Modality: Incorporating Ordering Source into the Equation

Feb. 2, 2004

Motivating Ordering Source: Graded Modality in Domain

(1) a. Michl must be the murderer.
 b. Michl is probably the murderer.
 c. There is a good possibility that Michl is the murderer.
 d. Michl might be the murderer.
 e. There is a slight possibility that Michl is the murderer.

- The detective will draw conclusions as to who the murderer is evidence available to him.

- The modal base, what the evidence provides, determines the accessible worlds.

- Some worlds among the epistemically accessible worlds are far-fetched than others with respect to the **normal course of**

  The worlds in the set of accessible worlds can be ordered in normal course of events.

- Can we use this ordering on the accessible worlds to capture notions of modality?

Definition of Ordering Source

- Ordering source: a function g from worlds to sets of propositi[...]

  Stereotypical ordering source in $w = g(w)$ = a set of propositi[...] represents the normal course of events in w.

- $g(w)$ induces partial ordering $\leq_{g(w)}$ on W:

  For all $u, v, w \in W$, and for any set of propositions $g(w)$:
  $u \leq_{g(w)} v$ iff $\{p : \ p \in g(w) \text{ and } v \in p\} \subseteq \{p : \ p \in g(w) \text{ and }$ [...]

  A world u is at least as close to the ideal represented by $g(w)$ [...] all propositions in $g(w)$ which are true in v are true in u as we[...]

- Modal base and ordering source together determine the relev[...] possible worlds to be considered for evaluating modal senten[...]

An Account of Graded Modality in Epistemic D

- A proposition p is a necessity ('must') in world w with respect
  base f and an ordering source g iff:

  for all $u \in \bigcap f(w)$, there is a $v \in \bigcap f(w)$ such that $v \leq_{g(w)} u$
  $z \in \bigcap f(w)$, if $z \leq_{g(w)} v$, then $z \in p$.

  (p is a necessity iff it is true in all accessible worlds which com
  ideal established by the ordering source.)

- p is a weak necessity ('probably') in w with respect to f and g
  (i) for all $u \in \bigcap f(w)$ and $u \in \neg p$, there is a $v \in \bigcap f(w)$ such
  and $v \in p$;
  (ii) it is not the case that for all $u \in \bigcap f(w)$ and $u \in p$, there is
  such that $v \leq_{g(w)} u$ and $v \in \neg p$.

- p is a good possibility in w with respect to f and g iff there is a
  $u \in \bigcap f(w)$ such that for all $v \in \bigcap f(w)$, if $v \leq_{g(w)} u$, then v

- p is a possibility ('might') in w with respect to f and g iff there i
  such that it comes closest to the ideal established by $g(w)$ an

An Account of Graded Modality in Epistemic Dom

- The semantics of modality as defined can also capture the en
  properties among modal expressions, keeping f and g param

<div align="center">

Michl must be the murderer

$\Downarrow$

Michl is probably the murderer

$\Downarrow$

There is a good possibility that Michl is the murde

$\Downarrow$

Michl might be the murderer.

</div>

Strength in Claims

(2)   a.   She climbed Mount Toby.
      b.   She must have climbed Mount Toby.

- Possibility 1:

  Modal base: $f(w)$ = in view of what we know in w.
  Ordering source: $g(w) = \emptyset$ (empty).

  Accessible worlds that come closest to the ideal provided by g
  including w.

  Result: (2b) entails (2a). Wrong prediction!

- Possibility 2:

  Modal base: $f(w)$ = in view of what we know in w.
  Ordering source: $g(w)$ = a non-empty set of propositions.

  Accessible worlds that come closest to the ideal provided by g
  may not include w.

  Result: (2b) does not entail (2a). Correct prediction!

Two Modal Bases: Epistemic and Circumsta

- Epistemic: Given all the facts and evidence, what might/must might/must be the case.

    (3)   There might be hydrangeas growing here.

    Ordering source: stereotypical conversational background like provided by normal course of events

- Circumstantial: Given the relevant facts, what can or must be

    (4)   Hydrangeas can grow here.

    Ordering source: normative conversational backgrounds like provides, what is good for you, what is moral, what is normal, etc.

    (5)   John should exercise everyday.

An Account of Inconsistencies

- Judgments in an imaginary country

  Every judge agrees that murder is a crime.

  Judge A decided that owners of goats are liable for damage th
  inflict on flowers and vegetables.

  Judge B decided that owners of goats are not liable for damag
  inflict on flowers and vegetables.

- Modal base: $f(w) = \emptyset$ (empty).
  Accessible worlds = $\bigcap f(w)$ = the set of all possible worlds.

- Ordering source: $g(w)$ = what the law provides
  {Murder is a crime, Goat owners are liable, Goat owners are r

- The set of all possible worlds can be partitioned into three typ

Type 1	Type 2	Type 3
worlds in which murder is not a crime	worlds in which murder is a crime and goat owners are liable	worlds in der is a cr owners ar

- Type 2 and 3 worlds come closest to the ideal set by the order

  Accessible worlds that come closest to the ideal provided by g
  = Type 2 $\cup$ Type 3.

- Makes the correct predictions

  (6) In view of what the judgments provide

      a. Murder is necessarily a crime.

      b. Murder is necessarily not a crime.

  (7) In view of what the judgments provide

      a. Owners of goats are possibly liable for damage cau
         animals.

      b. Owners of goats are possibly not liable for damage
         animals.

An Account of Samaritan Paradox

- Rethinking the semantics of conditionals

  (8) If a murder occurs, the murderer must go to jail.

  $[\![\text{if } \alpha, \text{ must } \beta]\!]^{f,g} = [\![\text{ must } \beta]\!]^{f',g}$, where for all $w \in W$,
  $f'(w) = f(w) \cup [\![\alpha]\!]^{f,g}$

  The proposition p from the *if*-clause further restricts the acces
  p-worlds.

- Modal base: $f'(w) = \emptyset \cup [\![\text{A murder occurs}]\!]$
  Accessible worlds = $W \cap [\![\text{A murder occurs}]\!]$ = a set of worlds
  murder occurs.

- Ordering Source: $g(w)$ = what the law provides
  {No murder occurs, If a murder occurs, the murderer goes to

- The accessible worlds that come closest to what the law provi
  restricted to the worlds in which a murder has occurred.

  (8) is predicted to be true in w if the murderer goes to jail in al
  accessible from w in which a murder has occurred.

Division of Labor between Semantics and Prag

- In Kratzer's (1990) system, both semantics and pragmatics ar modeling the meaning of modality.

    Semantics provides the general structure and the ingredients representation of modality, such as a modal base f and an or

    But the contents of f and g are determined by pragmatics, the from the discourse context and world knowledge.

- This is an example of how semantics and pragmatics interact full meaning of a natural language expression.