

Modality: A Standard Analysis

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Meaning and Grammar, Ch. 5.3.2; Kratzer 1991, pp. 63

Modality

- Modality has to do with necessity and possibility of situations.
- Grammatical means of expressing modality

Modal auxiliaries

- (1) New structures **must/can** be generated.

Adjectives

- (2) Winning the race is **impossible**.

Adverbs

- (3) **Possibly**, we will win the race.

Nouns

- (4) Winning the race is a remote **possibility**.

Suffixes

- (5) Such thoughts are **expressible** in any human language.

Simple present tense

- (6) This car **goes** 200 miles an hour.

Ambiguity in Modal Expressions

- Epistemic

- (7) In view of what the available evidence is
 - a. Jockl must be the murderer.
 - b. Jockl may be the murderer.

- Deontic

- (8) In view of what the law provides
 - a. Jockl must go to jail.
 - b. Jockl may smoke in the building.

- Circumstantial

- (9) In view of the relevant circumstances
 - a. Jockl must sneeze.
 - b. Jockl can lift the rock.

⇒⇒ Modal expressions themselves are not ambiguous. Whether interpreted as deontic, epistemic or circumstantial depends on the **conversational background** provided by the context.

Applying the Semantics of \Box and \Diamond in IPC to English

- \Box and \Diamond in IPC are logical necessity and logical possibility.

$[[\Box\phi]]^{M,w,g} = 1$ iff for all $w' \in W$ $[[\phi]]^{M,w',g} = 1$.

$[[\Diamond\phi]]^{M,w,g} = 1$ iff there exists at least one $w' \in W$ such that

(10) 3 cannot be divided by zero.

$\neg\Diamond[3 \text{ is divided by zero}] =$ It is not the case that there is a $w \in W$ such that 3 is divided by zero in w .

(11) Two plus two must make four.

$\Box[\text{two plus two makes four}] =$ For all $w \in W$, two plus two makes four in w .

- All possible worlds are considered in evaluating logical necessity and logical possibility. But *must* and *can* in English do not always work that way.

They sometimes form **contingent** modal statements, those whose truth values depend on what the world of evaluation looks like.

Applying the Semantics of \Box and \Diamond in IPC to English (cont.)

- *Can* in English

(12) Michael Jackson can't sing.

$\neg\Diamond[\text{mj sings}] =$ There is no $w \in W$ such that MJ sings in w .

This means that there is no logically possible world in which MJ sings. But clearly, one can conceive of a possible circumstance in which MJ sings.

- *Must* in English

(13) Michael Jackson must go to jail.

$\Box[\text{mj goes to jail}] =$ For all $w \in W$, MJ goes to jail in w .

This means that MJ goes to jail in all possible worlds. But clearly, one can conceive of a possible circumstance in which MJ doesn't go to jail.

- Only a **relevant subset of possible worlds** are considered for these modal sentences, which is determined by the **conversational background** provided by the context.

Conversational Background

- *what the law provides, what we know, what is provided by the* etc.

- What the law provides in a particular possible world is a set of

$$\llbracket \text{what the law provides} \rrbracket^{M,w} = \{p_1, p_2, p_3, \dots\}$$

- What the law provides could be different from one possible world

$\llbracket \text{what the law provides} \rrbracket^M =$ a function f from a set of possible worlds to a set of propositions.

A conversational background in $w = f(w) = \{p_1, p_2, p_3, \dots\}$

- A conversational background in w (= **modal base**) uniquely determines a relevant subset of possible worlds (= **accessible worlds**) to be used for evaluating modal sentences in w .

A proposition p is a set of possible worlds in which it is true.

$$\text{Accessible worlds} = \bigcap f(w) = \bigcap \{p_1, p_2, p_3, \dots\}$$

Meaning of Modals: *Must*

- Epistemic conversational background:

$[[\text{Jockl must be the murderer}]]^w = 1$ in w iff it follows from what we know in w that Jockl is the murderer.

$= 1$ in w iff in all the worlds in which all the propositions that we know in w are true, Jockl is the murderer.

- Deontic conversational background:

$[[\text{Jockl must go to jail}]]^w = 1$ in w iff it follows from what the law provides in w that Jockl goes to jail.

$= 1$ in w iff in all the worlds in which all the propositions that the law provides in w are true, Jockl goes to jail.

- Circumstantial conversational background:

$[[\text{Jockl must sneeze}]]^w = 1$ in w iff it follows from what the circumstance provides in w that Jockl sneezes.

$= 1$ in w iff in all the worlds in which all the propositions that the circumstance provides in w are true, Jockl sneezes.

