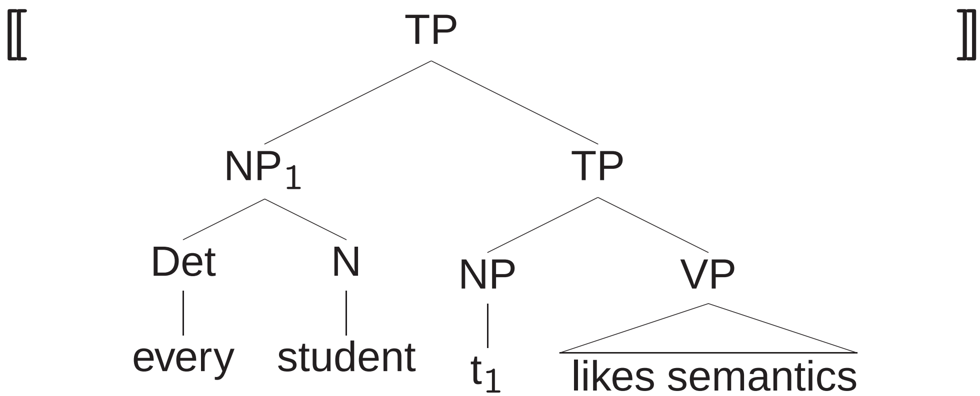
Generalized Quantifiers

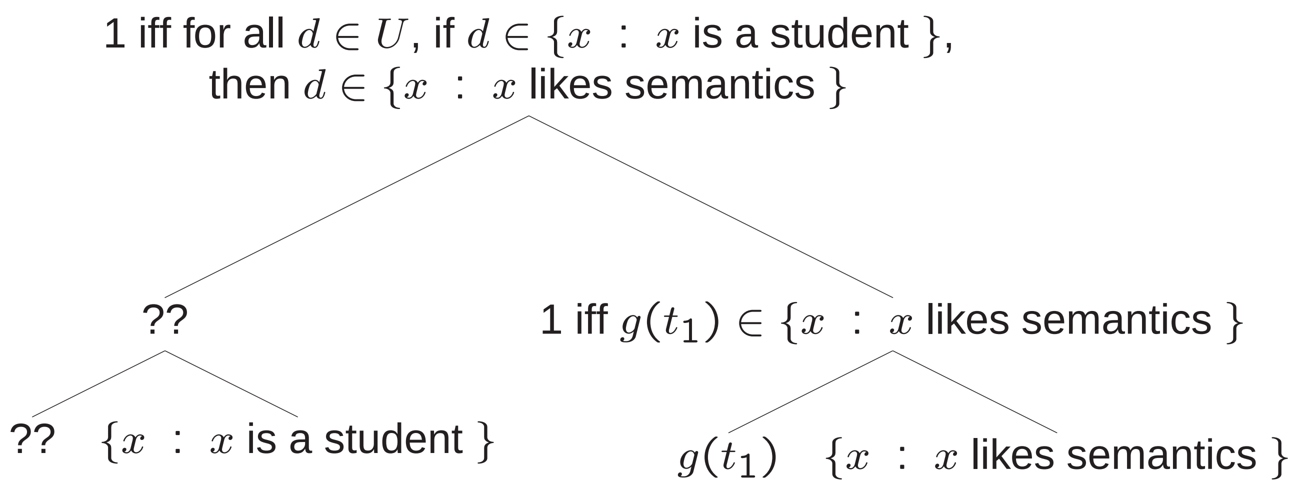
Ling 406/802; Spring 2005

Meaning and Grammar, Ch 9.1 - 9.2

Semantic Value of ***every student***

(1) Every student likes semantics.





Semantic Value of *every student* (cont.)

- Think of ⟦every student⟧ as a set of sets to which all the students belong.

  ⟦every student⟧ = $\{$ $\{x$: x drinks$\}$, $\{x$: x hates phonology$\}$, $\{x$: x owns a computer$\}$, $\{x$: x likes semantics$\}$, ...$\}$

  ⟦every student⟧ = $\{X$: ⟦student⟧ $\subseteq X\}$, where $X \subseteq U$

- Then, ⟦every⟧ can be defined as:

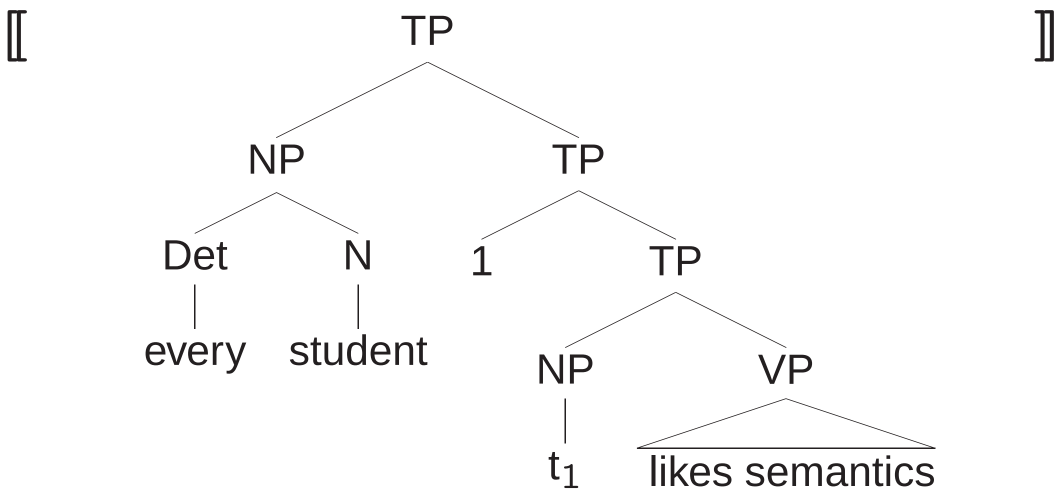
  ⟦every⟧ = $\{< X, Y >$: $Y \subseteq X\}$, where $X \subseteq U$ and $Y \subseteq U$

