

(b) Give the definition and calculate certainty equivalent. $CE = 50$

(c) Compare and discuss results in parts (a) and (b). *What are this person's preferences toward risk?*