Assignment 4: Predicate Logic

Ling 324; Fall 2007
Due on Nov. 9 in class

Your answers should be clear and well-organized, and written in full sentences in proper English when asked to provide explanations. Please type your answers.

1. Consider the sentences in (i) and (ii).

   (i) Jill likes Bill.

   (ii) Bill, Jill likes.

   What can you say about the difference in syntactic structure between (i) and (ii)? What can you say about the difference in meaning between (i) and (ii)? Now, translate the two sentences into predicate logic formulas.

2. Translate the following English sentences to predicate logic formulas. Choose your own constants, variables and predicate letters, providing the key.

   For example: Every cat is black or white.

   Key: C(x): x is a cat; B(x): x is black; W(x): x is white.

   Formula: $\forall x[C(x) \to [B(x) \vee W(x)]]$

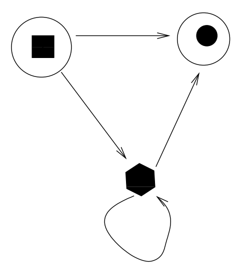
   (a) Any pet either loves itself or some person.
   (b) Dogs will eat anything.
   (c) Some sleepy student didn't answer any questions.
   (d) No dog except Fido barked.
   (e) Every student who takes LING-324 loves it. (HINT: Treat 'LING-324' as a proper name.)

3. Draw a syntactic tree for each of the following predicate logic formulas. And then specify the scope of each quantifier, and identify all bound and free occurrences of variables. (But don't worry about any variable within a quantifier phrase.)

   (a) $\forall x Q(x, y) \wedge P(x)$
   (b) $\forall x \neg [P(x) \to \exists x \forall z Q(x, y, z)]$
   (c) $\exists x Q(x, y) \wedge \forall y P(y, x)$

4. Take the model M_1 that we went over in class. As we did in class, let us take a language in Predicate Logic such that the unary predicate A denotes the set of individuals with a circle around, and the binary predicate R denotes the relation encoded by the arrows.

M1



U(niverse) = {s, c, h}
V(A) = {s, c}
V(R) = {$<$s,c$>$, $<$s,h$>$, $<$h,c$>$, $<$h,h$>$}

Draw the syntactic trees and spell out the compositional semantic interpretation of the following predicate logic formulas, and determine the truth value of the formulas in M_1. Explain how you arrived at the truth values.

(a) $\exists x[R(x,x)]$

(b) $\forall x \exists y[R(x,y)]$

(c) $\exists x[A(x) \wedge \forall y[R(y,x)]]]$

5. Consider the model M_3 described below.

U_3 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

V(j) = 1

V(m) = 10

V(n) = 6

V(P) = {x: x is an even number in U_3} = {2, 4, 6, 8, 10}

V(Q) = {x: x is an odd number in U_3} = {1, 3, 5, 7, 9}

V(K) = {$< x, y >$: $x < y$ (x is less than y), where x and y are elements of U_3}

V(L) = {$< x, y, z >$: $x + y = z$, where x, y, z are elements of U_3}

Let assignment function g be specified as follows:
$g(x_1)$ = 2, $g(x_2)$ =4, $g(x_3)$ = 9, and for all $n \geq 4$, $g(x_n)$ = 3.

(a) Evaluate the truth values of the following predicate logic formulas, using M_3. Explain how you arrived at the truth values in prose (including arithmetic statements).

i. L(x_2, x_1,n)

ii. $\forall x_1$L$(x_1,$ j$, x_1)$

iii. $\forall x_2 \exists x_3$[K$(x_2, x_3) \vee x_2 =$ m]

iv. $\neg$ P(j) $\leftrightarrow$ Q(j)

v. $\exists x_1 \exists x_2 \exists x_3 \neg[\mathrm{L}(x_1, x_2, x_3) \leftrightarrow \mathrm{L}(x_2, x_1, x_3)]$

(b) Using only the constants and predicates specified in the description of M_3 (including the fact that the Universe U_3 contains just the numbers from 1 to 10), translate the following sentences of English into predicate logic formulas.

i. There is something that is odd and not even, and there is something that is [not odd] and [even].

ii. Everything is greater than or equal to itself.

iii. Everything has something less than it.

iv. All numbers except 10 are odd numbers.