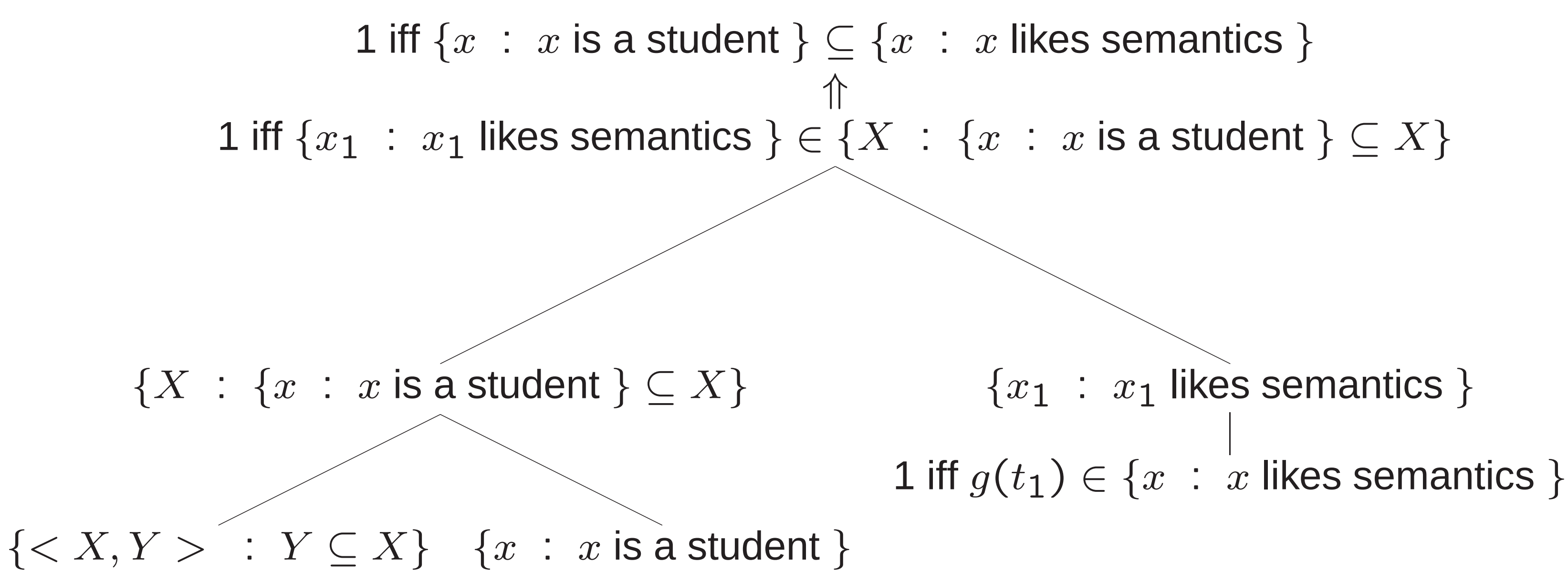
- And ⟦every student likes semantics⟧ can be defined as:

  1 iff ⟦likes semantics⟧ $\in$ ⟦every student⟧

  1 iff $\{x$: x likes semantics$\}$ $\in$ $\{$ $\{x$: x drinks$\}$, $\{x$: x hates phonology$\}$, $\{x$: x owns a computer$\}$, $\{x$: x likes semantics$\}$, ...$\}$

Compositionals Semantics for *Every student likes semantics*





Semantic Value of *some student*

- $[\![$some student$]\!]$ as a set of sets to which at least one student belongs.

