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Mass nouns and non-singular logic

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1. Introduction[1]

A singular term of a natural language is a term that, each time it is used to refer within an utterance, refers to a single individual. Thus, both *John* and *this cat* are singular terms, as we can see for instance in the following sentence, uttered in appropriate contexts:

John fed this cat.

By contrast, there is intuitively no reason to think that *John and Bill* and *these cats* are singular terms. Each expression may refer to several things, and may thus be said to be a non-singular term:

John and Bill fed these cats.

(Typically, a plural expression refers to more than one thing. However, it can also refer to one thing only, as in the following sentence: *The morning star and the evening star are the same star.*)

What about mass nouns, like *wine* and *silverware*? Consider the sentence:

This wine costs a hundred euros.

If there is one bottle of wine in front of us, it seems that one thing is being referred to. However, if there are six bottles of wine, it seems, intuitively, that six things are being referred to. The goal of this paper is to see whether a formal account of the semantics of mass nouns can pay justice to this intuition.

This is done in light of relatively recent works in logic and on plurals. A dilemma put forward by Schein (1993, 2006) and Rayo (2002, 2006) suggests that, in order to characterize the semantics of plurals, we should not use predicate logic, but non-singular logic, a formal language whose terms may refer to several things at once. What about mass nouns, then? Should non-singular logic be used to represent the truth-conditions of sentences in which mass nouns appear?

We explore several issues that are linked to this question. We first present the data that a semantics of mass nouns must account for (§2). We explain how the dilemma arises for plurals (§3), and we show that it also applies to mass nouns (§4). We then use non-singular logic to develop a semantics of mass nouns that treats them as non-singular terms, which may refer to several things at once (§5).

In one respect, the account we put forward resembles set-theoretic approaches, which would assign a set of things as the denotation of mass noun phrases. Indeed, the non-singular semantics we shall propose assigns some things (rather than a set of things) as the denotation of such phrases. Now, various kinds of sentences have been seen as presenting difficulties for set-theoretic approaches (cf. Bunt (1985) and Pelletier and Schubert (2003)). Chief among them are the following:

The gold on the table weighs seven ounces. (Bunt 1985)

All phosphorus is either red or black. (Roeper 1983)

The clay that made up those three bowls is identical with the clay that now makes up these two statues. (Example inspired by Cartwright 1965)

We show in §5 that the non-singularist account we propose is able to deal with such sentences.

[1] Acknowledgements suppressed for blind review.

2. Data that a semantics of mass nouns must account for

2.1. General observations

A semantics of mass nouns should specify the truth conditions of the sentences in which mass nouns appear. It should be able to do so for the various kinds of noun phrase that mass nouns can head: definite and demonstrative noun phrases (*this M, the M that Qs*, e.g., *the gold that is on the table*), and quantified noun phrases (*some / a lot of / all / any M*, e.g., *some gold*). In particular, it should tell us what definite and demonstrative noun phrases refer to, and what quantified noun phrases quantify over.

At the same time, the semantics should account for the various kinds of construals to which sentences containing mass nouns are liable: so-called *collective*, *distributive*, and *"intermediary" construals* (cf. Gillon 1992). We characterize these construals in the next section (§2.2), in connection with each kind of noun phrase.

Such construals are also observed for sentences containing plurals. This tells us that the semantics of mass nouns and the semantics of plurals must share some significant features, though they need not be identical.[2]

Let us indicate one of these properties here: mass nouns and plurals *refer cumulatively* (cf. Quine 1960 for mass nouns). Consider a mass noun *M*. Suppose that, in a given circumstance, we can truly say, of something x that *This is M* (with *this* referring to x) and of something distinct, y, that *This is M* (with *this* referring to y). Then, in the same circumstance, we can refer to x and y together, and say of x and y that *This is M.* This characteristic of mass nouns is called *cumulative reference*. Plurals have the same property. Let *Ns* be a plural count noun. If these are Ns and those are Ns, then we can refer to these and those together, and say of all of them that they are Ns.

Sentences containing mass nouns, plurals and singular count nouns are also liable, in certain cases, to so-called *generic construals*: *Gold is expensive*, *Dinosaurs are extinct*, *The potato is highly digestible,* etc. Following Gillon (1990), we take these to correspond to a variety of independent phenomena, which are not specifically related to mass nouns and plurals.[3] So we will have nothing to say about them in this paper.

