Assignment 1: Set theory, Relations, Functions

Ling 324; Fall 2007
Due on Sept. 14 in class

Your answers should be clear and well-organized, and written in full sentences in proper English when asked to provide explanations. Please type your answers, or write *very* neatly.

1. Determine whether each of the following is true or false and explain why. Note that $\emptyset$ stands for the empty set.
   (a) $\emptyset \subseteq \emptyset$
   (b) $\emptyset \in \emptyset$
   (c) $\emptyset \in \{\emptyset\}$
   (d) $\emptyset \subseteq \{\emptyset\}$
   (e) $\{a, b, c\} \in \{a, b, \{a, b, c\}\}$
   (f) $\{a, b, c\} \subseteq \{a, b, \{a, b, c\}\}$
   (g) $\{a, b, c\} \in \wp(\{a, b, \{a, b, c\}\})$
   (h) $\{\{a, b, c\}\} \subseteq \wp(\{a, b, \{a, b, c\}\})$
   (i) $\{a, b, c, \{a, b, c\}\} - \{a, b, c\} = \{a, b, c\}$
2. Specify the following sets using a list notation.
   (a) $(\{1, 3, 5, 7\} \cup \{3, 1\}) \cap \{3, 5, 7\}$
   (b) $(\{1, 2\} - \{5, 7, 9\}) \cup (\{5, 7, 9\} - \{1, 2, 7\})$
   (c) $\wp(\{7, 8, 9\}) - \wp(\{7, 8\})$
   (d) $\wp(\{\emptyset\})$
3. Specify the following sets using a list notation.
   (a) $(\{2\} \times \{1, 2\}) \times \{1, 2, \{3\}\}$
   (b) $\{\emptyset, 1\} \times \{1, 2\}$
   (c) $\wp(\{1, 2, 3\}) \times \{1\}$
4. Using the set-theoretic equalities in 'Mathematical Preliminaries' lecture notes, show that the following set-theoretic expression is true for any sets X and Y.
   $X \cap (Y - X) = \emptyset$

5. Are the following statements true or false? Explain why.

   (a) $\{x : x = b\} = \{b\}$

   (b) $\{x : x$ is green$\} = \{x : x$ is red$\}$

   (c) $\{y : y \in B\} = B$

   (d) $\{y : y \in \{x : x \in A\}\} = \{x : x \in A\}$

   (e) $\{y : \{x : x$ likes $y\} = \emptyset\} = \{y : \{x : y$ likes $x\} = \emptyset\}$

6. Let M = {John, Mary, Peter};
   Let $taller\_than$ be a relation in M.
   Assume that John is taller than Mary, and Mary is taller than Peter.

   (a) Specify $taller\_than$ relation in M using a predicate notation.

   (b) Specify $taller\_than$ relation in M using a list notation.

   (c) Is $taller\_than$ relation in M reflexive? Explain why.

   (d) Is $taller\_than$ relation in M symmetric? Explain why.

   (e) Is $taller\_than$ relation in M transitive? Explain why.

   (f) Specify $taller\_than'$ in M (complement of $taller\_than$) first using a list notation and then using a predicate notation.

   (g) Specify $taller\_than^{-1}$ in M (inverse of $taller\_than$) first using a list notation and then using a predicate notation.

7. The following are relations from {1,2,3,4} to $\{a, b, c, d\}$. Which of the following relations are (total) functions? For the ones that are (total) functions, describe the kind of function each one is (one-to-one vs. many-to-one, onto or into).

   a. $\{< 1, a >, < 2, b >, < 3, c >, < 4, d >\}$

   b. $\{< 1, a >, < 2, b >, < 3, c >, < 4, c >\}$

   c. $\{< 1, a >, < 2, b >, < 3, c >, < 3, d >\}$

   d. $\{< 1, a >, < 2, b >, < 3, c >\}$

8. Give the characteristic functions of the following sets with respect to the universe {1,2,3,4}. Specify them first as set of pairs, and then in a notation using arrows.

   (a) { }
   (b) {2}
   (c) {1,4}
   (d) {1, 2, 3, 4}