  $[\![$some student$]\!]$ = $\{$ $\{x$: x got an A in syntax$\}$, $\{x$: x failed phonology$\}$, $\{x$: x takes a bus$\}$, $\{x$: x likes semantics$\}$, ...$\}$

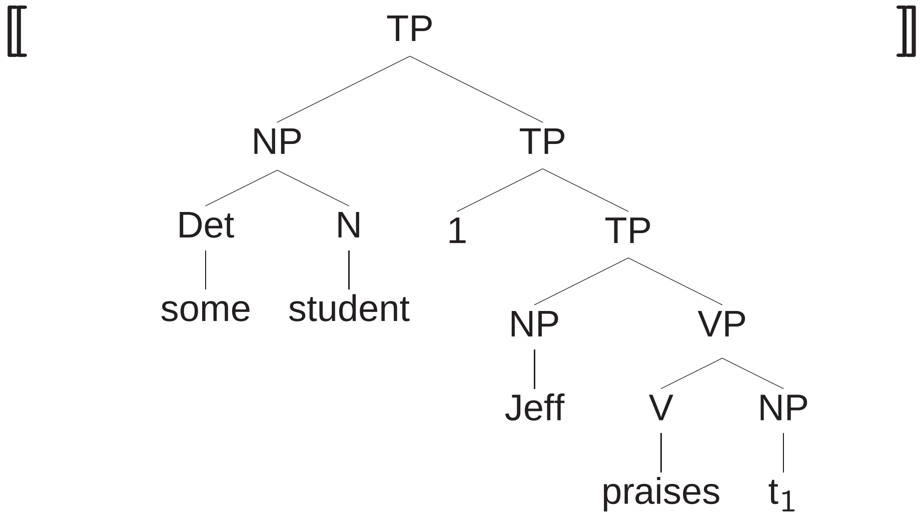
  $[\![$some student$]\!]$ = $\{X$: $[\![$student$]\!]$ $\cap X \neq \emptyset\}$, where $X \subseteq U$

- Then, $[\![$some$]\!]$ can be defined as:

  $[\![$some$]\!]$ = $\{< X, Y >$: $Y \cap X \neq \emptyset\}$, where $X \subseteq U$ and $Y \subseteq U$

- How should we define $[\![$Jeff praises some student$]\!]$?

Compositional Semantics of *Jeff praises some student*



$$1 \text{ iff } \{x : x \text{ is a student }\} \cap \{x_1 : \text{ Jeff praises } x_1\} \neq \emptyset$$

$$\Uparrow$$

$$1 \text{ iff } \{x_1 : \text{ Jeff praises } x_1\} \in \{X : \{x : x \text{ is a student }\} \cap X \neq \emptyset\}$$

$$\{X : \{x : x \text{ is a student }\} \cap X \neq \emptyset\} \qquad \{x_1 : \text{ Jeff praises } x_1\}$$

$$\{< X, Y > : Y \cap X \neq \emptyset\} \quad \{x : x \text{ is a student }\} \qquad 1 \text{ iff Jeff} \in \{x : x \text{ praises } g(t_1)\}$$

Semantic Value of Other Generalized Quantifiers and Determiners

Generalized quantifiers refer to full NPs, like *every student*, *some student*, *no student*, *most students*, etc. Determiners refer to *every, some, no, the, most*, etc.

For every $X \subseteq U$ and $Y \subseteq U$:

- ⟦no N⟧ = $\{X \ : \ ⟦\text{N}⟧ \cap X = \emptyset\}$

  ⟦no⟧ = $\{< X, Y > \ : \ Y \cap X = \emptyset\}$

- ⟦the N⟧ = $\{X \ : \ \text{for some } u \in U, \ ⟦\text{N}⟧ = \{u\} \text{ and } u \in X\}$

  ⟦the⟧ = $\{< X, Y > \ : \ \text{for some } u \in U, \ Y = \{u\} \text{ and } u \in X\}$