Finally, let us discuss one property that various authors have ascribed to mass nouns, the property of *having minimal parts.* Let us say that x is a *minimal part* for a mass noun *M* if *M* applies to x (i.e. the sentence *This is M* is true when the demonstrative *this* refers to x), but *M* applies to no part of x (there is no part y of x such that the sentence *That is M* is true when *that* refers to y). A collective mass noun like *furniture* has minimal parts: each piece of furniture is a minimal part for the noun. However, a mass noun like *time* is plausibly taken not to have minimal parts: any time x has some time y as part. The same seems to be true of a mass noun like *space.*[4] Consider now mass nouns that name substances, like *water* and *gold.* Modern physics suggests that they have minimal parts, namely their individual molecules. However, it is unclear whether the customary meaning of a word like *water* has changed because of the discoveries of modern science. Be this as it may, some mass nouns seem to have minimal parts (like *furniture* and *jewelry*), while others do not (like *time* and *space*). We can conclude that grammar is mute on this score: the grammar of a language like English says nothing

[2] This is well known in the literature; see for instance Link (1983), Gillon (1992) and Chierchia (1998).

[3] For reviews of the literature on generic construals, see Krifka et al. (1995) and Pelletier and Asher (1997).

[4] It is at least very much unclear whether these nouns have minimal parts. Therefore, the grammar that the linguist hypothesizes should not forbid a mass noun to be without minimal parts.

about whether a mass noun should have minimal parts (this is also the conclusion of Gillon (1992)).

2.2. Collective, distributive and intermediary construals

As suggested by Gillon (1992), a sentence containing a mass noun may receive collective, distributive, and "intermediary" construals, modulo the meaning of the particular lexical items composing the sentence, context of speech and knowledge of the world.[5] While Gillon has given a few examples of these construals, we find that he has not considered the phenomena in enough detail. We therefore attempt to describe the phenomena somewhat more precisely. We first characterize the construals of sentences containing definite and demonstrative noun phrases. We then look at the construals of sentences containing quantified noun phrases.

Consider a sentence whose subject is a definite or demonstrative mass noun phrase, e.g., *This silverware costs a hundred euros*. The sentence may be true if the silverware costs, all together, a hundred euros: this is what is called the *collective construal* of the sentence. It may be true if each piece of silverware, by itself, costs a hundred euros: this is the *distributive construal.* It may also be true if the silverware demonstrated consists in two sets of silverware, each set of silverware costing, by itself, a hundred euros: this is what we shall call an *intermediary construal.*

The same range of construals may be observed with a mass noun like *wine*: *This wine costs a hundred euros*. An intermediary construal can be obtained, for instance, when the wine demonstrated consists of two cases of bottles of wine.

A simple clause may receive collective, distributive, and intermediary construals as long as at least one of the arguments of the verb is a mass expression (Gillon 1996). The mass noun phrase may be the subject, the object or the indirect object of the verb, or the object of a prepositional phrase complementing the verb. Take the sentence: *Bill ran through the furniture*. Its most likely construal is a collective one: Bill ran among the pieces of furniture. Imagine, however, that the furniture consisted in several chairs that were two meters high. Then a distributive construal becomes possible, whereby Bill literally ran through each piece of furniture. Alternatively, imagine that the furniture is divided into groups, each group being arranged to form a kind of *Arc of Triumph*. Then an intermediary construal becomes possible.

This example shows that the specific meanings of the verbal expression and its arguments, combined with knowledge of the world and context of speech, may render a type of construal more, or less, plausible.

We also see that intermediary construals are harder to get than distributive and collective construals, requiring specific information about the context in order to become available. Intermediary construals are often easier to get when the verb has several arguments, as in the following example due to Gillon (1992):

This fruit was wrapped in that paper.

An intermediary construal with respect to its first argument (*this fruit*) would be one where there are several pieces of paper, each enclosing several pieces of fruit.

