Tutorial 8. *Games of Asymmetric Information.* *Solutions.*

Problem 1. Suppose that half of population drives carefully and will get into an accident with probability 0.2. The other half of the drivers is not as careful and faces the probability of accident 0.5. Assume all drivers have the same income of \$400 per year. Getting into an accident results into damages of \$231. Drivers are risk-averse and have utility function $U = \sqrt{w}$. There is a risk-neutral insurance company in the city.

(a) For now assume that information is perfect and symmetric. What will happen if the insurance company offers full insurance at actuarially fair rates? What will be the total premiums for each type? Which type will chose to buy the insurance?

Actuarially fair rate is such that the premium for each dollar insured is equal to the expected payment by the insurance company, so the expected profits are zero (expected revenue=expected cost). When the insurance company sells full coverage to a high-risk driver, it knows that with probability 0.5 it will have to pay \$231, so it will break-even at total premium $0.5 \cdot 231 = 115.5$. *For the low-risk drivers the probability of the accident is only 0.2, so the expected cost of covering such driver is* $0.2 \cdot 231 = 46.2$. *Recall from the topic 'Choice under Uncertainty' that at the actuarially fair rates the risk-averse individuals maximize expected utility by fully insuring themselves, so both types will choose to buy insurance and will be better off compared to the situation when they are uninsured.* ***Check:*** *for the low-risk* $EU(uninsured) = 0.8\sqrt{400}+0.2\sqrt{169} = 18.6 < EU(insured) = \sqrt{400 - 46.2} = 18.8$*; for the high-risk* $EU(uninsured) = 0.5\sqrt{400} + 0.5\sqrt{169} = 16.5 < EU(insured) = \sqrt{400 - 115.5} = 16.9$
Notice that in this case efficiency requires that all the risk is born by the risk-neutral party, so the outcome above is efficient.

(b) Now let's make a more realistic assumption: each driver knows his type, but this information is not available to the insurance company. ***If*** all drivers decided to purchase the insurance, at what total premium would the insurance company break-even?

If all drivers purchased the insurance, the company's expected cost per customer would be $0.5 \cdot 115.5 + 0.5 \cdot 46.2 = 80.85$.

(c) Suppose the insurance company does charge the premium you calculated in part (b), which type will decide to buy the insurance?

At this premium the high-risk drivers would be happy to buy insurance, because the coverage is even cheaper than in part (a). For the low-risk drivers now $U(insured) = \sqrt{400 - 80.85} = 17.86$*, which is lower then their EU without insurance, so they will choose to stay uninsured. This also means that the insurance company will only sell insurance to high-risk drivers at the rate calculated in part (a).*

(d) Discuss your results.

In this case asymmetric information about the probability of an accident results in adverse selection: the price at which the good (insurance) can be sold is not acceptable to the 'high-quality' (low-risk) side of the trade agreement. Recall that in case of insurance government can make insurance compulsory, in that case the market will not collapse, however, the low-risk individuals will be worse-off compared to being uninsured.

Problem 2. Suppose that there are two types of workers in the labor market. For simplicity let's assume that the firms hire workers to participate on a project and it's a one-time interaction. A high-skill worker's productivity in the project is $a_H = 8,000$ and low-skill worker's productivity is $a_L = 3,000$. Workers know their type, but the firms don't. Assume that the labor market is competitive, so when a firm hires a worker, it offers wage equal to worker's expected productivity. Suppose that a local college offers a certificate program; completion of the program has no impact on workers' productivity. High-skill worker's cost of obtaining the certificate (counting both tuition and effort invested) is $c_H = 1,500$ and for the low-skill workers it is $c_L = 4,500$.

(a) Discuss possible equilibrium outcomes of this game. Will the outcome be efficient?