- ⟦most N⟧ = $\{X \ : \ |X \cap ⟦\text{N}⟧| > |X^- \cap ⟦\text{N}⟧|\}$

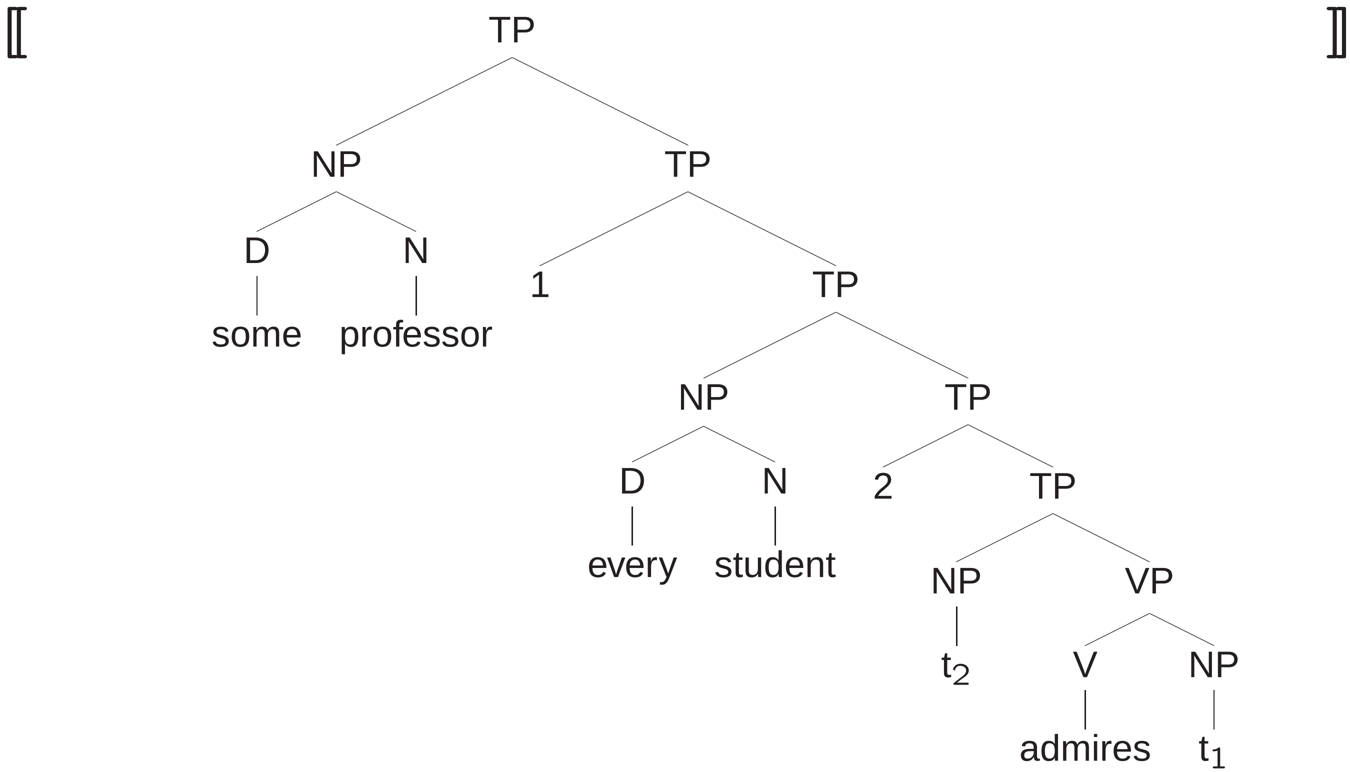
  ⟦most⟧ = $\{< X, Y > \ : \ |X \cap Y| > |X^- \cap Y|\}$

- ⟦two N⟧ = $\{X \ : \ |X \cap ⟦\text{N}⟧| \geq 2\}$ or $\{X \ : \ |X \cap ⟦\text{N}⟧| = 2\}$

  ⟦two⟧ = $\{< X, Y > \ : \ |X \cap Y| \geq 2\}$ or $\{X \ : \ |X \cap Y| = 2\}$

Two Generalized Quantifiers in a Sentence?

(2) Every student admires some professor.



Two Generalized Quantifiers in a Sentence? (cont.)

$$1 \text{ iff } \{x : x \text{ professor }\} \cap \{x_1 : \{x : x \text{ student }\} \subseteq \{x_2 : x_2 \text{ admires } x_1\}\} \neq \emptyset$$

$$\Uparrow$$

$$1 \text{ iff } \{x_1 : \{x : x \text{ student }\} \subseteq \{x_2 : x_2 \text{ admires } x_1\}\} \in \{X : \{x : x \text{ professor }\} \cap X \neq \emptyset\}$$

- $\{X : \{x : x \text{ professor }\} \cap X \neq \emptyset\}$
  - $\{< X, Y >: Y \cap X \neq \emptyset\}$
  - $\{x : x \text{ professor }\}$
- $\{x_1 : \{x : x \text{ student }\} \subseteq \{x_2 : x_2 \text{ admires } x_1\}\}$
  - $1 \text{ iff } \{x : x \text{ student }\} \subseteq \{x_2 : x_2 \text{ admires } g(t_1)\}$
  - $\Uparrow$
  - $1 \text{ iff } \{x_2 : x_2 \text{ admires } g(t_1)\} \in \{X : \{x : x \text{ student }\} \subseteq X\}$
    - $\{X : \{x : x \text{ student }\} \subseteq X\}$
      - $\{< X, Y >: Y \subseteq X\}$
      - $\{x : x \text{ student }\}$
    - $\{x_2 : x_2 \text{ admires } g(t_1)\}$
      - $1 \text{ iff } g(t_2) \in \{x : x \text{ admires } g(t_1)\}$

Syntax of a Fragment of English (F4)

Syntax of F4 is essientially the same as F3, except a few minor differences.

- Addition of more determiners

  Det $\longrightarrow$ the, a, every, some, no, most, two, ...

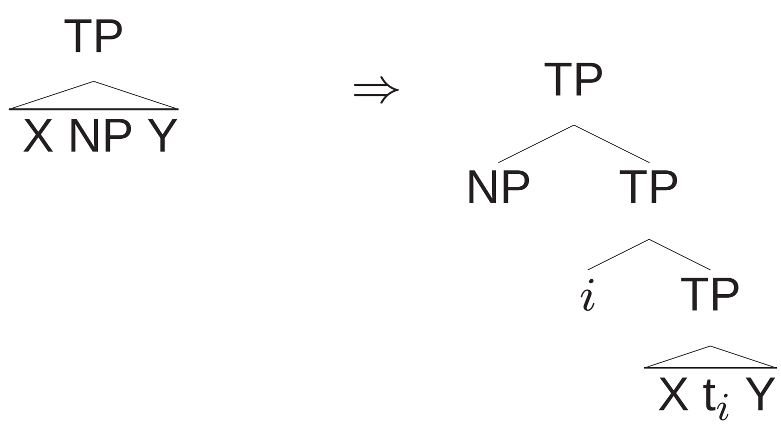
- Addition of more syntactic rules

  NP $\longrightarrow$ N CP (to handle relative clauses)

  VP $\longrightarrow$ VP Conj VP

  V $\longrightarrow$ V Conj V

- Rule for Quantifier Raising (QR):



Translating Syntax of F4 to Logical Representation

- Assuming that D is a determiner category, and GQ is a generalized quantifier category:

  If α' is in D and β' is in 1-place predicate, $\alpha'(\beta')$ is in GQ.