In all the examples we have given so far, intermediary construals correspond to *partitions* of the denotation of the mass noun phrase. It is well known that plurals may also receive distributive, collective, and intermediary construals. In their case,

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[5] Remark: sentences containing plurals also receive distributive, collective and intermediary construals. This is well documented by Gillon (1992, 1996) and Schwarzschild (1996). The reader may confirm this for himself or herself by replacing mass nouns by plurals in the examples we give below.

intermediary construals may correspond not only to partitions, but also, more generally, to *covers* (Gillon 1992).[6] Imagine that any two of three men composed one opera together. This may make the sentence *These men wrote these operas* true. The three pairs of men do not correspond to a partition of the three men, but to a cover.

Can we find similar cases with mass nouns? This is harder than with plurals. Still, consider the sentence: *These men stole that gold.* It may be that some gold was repeatedly part of the gold stolen jointly by some of the men. The situation would correspond to a cover of the gold that is not a partition. Likewise, with *This livestock carried that furniture,* it may be that some pieces of furniture were repeatedly part of the furniture carried jointly by some of the livestock. We conclude that the semantics of mass nouns should leave room, not only for partitions, but also for all kinds of covers.

Let us now look at quantified mass noun phrases, starting with existential ones, like *a lot of M* and *some M.* Consider first sentences with *a lot of.* The sentence *Bill ran through a lot of furniture* asserts that there was a lot of furniture, and that Bill ran through it. It may receive a collective construal: Bill ran among many pieces of furniture. It may also receive a distributive construal: Bill ran through each of many pieces of furniture. Moreover, a sentence like *A lot of fruit was wrapped in a lot of paper* may receive an intermediary construal, for instance with respect to its first argument (*a lot of fruit*): there were many pieces of fruit and many pieces of paper, each piece of paper enclosing some pieces of fruit.

Consider now sentences with *some.* The sentence *Bill ran through some furniture* asserts that there was some furniture, and that Bill ran through it. In other words, relative to a certain cover of the denotation of *furniture*, there is at least one element of this cover such that Bill ran through it.

Let us now look at quantified mass noun phrases that are universal, like *all M* and *any M.* Imagine a man who goes into a shop and looks carefully at what is sold. Later, he reports to his wife:

All silverware costs a hundred euros.

A distributive construal is available: each piece of silverware costs a hundred euros. An intermediary construal also seems to be possible: what is sold consists in several sets of silverware and each set costs a hundred euros. With some stretch, one may perhaps imagine a collective reading, but it seems to be parasitic on the construal obtained with the phrase *all the silverware*: *All the silverware costs a hundred euros.* A collective construal with *all silverware* appears in fact to be infelicitous.

Likewise, distributive and intermediary construals (but not collective construals) seem to be available with phrases of the form *any M.*

Having characterized the main data that a semantics of mass nouns must account for, we turn to the dilemma that has been presented for semantic of plurals based on predicate logic, before examining whether the dilemma applies for mass nouns as well.

3. The dilemma for plurals

3.1. The dilemma

The constants and variables of predicate logic are singular in the following sense. Under any interpretation, a constant is interpreted as one individual, and under any assignment, a variable is interpreted as one individual.

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[6] Gillon considers only *minimal covers* (no element of the cover should be part of another element), which he calls *aggregations.* We prefer to use the more general notion of cover. Nothing of substance hinges on that choice.

Now, the representation of a plural expression like *John and Bill* in predicate logic is plausibly taken to be a constant. So, it must be interpreted as an individual in the domain of interpretation. Starting with Bolzano and Frege, two kinds of proposals have been made for what this individual may be: it may be a set, like the set whose members are John and Bill, or a whole, like the mereological sum of John and Bill.

Recently, however, Schein (1993, 2006) and Rayo (2002, 2006) have put forward a general argument to show that representing plural expressions in predicate logic is unsatisfactory.

On the one hand, if we use sets to represent plurals, we run into the contradiction discovered by Russell in naive set theory. Indeed, the sentence *There are some sets such that any set is one of them just in case it is a set that is not a member of itself* seems to be comprehensible and true, yet it cannot be represented using sets, for the resulting logical representation would be contradictory (cf. Boolos 1984; see also §4.1 for more details).

On the other hand, if we use mereological sums, the semantics turns out to be too weak. Indeed, there exist count nouns *M* and *N* such that an M is not an N, but the Ms and the Ns have the same mereological sum. Then, for any given predicate *P*, the semantics attributes the same truth-value to the sentences *The Ms P* and *The Ns P*. With certain predicates, however, these sentences have, intuitively, opposite truth-values.