At first let's check whether a ***separating equilibrium***[1] *exists. Suppose that firms believe that having a certificate is a credible signal of a worker being high-skilled. Then the firms will offer wage $w_H = 8,000$ to all workers who choose to obtain it and $w_L = 3,000$ to all without certificate. Denote level of education e, which is 1 if worker obtained the education and zero otherwise. For high-skill worker payoff of getting education is then $\pi_H(e = 1) = 8,000 - 1,500 = 6,500$, which is strictly better than not getting education $\pi_H(e = 0) = 3,000$. Given the beliefs of the firms, high-skill workers find it in their interest to choose e = 1. We can rephrase this condition as 'the expected gain in future income is greater than the cost of education, so high-skill workers will choose to obtain the certificate'; mathematically $c_H < w_H - w_L$). Let's look at the low-skilled workers. Given the beliefs of the firms, $\pi_L(e = 1) = 8,000 - 4,500 = 3,500 > \pi_L(e = 0) = 3,000$, so low-skilled worker's best strategy is to obtain the certificate as well. Recall from the lecture that we discussed that in signalling games beliefs are a part of equilibrium and players' beliefs must not contradict the reality. When we check whether the firms' beliefs are realistic we find that the answer is 'no': eventually firms see that all job applicants have the certificate and half of them is unskilled, so the firms will update their beliefs and offer the same wage to all workers, which by zero-profit condition must be equal to the expected productivity: $w_{avg} = 0.5 \cdot 8,000 + 0.5 \cdot 3,000 = 5,500$. When workers realize that having the certificate has no impact on the wage offered by the firms, they choose not to get the education.*
In conclusion, given the parameters of the model the separating equilibrium does not exist; in the ***pooling equilibrium*** [2] *nobody obtains the education. Equilibrium is efficient because in this model education is a costly activity that does not result in any social gain.*

[1]In separating equilibrium different types of players (workers) choose different strategies (education).
[2]In pooling equilibrium all players choose the same action.

(b) Based on your results in part (a), if you were the program head in the college, how could you improve enrollment?

In this model education only has private value to the workers if it can be used as a credible signal of being a high-skill worker. The signal is credible if it is cheap enough for high-skill workers to obtain $c_H < a_H - a_L$. *Recall that* a_H *and* a_L *are the wages that worker will get depending on education, so what this inequality really means is that the cost of obtaining education for high-skill workers is smaller than the potential difference in income in the separating equilibrium. Also this signal must be costly enough for the low-skill workers:* $c_L > a_H - a_L$ *, so that in the separating equilibrium this type of workers chooses* $e = 0$. *In part (a) nobody chose to obtain the certificate because it was not a credible signal of a worker's skill, so if the program head wants people to enroll in the program he must redesign it according to the equations above. When you think about it, having a really expensive but easy to pass program is not necessarily a good idea, because if it is only a matter of paying the fees, the inequalities above may still not be satisfied. However, making the program more challenging in terms of contents (which could be done in combination with increasing tuition fees) could achieve the goal if sufficient difference between* c_H *and* c_L *can be generated.*

Problem 3. Patricia owns a boutique and hired Alfred as a salesperson for her store. For simplicity assume that the daily revenue can be either $X_H = 60$ (in the context of the lecture this means that 'the project is a success') or $X_L = 20$ (the project 'fails'). The revenue depends on two things: demand and Alfred's effort[3]. Alfred can work hard $e = 1$ or be lazy $e = 0$. If Alfred works hard the probability of success is 0.8; if Alfred is lazy the probability of success is 0.25. Patricia, who is the principal, is risk-neutral and maximizes her (expected) profits $\pi_P = X - w$, where w is the wage she will pay to Alfred. Alfred (agent) is risk-averse and has payoff $\pi_A = \sqrt{w} - e$ (notice that this is his utility of wealth net of the cost of exerting the effort). Instead of working in the boutique Alfred could flip burgers in McDonalds for 16 dollars a day which does not involve any effort, so his reservation utility is $\overline{U} = \sqrt{16} = 4$.

(a) Suppose that the effort is observable. Find the profit-maximizing wage if (i) Patricia wants Alfred to be lazy (ii) Patricia wants Alfred to work hard. Which level of effort would Patricia choose?

*If effort is observable, then in order to max her profits Patricia only needs to offer the lowest acceptable wage to Alfred in either case. This condition is what we called in the lecture the **participation constraint-** wages offered by the principal should be high enough so that agents payoff from participating in the project is at least as high as agent's reservation unitity (value of the outside option). If P wants A to be lazy, she just needs to pay him $16, which results in payoff of 4, same as what he'll get in MacDonald's. If she wants him to work hard, then she needs to satisfy* $\sqrt{w} - 1 = 4$, *so* $w = 25$ *(that is provided he works hard, which she can verify, and pay him nothing otherwise). P wants to max her profits, so she'll choose the level of effort that results in higher expected profits for her.*
If she contacts Alfred to exert low effort level her expected payoff is $\pi_P(e = 0) =$

[3]This means that when effort is unobservable and the revenue is low, there is no way for Patricia to tell whether the 'project failed' because Alfred was lazy or because it was a bad day and the demand was low.