  If α' is in GQ and β' is in 1-place predicate, $\alpha'(\beta')$ is a well-formed formula.

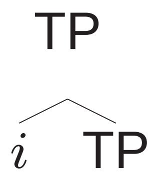
- $[\,[_{\text{Det}}\ \alpha]\,]' = \lambda Y \lambda X[\alpha'[Y][X]]$ $\qquad$ $[\,[_{\text{Det}}\ \text{every}]\,]' = \lambda Y \lambda X[\text{every}'[Y][X]]$

- $[\,[_{\text{NP}}\ \text{Det}\ \text{N}]\,]' = \lambda X[\text{Det}'[\lambda x[\text{N}'(x)]][X]]$

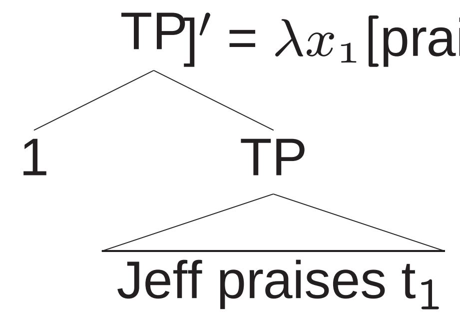
  $[\,[_{\text{NP}}\ [_{\text{Det}}\ \text{every}]\ [_{\text{N}}\ \text{student}]]\,]' =$

  $= \lambda X[\text{every}'[\lambda x[\text{student}'(x)]][X]]$

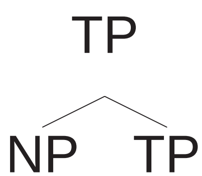
Translating Syntax of F4 to Logical Representation (cont.)

- [

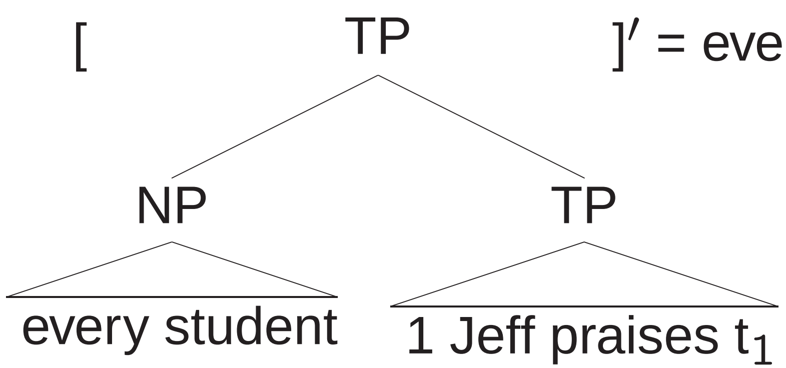
]$' = \lambda x_i$[TP]$'$

  [

]$' = \lambda x_1$[praise$'$(j$', x_1$)]

- [

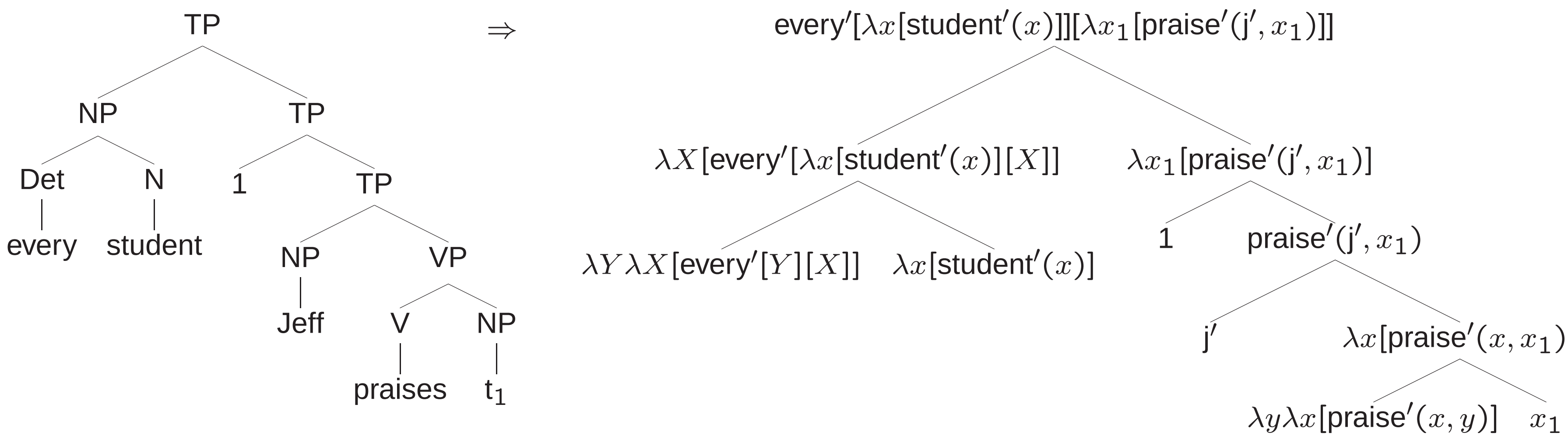
]$'$ = NP$'$(TP$'$)