Let us consider an example given by Rayo (2002 and p.c.). Suppose that there are several piles of grains, which are scattered, i.e. spatially separated from one another in irregular fashion. Then, the sentence *The piles of grains are scattered* is true, while *The grains are scattered* seems to be false. (We do not consider an alternative understanding of the sentence, where each of the piles has been scattered and destroyed in the process, only grains remaining.) One may object that a pile of grains is not identical with the grains that make it up: if the grains are dispersed, the pile ceases to exist, while the grains continue to exist. Therefore, the mereological sum of the piles of grains is distinct from the sum of the grains. However, take the sentence *The mereological sums, each of which is the mereological sum of the grains making up a pile, are scattered.* In the circumstance imagined, it is true, while *The grains are scattered* is false. But the sum of the mereological sums of grains is identical with the mereological sum of all the grains, so the semantics must attribute the same truth-value to both sentences.

It is possible to resist this argument. Concerning the first horn of the dilemma, several researchers defend an *open-ended conception of sets*, whereby it is impossible to refer to all sets at once (see Glanzberg 2004, and Rayo and Uzquiano 2006). The second horn of the dilemma is likely to be resisted by someone who admits the thesis of *composition as identity*. This thesis may be expressed as follows (cf. van Inwagen 1994). Let the ds be any things. Suppose that they have a mereological sum t, so that we may say that the ds compose t. Then they are identical with their mereological sum.[7] Suppose that composition as identity were true. Then, how could there be a difference between referring to several things at once and referring to their mereological sum, if the sum of these things was identical with these things?

In this work, however, we take the argument to be sound. Indeed, together with Williamson (2003), we think that absolutely unrestricted quantification is genuinely possible, hence it is possible to refer to all sets; and together with van Inwagen (1994)

[7] Remark: the thesis is sometimes expressed using a narrower notion of composition, notably by van Inwagen (1990) himself. In *Material beings*, van Inwagen states that the ds compose t just in case no two ds overlap and t is the mereological sum of the ds. This terminological difference is unimportant here.

and Yi (1999), we reject the thesis of composition as identity (see the discussion at the end of §4.3). So, plurals should not be represented in predicate logic. An attractive alternative is offered by *non-singular logic* (Rayo and Yi use the expression *plural logic* instead; see Rayo 2002, 2006, Linnebo 2004, Yi 2005, 2006, and McKay 2006). Non-singular logic possesses non-singular constants and variables, which may be interpreted as one or several individuals of the domain of interpretation. This makes for a natural representation of the semantics of plurals.

3.2. Non-singular logic

We give here a brief sketch of the main characteristics of non-singular logic, referring the reader to the authors above for a detailed exposition.

As we said, predicate logic has constants and variables that are singular. Under any interpretation and variable assignment, a term (be it a constant or a variable) is interpreted as one individual of the domain of interpretation. A formula consisting of a predicate whose argument is a constant is true just in case the constant is interpreted as one individual that satisfies the property expressed by the predicate.

Non-singular logic possesses singular and non-singular constants and variables. Under any interpretation and variable assignment, a term (a constant or a variable) that is not singular may be interpreted as one or more individuals of the domain of interpretation. A formula consisting of a predicate whose argument is a non-singular constant is true just in case the constant is interpreted as one or more individuals that jointly satisfy the property expressed by the predicate.

By convention, variables like ‘x’, ‘y’ and ‘z’ are singular variables, while variables ending with an ‘s’, like ‘xs’, ‘ys’ and ‘zs’, are non-singular. Quantifiers apply both to singular and non-singular variables. ‘$\exists x\ (Px)$’ means: there is some thing that satisfies P. $\exists xs\ (Pxs)$’ means: there are one or more things that jointly satisfy P. A predicate may have one or more argument places, and in each argument place, it may accept singular terms, non-singular terms, or both.

The predicate *among* plays a special role in non-singular logic, where it is treated as a logical predicate, together with identity. We note it ‘$\angle$’. ‘$x \angle xs$’ means that x is among the xs (it is one of the xs). The following are *axiom schema* of non-singular logic:

(NSC) $\exists x\ (Qx) \rightarrow \exists zs\ (\forall y\ (y \angle zs \leftrightarrow Qy))$ *Non-singular comprehension*

If some thing x Qs, then there are some things, the zs, such that some thing y Qs just in case it is among them.