Meaning of Modals: *May, Can*

- Epistemic conversational background:

$[[\text{Jockl may be the murderer}]]^w = 1$ in w iff it is consistent from what we know in w that Jockl is the murderer.

$= 1$ in w iff in some world in which all the propositions that we know in w are true, Jockl is the murderer.

- Deontic conversational background:

$[[\text{Jockl may smoke in the building}]]^w = 1$ in w iff it is consistent with the law that provides in w that Jockl smokes in the building.

$= 1$ in w iff in some world in which all the propositions that the law provides in w are true, Jockl smokes in the building.

- Circumstantial conversational background:

$[[\text{Jockl may sneeze}]]^w = 1$ in w iff it is consistent with what the circumstance provides in w that Jockl sneezes.

$= 1$ in w iff in some world in which all the propositions that the circumstance provides in w are true, Jockl sneezes.

Meaning of Modals: More Formally

- $\llbracket \text{must } \phi \rrbracket^w = 1$ iff for all $w' \in \cap f(w)$, $\llbracket \phi \rrbracket^{w'} = 1$
 $= 1$ iff $\cap f(w) \subseteq \{w' : \llbracket \phi \rrbracket^{w'} = 1\}$
- $\llbracket \text{can } \phi \rrbracket^w = 1$ iff there is a $w' \in \cap f(w)$, $\llbracket \phi \rrbracket^{w'} = 1$.
 $= 1$ iff $\cap f(w) \cap \{w' : \llbracket \phi \rrbracket^{w'} = 1\} \neq \emptyset$
- Meaning of *must* and *can* are related to each other in a certain way
 - $\text{must } \phi = \text{neg can } [\text{neg } \phi]$
(14) a. We must rehearse for the play.
b. We cannot not rehearse for the play.
(Cf. $\forall x P(x) = \neg \exists x \neg P(x)$, $\Box \phi = \neg \Diamond \neg \phi$)
 - $\text{can } \phi = \text{neg must } [\text{neg } \phi]$
(15) a. We can rehearse for the play.
b. It's not the case that we must not rehearse for the play.
(Cf. $\exists x P(x) = \neg \forall x \neg P(x)$, $\Diamond \phi = \neg \Box \neg \phi$)

What has the Standard Analysis Accomplished

- Captures the apparent ambiguity of modal expressions
- Accounts for the contingency of modal statements
- Captures the duality of *must* and *can*

QUESTION: As we have seen before, English modals also express possibility and logical necessity. Within the standard analysis, how do we characterize the conversational background of modals that express possibility or logical necessity?

- (16)
- a. 3 cannot be divided by zero.
 - b. Two plus two must make four.

Shortcomings: Inconsistencies

- Judgments in an imaginary country

Source of law in this imaginary country is the judgments made which are handed down.

Every judge agrees that murder is a crime.

Judges disagree on certain issues.

Judge A decided that owners of goats are liable for damage they inflict on flowers and vegetables.

Judge B decided that owners of goats are not liable for damage they inflict on flowers and vegetables.

- Conversational background based on *what the judgments provide*

{Murder is a crime, Owners of goats are liable for damage they inflict on flowers and vegetables, Owners of goats are not liable if their animals inflict on flowers and vegetables}

- Predictions made by the standard analysis on necessity modal logic

(17) In view of what the judgments provide

- a. Murder is necessarily a crime.
- b. Murder is necessarily not a crime.

Wrongly predicts that both examples in (17) are true. According to intuition, only (17a) should come out to be true.

- Predictions made by the standard analysis on possibility modal logic

(18) In view of what the judgments provide

- a. Owners of goats are possibly liable for damage caused by their animals.
- b. Owners of goats are possibly not liable for damage caused by their animals.

Wrongly predicts that both examples in (18) are false. According to intuition, both should come out to be true.

Shortcomings: Samaritan Paradox in Conditional

- A conversational background based on *what the law provides*
{No murder occurs, If a murder occurs, the murderer will go to jail.
- Predictions made by the standard analysis of modality and conditionals:
(19) It is necessary that
 - a. if a murder occurs, the murderer will go to jail.
 - b. if a murder occurs, the murderer will be knighted.
 - c. if a murder occurs, the murderer will be given \$100.
 - d. if a murder occurs, the murderer will be fined \$100.

Any old conditional whose antecedent is the proposition *a murder occurs* comes out to be true!

Shortcomings: Graded Notions of Modality

- Modal expressions are gradable. But the standard analysis cannot capture the notion of graded modality.

- (20)
- a. Michl is probably the murderer.
 - b. There is a good possibility that Michl is the murderer.
 - c. Michl might be the murderer.
 - d. There is a slight possibility that Michl is the murderer.

In the standard analysis, there is no difference in truth conditions between the examples in (20).