  [

]$'$ = every$'$[λx[student$'(x)$]] [λx_1[praise$'$(j$', x_1$)]]

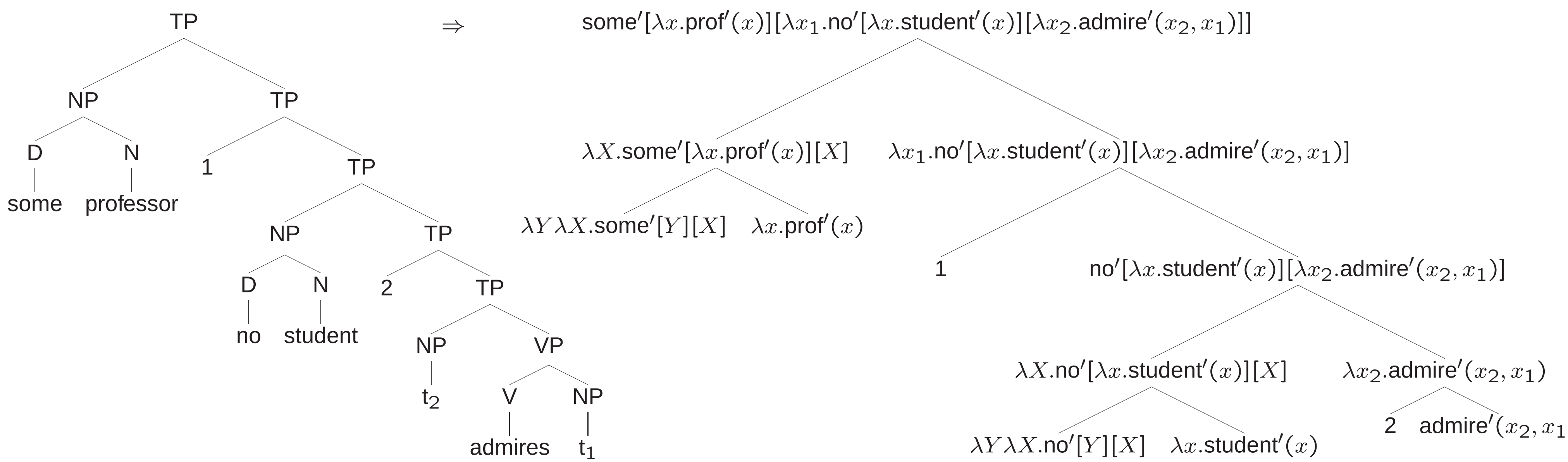
Putting the Logical Representations Together

(3) Jeff praises every student.



Putting the Logical Representations Together (cont.)

(4) No student admires some professor.



Truth-conditional Interpretion of Logical Representations

Assume a model M with domain of universe U, and an arbitrary assignment g.

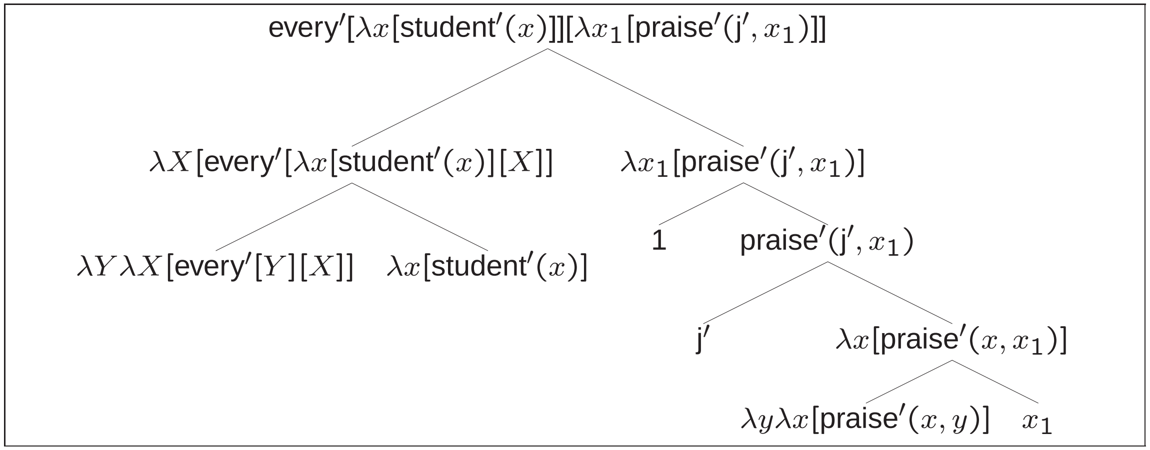
1. (a) If α' is a D and β' is a 1-place predicate, $[\![\alpha'(\beta')]\!]^{M,g} = [\![\alpha']\!]^{M,g}([\![\beta']\!]^{M,g})$.
   (b) If α' is a GQ and β' is a 1-place predicate, $[\![\alpha'(\beta')]\!]^{M,g} = 1$ iff $[\![\beta']\!]^{M,g} \in [\![\alpha']\!]^{M,g}$.