(NSI) $\forall x\ (x \angle ys \leftrightarrow x \angle zs) \rightarrow (Qys \leftrightarrow Qzs)$ *Non-singular indiscernibility*

For any things ys and any things zs, if they have the same things among them, then they satisfy the same formulas. (In all formulas, we drop initial universal quantifiers to enhance readability. Thus, in (NSI), the initial quantifiers ‘$\forall ys$’ and ‘$\forall zs$’ have been dropped.)

The theory of truth and the model theory of non-singular logic differ from those employed for predicate logic. The metalanguage in which such theories are expressed possesses terms that can refer to several things at once, and predicates that can be satisfied collectively by several things. For predicate logic, a model is specified using sets. In particular, a set fixes the universe of discourse (what objects the quantifiers range over), and a subset of this set specifies the interpretation of a one-place predicate. For non-singular logic, the model theory does without sets, in order to avoid set-theoretic paradoxes like the paradox mentioned in the last section. The universe of discourse is not fixed by a set of things, but by these things themselves. Using a non-singular constant in the metalanguage, these things may be referred to as, say, the ts.

What about predicates? How can their interpretation be specified? McKay (2006) and Yi (2006) use worldly properties and relations in order to interpret predicates. A formula like 'Pj' is true if and only if the object that interprets the constant 'j' has the property that interprets the predicate 'P'. Doing so has the following limitation. The metalanguage refers to properties and relations. The object language, however, cannot refer to all the properties and relations that there are, for this would generate a Russellian paradox. Therefore, it is not possible, in the object language, to refer to and quantify over absolutely everything there is.

As indicated in §3.1, with Rayo (2002, 2006) and Williamson (2003), we held that absolutely unrestricted quantification is genuinely possible. Therefore, we cannot follow McKay and Yi. Rayo (2006) develops an alternative that allows for absolutely unrestricted quantification. It consists in using languages that contain not only non-singular terms, but also terms that are "hyper-non-singular" (or "super-plural", as Rayo qualifies them), "hyper-hyper-non-singular", etc. A non-singular term may refer to several things. Therefore, we may say, in a somewhat misleading way, that it may refer to a "plurality" of things. (This is misleading since the expression 'a plurality' suggests that one thing is referred to, while a non-singular term simply refers to several things.) A hyper-non-singular term may refer to several "pluralities". Likewise, a "hyper-hyper-non-singular" term may refer to several "pluralities of pluralities". Hyper-non-singular terms allow one to formulate a principle of *hyper-non-singular comprehension*:

(HNSC) $\exists$xs (Qxs) $\rightarrow \exists$zss ($\forall$ys (Qys $\leftrightarrow$ ys $\angle$ zss)) [8]

If some things Q, then there are some "pluralities" (the zss) such that some things Q just in case they are among these "pluralities".

Thus, given a (collective) predicate, we can use a hyper-non-singular term to refer to the "pluralities" that the predicate applies to.

Rayo (2006) establishes a series of results concerning the model theories of various languages. The model theory of a language that contains singular and non-singular terms and allows for absolutely unrestricted quantification cannot be expressed in a language of the same type, but it can be expressed in a language that also contains hyper-non-singular terms. Likewise, the model theory of a language that contains singular, non-singular, and hyper-non-singular terms and allows for absolutely unrestricted quantification cannot be expressed in a language of the same type, but it can be expressed in a language that also contains hyper-hyper-non-singular terms.

As we will show, in order to specify the truth-conditions of sentences containing mass nouns and plurals, we will need to be able to refer to several "pluralities". We will do so using hyper-non-singular terms.

[8] 'ys $\angle$ zss' is true if and only if the non-singular term 'yz' denotes some things (a "plurality") that are among the several "pluralities" denoted by the hyper-non-singular term 'zss'. We find it convenient to use the same symbol '$\angle$' as before. But we are really employing two distinct logical primitives: one that says that some thing is among some things ('x $\angle$ ys'), the other that says that a "plurality" is among several "pluralities" ('ys $\angle$ zss').

4. Does the dilemma found with plurals apply to mass nouns?

We now want to examine the situation in the case of mass nouns. We consider two questions in turn.