$0.25 \cdot 60 + 0.75 \cdot 20 - 16 = 30 - 16 = 14$. *If she contacts Alfred to exert high effort level, her expected payoff is* $\pi_P(e = 1) = 0.8 \cdot 60 + 0.2 \cdot 20 - 25 = 52 - 25 = 27$. *Patricia will prefer Alfred to work hard. Think about what the numbers tell you about Patricia's choice: if A works hard the expected revenue increases from 30 to 52; to make A work hard P has to raise wage from 16 to 25, since the expected revenue increases by more than the increase in wage, for P it is profitable to contract A to work hard.*

(b) Now assume that the effort is unobservable. Patricia can no longer make wage contingent on effort, so she realizes that to create incentive for Alfred to work hard she has to give him a bonus in case the sales turn out to be high. Suppose the bonus scheme in this case works as follows: if sales are low, she will pay Alfred w_L as you calculated in part (a) for high effort. If sales are high, Alfred will receive $w_H = w_L + B$, where B is the bonus. Calculate the lowest bonus that will induce $e = 1$. Will Patricia choose Alfred to work hard now?

In part (a) Patricia's question was: what is the (lowest) wage that I need to pay Alfred to make him work for me, or, in formal language, her concern was to satisfy Alfred's participation constraint. When effort level is unobservable, if she wants Alfred to work hard, she needs to design the pay schedule so that Alfred will choose to work hard, or, she has to satisfy ***incentive-compatibility constraint****- make wage conditional on the outcome of the project, and choose wages (in this case bonus) so that for A expected payoff from working hard is greater or equal than his payoff of e=o. So, the size of the bonus B should be such that Alfred's BR is to choose* $e = 1$. *Mathematically,* $0.8\sqrt{25 + B} + 0.2\sqrt{25} - 1 \geq 0.25\sqrt{25 + B} + 0.75\sqrt{25}$, *rearrange to obtain* $0.55\sqrt{25 + B} \geq 3.75$ *resulting in* $B \geq 21.5$. *Since Patricia wants to max her profits, she'll offer the lowest acceptable bonus.*
In this specific case, given the wording of this question, when Patricia chooses low effort level, her expected profits are exactly the same as in part (a). If Patricia decides to offer the bonus scheme to make Alfred work hard, her expected profits are $0.8 \cdot (60 - 46.5) + 0.2 \cdot (20 - 25) = 9.8$. *So for her it does not make sense to encourage Alfred to work hard even though it is efficient to do so.*
The proper solution that would follow the logic of the lecture is that P has to choose w_1 *- wage in case of success (high revenue) and* w_o *- wage in case of failure so that both participation and incentive-compatibility constraints are binding. In this case* $w_1 = 28.77$ *and* $w_0 = 12.57$, *then the Principal's profit is 26.47. That means the principal still wants to induce e = 1. Those of you who consider enrolling into Masters in Economics after completing your BA, can try to derive those: re-label variables* $y = \sqrt{w_1}$ *and* $z = \sqrt{w_o}$ *and sub into the constraints, so now you have two linear equations with two variables z and y that you can solve, and find the corresponding wages.*
Concluding remarks. As we discussed in the lecture, moral hazard can cause inefficiency. (i) if P wants to impose e=1 he must design a variable pay where wage depends on the outcome of the project, so the agent is exposed to risk (all risk should be born by the risk-neutral P). (ii) exposing agent to risk means that P will have to offer higher expected wage to the A to satisfy participation constraint (compared to the observable effort), which lowers profitability of the project to P and he may choose e=0 when e=1 is the efficient effort level. Solutions to moral hazard: P can monitor agent if monitoring is

not too costly; in this problem Patricia could rent the boutique to Alfred, then A would be a residual claimant to the profits: he pays fixed fee (say per month) to Patricia and keeps all the remaining revenue to himself - now Alfred would choose efficient effort level - as long as the difference in the expected revenue is sufficiently high to compensate the cost of effort, he'll chose to work hard (but he is still exposed to risk, so there is inefficiency arising from that).

Problem 4. Suppose that there are two manufacturers producing a similar product, such that for the consumers the quality of the good is impossible to verify before they actually purchase the product (some electronic device will make a good example). Discuss the potential inefficiency caused by asymmetric information in this case. What solution do markets offer in this case? *(Hint: the cost of high-quality product is higher than the cost of producing low-quality good; naturally consumers would be willing to pay higher price for a high-quality good; how can producers of high-quality product distinguish themselves from producers of low-cost product?).*

Mathematically the answer is very similar to part (b) of problem 2: there must be a signal cheap enough for the high-quality producers to provide and costly enough for the low-quality producers not to provide. Warranty can be such a signal. In this context firms can also offer the warranty for different time periods: if a product is low-quality it has a higher probability of breaking at each moment, so the longer the warranty the higher is the cost for the company. Then the high-quality producers can signal quality by offering warranty for long enough time period so that the low-quality producers will not find it in their interest to match it.