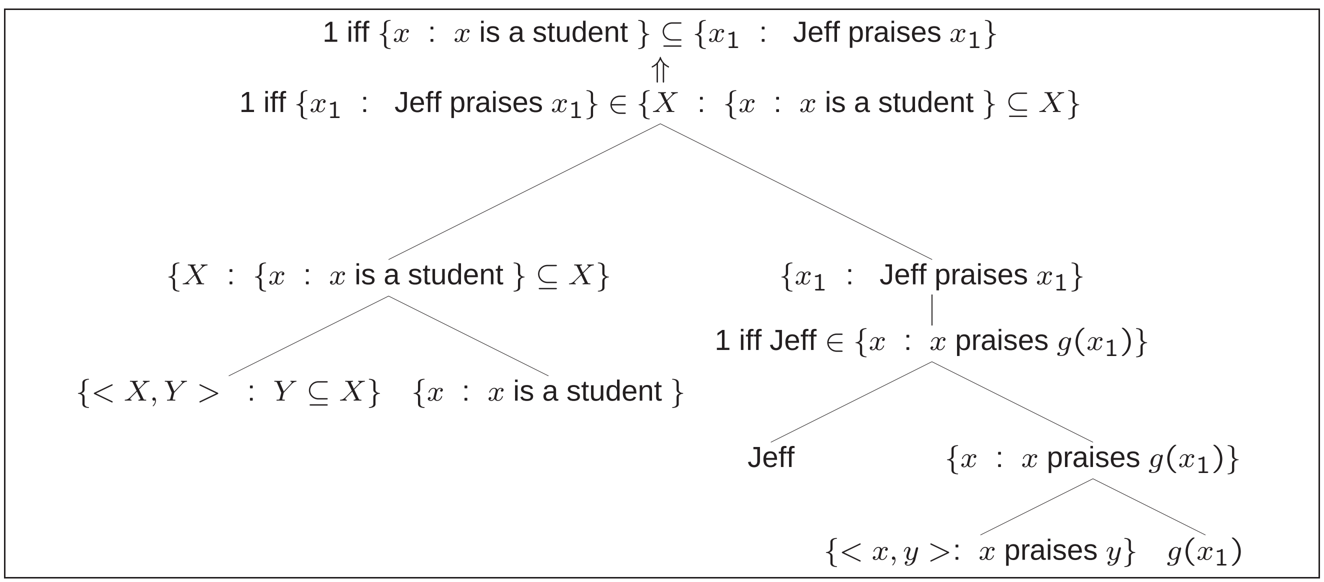
2. For every $X \subseteq U$ and $Y \subseteq U$,
   (a) $[\![\lambda Y \lambda X [\text{every}'[Y][X]]]\!]^{M,g} = \{< X, Y > \; : \; Y \subseteq X\}$
   (b) $[\![\lambda Y \lambda X [\text{some}'[Y][X]]]\!]^{M,g} = \{< X, Y > \; : \; Y \cap X \neq \emptyset\}$
   (c) $[\![\lambda Y \lambda X [\text{no}'[Y][X]]^{M,g}]\!] = \{< X, Y > \; : \; Y \cap X = \emptyset\}$
   (d) $[\![\lambda Y \lambda X [\text{the}'[Y][X]]]\!]^{M,g} = \{< X, Y >: \; \exists u \in U, \; Y = \{u\} \text{ and } u \in X\}$
   (e) $[\![\lambda Y \lambda X [\text{most}'[Y][X]]]\!]^{M,g} = \{< X, Y > \; : \; |X \cap Y| > |X^{-} \cap Y|\}$
   (f) $[\![\lambda Y \lambda X [\text{two}'[Y][X]]]\!]^{M,g} = \{< X, Y > \; : \; |X \cap Y| \geq 2\}$ or $\{X \; : \; |X \cap Y| = 2\}$

3. (a) $[\![\lambda x [\phi']]\!]^{M,g} = \{d \in U : [\![\phi']\!]^{M,g[d/x]} = 1\}$
   (b) $[\![\lambda y \lambda x [\phi']]\!]^{M,g} = \{< d_1, d_2 > : [\![\phi']\!]^{M,g[[d_2/y]d_1/x]]} = 1\}$