4.1. Do we run into a Russellian contradiction if the semantics of mass nouns is based upon sets and predicate logic?

Suppose that we have a semantics of mass nouns based upon sets and predicate logic. Do we thereby run into a Russellian contradiction?

Let us first see more precisely how Schein and Rayo obtain a contradiction with plurals. Consider the following sentence schema, where *N* is a count noun and *P* is a verbal expression:

There are some Ns such that, for any y, y is one of these Ns just in case y is an N that does not P.

We get an intelligible and true instance of the schema if we replace *N* by *animal* and *P* by *have a tail*:

There are some animals such that, for any y, y is one of these animals just in case y is an animal that does not have a tail.

Suppose that plurals are represented using sets. Then the representation of the sentence schema becomes:

$\exists x\ (\mathrm{set}(x) \wedge \forall y\ (y \in x \leftrightarrow Ny \wedge \neg Py))$

For example:

There is a set x such that, for any y, y is a member of x just in case y is an animal and y does not have a tail.

So far, so good. Yet, replace *N* by *set* and *P* by *is a member of itself*. This instance of the schema is still intelligible and true:

There are some sets such that, for any y, y is one of these sets just in case y is a set that is not a member of itself.

But if we represent plurals using sets, we obtain a Russellian contradiction:

$\exists x\ (\mathrm{set}(x) \wedge \forall y\ (y \in x \leftrightarrow \mathrm{set}(y) \wedge y \notin y))$

Indeed, if we suppose that x is a member of itself, we conclude that x should not be a member of itself. And if we suppose that x is not a member of itself, we conclude that x should be a member of itself.

Remark: This argument presupposes that the domain of the existential quantifier '$\exists x$' is the same as that of the universal quantifier '$\forall y$'. As already said, with Williamson (2003), we suppose that it is possible to quantify over absolutely everything there is. In particular, we suppose that, in the above English sentence, it is possible to quantify over all the sets that there are. This ensures that the set x that is used in the representation of the sentence is among the sets that are talked about in the sentence.

Can we generate a similar contradiction using mass nouns instead of singular and plural count nouns, like *set* and *sets*? Let us take a mass expression *M* and consider the following sentence schema:

There is some M such that any M that does not P is part of it and no M that Ps is part of it.

Replacing *M* by *silverware* and *P* by *have a blade*, we get:

There is some silverware such that any silverware that does not have a blade is part of it and no silverware that has a blade is part of it.

The sentence is understandable. Its most salient interpretation, the one that interests us, may be paraphrased as:

There is some silverware such that any piece of silverware that does not have a blade is part of it and no piece of silverware that has a blade is part of it.

Under this interpretation, and assuming that there is some silverware that does not have a blade (e.g. a spoon), the sentence is both intelligible and true.

Using sets and predicate logic, it may be represented as follows:

$\exists x\, (\mathrm{set}(x) \wedge \forall y\, (y \in x \leftrightarrow \mathrm{silverware}(y) \wedge \neg\mathrm{has\text{-}a\text{-}blade}(y)))$

Given the meanings of *silverware* and *have a blade*, only a piece of silverware is some silverware that can have a blade or fail to have one. So, as the paraphrase given above indicates, this is equivalent to:

$\exists x\, (\mathrm{set}(x) \wedge \forall y\, (y \in x \leftrightarrow \mathrm{piece\text{-}of\text{-}silverware}(y) \wedge \neg\mathrm{has\text{-}a\text{-}blade}(y)))$

Using this sentence schema, can we get a contradiction? Let us try with an hypothetical mass noun, *setware*, which denotes sets. This mass noun functions just like the mass nouns *livestock* and *silverware*. Some livestock, we may say, is one or more animals of the farm. Likewise, some setware, we may say, is one or more sets. Replacing *M* by *setware* and *P* by *is a member of itself* in our sentence schema, we get:

There is some setware such that any setware that is not a member of itself is part of it and no setware that is a member of itself is part of it.

The sentence is understandable. Its most salient interpretation, the one that interests us, may be paraphrased as:

There is some setware such that any set that is not a member of itself is part of it and no set that is a member of itself is part of it.

Under this interpretation, and assuming that there is some setware that is not a member of itself, the sentence is thus intelligible and true.