Interpreting Logical Representations Compositionally

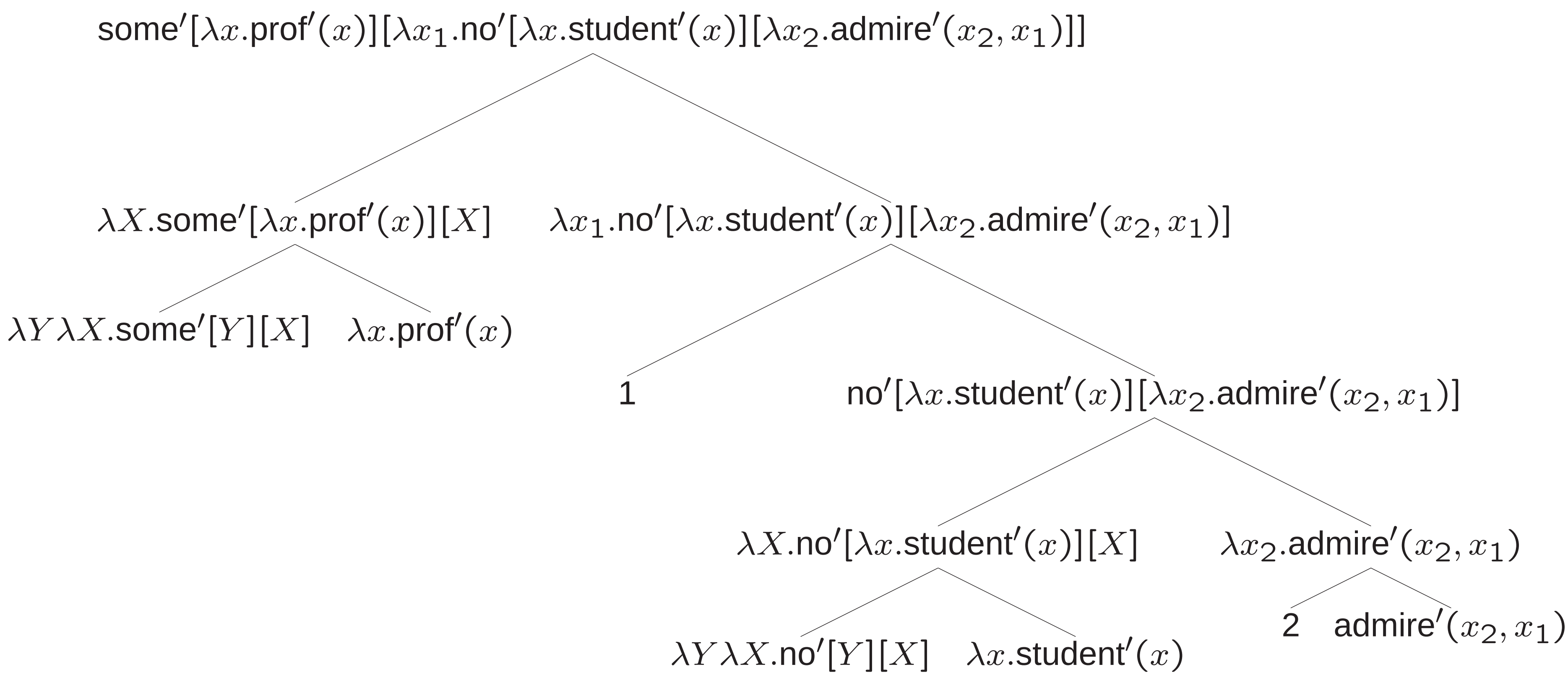
(5) Jeff praises every student.





Interpreting Logical Representations Compositionally (cont.)

(6) No student admires some professor.



Interpreting Logical Representations Compositionally (cont.)

$$1 \text{ iff } \{x : x \text{ professor } \} \cap \{x_1 : \{x : x \text{ student } \} \cap \{x_2 : x_2 \text{ admires } x_1\} = \emptyset\} \neq \emptyset$$

$$\Uparrow$$

$$1 \text{ iff } \{x_1 : \{x : x \text{ student } \} \cap \{x_2 : x_2 \text{ admires } x_1\} = \emptyset\} \in \{X : \{x : x \text{ professor } \} \cap X \neq \emptyset\}$$

$$\{X : \{x : x \text{ professor } \} \cap X \neq \emptyset\}$$

$$\{< X, Y >: Y \cap X \neq \emptyset\} \quad \{x : x \text{ professor } \}$$

$$\{x_1 : \{x : x \text{ student } \} \cap \{x_2 : x_2 \text{ admires } x_1\} = \emptyset\}$$

$$1 \text{ iff } \{x : x \text{ student } \} \cap \{x_2 : x_2 \text{ admires } g(x_1)\} = \emptyset$$

$$\Uparrow$$

$$1 \text{ iff } \{x_2 : x_2 \text{ admires } g(x_1)\} \in \{X : \{x : x \text{ student } \} \cap X = \emptyset\}$$

$$\{X : \{x : x \text{ student } \} \cap X = \emptyset\}$$

$$\{< X, Y >: Y \cap X = \emptyset\} \quad \{x : x \text{ student } \}$$

$$\{x_2 : x_2 \text{ admires } g(x_1)\}$$

$$1 \text{ iff } g(x_2) \in \{x : x \text{ admires } g(x_1)\}$$