It is represented as:

$\exists x\, (\mathrm{set}(x) \wedge \forall y\, (y \in x \leftrightarrow \mathrm{setware}(y) \wedge y \notin y))$

Given the meanings of *setware* and *be a member of itself*, only a set is some setware that can be a member of itself or fail to be. So, as the paraphrase given above indicates, this is equivalent to:

$\exists x\, (\mathrm{set}(x) \wedge \forall y\, (y \in x \leftrightarrow \mathrm{set}(y) \wedge y \notin y))$

So, using the mass noun *setware*, we are able to reproduce the Russellian paradox found with plurals. Of course, this mass noun is an hypothetical one. But it functions in the same way as collective mass nouns like *livestock* and *silverware*. It is therefore a possible and legitimate addition to English. The fact that we get a contradiction when we try to represent a sentence where it appears using sets suggests that, just as in the case of plurals, a semantics of mass nouns should not be based solely on predicate logic and sets.

4.2. Can a semantics of mass nouns be based upon mereological sums and predicate logic?

Let us now look at our second question. Mereology may be used in various ways in connection with the semantics of mass nouns. (We state the axioms of extensional mereology in §5.1, and we define a notion of generalized mereological sum at the end of §5.3.) One possibility (which can partly be traced to Quine (1960), though he does not adopt it) consists in translating a sentence like *This is wine* as '$c < W$', where 'c' denotes what is demonstrated and 'W' denotes the mereological sum of all the wine that there is. The problem with this is that there are some parts of wine (e.g. atoms) that do not count as wine, a problem which is even clearer in the case of mass nouns like *furniture*: the leg of a chair is not furniture. Therefore, we will not consider this possibility any further in this paper.

The question we are asking is the following. Can we devise a satisfactory semantics of mass nouns if we use predicate logic and identify the denotation of a mass noun phrase with a certain mereological sum? In the case of plurals, the corresponding question receives a negative answer. How do things stand in the case of mass nouns?

The first thing we should ask ourselves is whether the Russellian paradox found when using sets also arises when using mereological sums. Let us start with the sentence with the word *silverware*:

There is some silverware such that any silverware that does not have a blade is part of it and no silverware that has a blade is part of it.

Using the parthood relation (≤), it is represented as:

$\exists x$ (silverware(x) $\wedge$ $\forall y$ (silverware(y) $\rightarrow$ ($\neg$has-a-blade(y) $\leftrightarrow$ y $\leq$ x)))

In this representation, 'x' is intended to denote a certain mereological sum, the mereological sum of anything that is silverware and that does not have a blade, taking this mereological sum to be some silverware. This is the sum of any piece of silverware that does not have a blade. The above is thus also equivalent to:

$\exists x$ (silverware(x) $\wedge$ $\forall y$ (piece-of-silverware(y) $\rightarrow$ ($\neg$has-a-blade(y) $\leftrightarrow$ y $\leq$ x)))

Let us see now what happens for the sentence with the word *setware*:

There is some setware such that any setware that is not a member of itself is part of it and no setware that is a member of itself is part of it.

It is represented as:

$\exists x$ (setware(x) $\wedge$ $\forall y$ (setware(y) $\rightarrow$ (y is not a-member-of-itself $\leftrightarrow$ y $\leq$ x)))

Here, 'x' is intended to denote the mereological sum of anything that is setware and that is not a member of itself, i.e. the sum of any set that is not a member of itself. The representation is thus equivalent to:

$\exists x$ (setware(x) $\wedge$ $\forall y$ (set(y) $\rightarrow$ (y is not a-member-of-itself $\leftrightarrow$ y $\leq$ x)))

No contradiction necessarily arises from these representations.

Uzquiano (2006a, 2006b) shows that various combinations of plausible, additional assumptions concerning set theory and mereology do land us into paradox. We do not have enough space to present Uzquiano's results. But they are less dramatic than the contradictions obtained in §3.1 and §4.1. Though the assumptions in question are relatively plausible, we are not forced to adopt all of them, and there are in fact various ways to escape paradox, which Uzquiano discusses (while endorsing only one of them).

The account sketched in this section is committed to the existence of mereological sums of sets, and, by parity of reasoning, to the existence of mereological sums of whatever one may name by a plural expression (since we may postulate the existence a mass noun corresponding to any plural expression). While friends of mereology argue that mereology is indeed topic neutral and universally applicable, this is not indisputable. From his investigations, Uzquiano precisely concludes that we should not take mereology to apply to sets. However, as just said, one may evade the paradoxes that Uzquiano is concerned with in various ways. One of those, adopted by Lewis (1991), consists in adopting a set theory with proper classes, where a proper class cannot be the member of a singleton set. This might have a cost concerning the foundations of mathematics, as Uzquiano thinks, but at least this position is consistent and not self-contradictory.

The second question we need to examine is whether we can identify the same kind of semantic weakness as the one observed with plurals. For such a weakness to be present, there would have to be two mass nouns, *M* and *N*, such that some M is not (always) some N, but the M and the N have the same mereological sum. Then, for any predicate given P, the semantics would attribute the same truth-value to the sentences *The M Ps* and *The N Ps*. However, there might be predicates with which these sentences have, intuitively, opposite truth-values.

Let us consider a first example due to Parsons (1970). Suppose that all wood is used to make up furniture and all furniture is made of wood. If the mereological sum of the wood is identical with the mereological sum of the furniture, then all sentences of the form *The wood Ps* and *The furniture Ps* are predicted to have the same truth-value. Yet, it might well be that *The furniture is heterogeneous* is true, intuitively, while *The wood is heterogeneous* is false.

However, as in the case of grains and piles of grains, one may object that some furniture and the wood that make it up are distinct: if the furniture is broken, it ceases to exist, while the wood does not. Therefore, the mereological sum of the wood is distinct from the mereological sum of the furniture.

What about considering, instead of furniture, the mereological sum of the wood making up a piece of furniture? This is not exactly what we want. We need to use an expression that is mass, both syntactically and semantically. In particular, it must satisfy the property of cumulative reference mentioned in §2.1; this imposes restrictions on what new mass nouns we may countenance. So let us define the mass term *woodware* as follows. x is some *woodware* just in case x is the mereological sum of the wood making up some furniture. The sentence *The woodware Ps* then means the same as *The mereological sum of the wood making up some furniture Ps*. Can we find a predicate *P* such that *The wood Ps* and *The woodware Ps* would intuitively have opposite truth-values? We have not been able to construct such a case, although we cannot be sure that this is impossible.

There is, however, one respect in which a semantics of mass nouns based upon mereological sums would be too weak. We saw in §2.2 that sentences containing mass nouns are liable to collective, distributive and intermediary construals. This was for instance the case for the sentence *This silverware costs a hundred euros*. The sentence may be true if the silverware costs, all together, a hundred euros: this is the *collective construal* of the sentence. It may be true if each piece of silverware, by itself, costs a hundred euros: this is the *distributive construal*. It may also be true if the silverware demonstrated consists in two sets of silverware, each set of silverware costing, by itself, a hundred euros: this is an *intermediary construal*.

To capture intermediary construals, a notion of *cover* of the denotation of a mass noun phrase akin to the notion used by Gillon (1992) is needed, and to express this notion, the apparatus of sets, or something as expressive, is required. This notion may be characterized as follows. A set X is an M-cover^{s} of a set Z just in case:

i) Each element of X is some M (e.g., each element is some wine);

ii) The mereological sum of the elements of X is identical with the mereological sum of the elements of Z.

This definition uses both the notion of set and the notion of generalized mereological sum (the latter is characterized at the end of §5.3). What is important here is that a cover is a set of things (which satisfies certain additional conditions). Using mereology and predicate logic alone, we could not refer to such a set. The notion of set, or something as expressive, like non-singular quantification in non-singular logic, is needed. Condition ii) of the present definition uses mereological sums, but, as we will see in §5.3, it can be replaced by a condition that does not use them, in a non-singularist account.

4.3. Discussion

What precedes shows that a semantics of mass nouns based on mereological sums and predicate logic would be too weak, since it would be incapable of capturing the intermediary construals that sentences containing mass nouns may receive. Before that, we had seen that a semantics formulated in predicate logic and based upon sets would lead to a Russellian contradiction.

We reached the latter conclusion by considering the hypothetical mass noun *setware*. Although this lessens the intuitive force of the argument, we think that the argument goes through just as well. After all, the count noun *set* itself must have appeared at some time in the vocabulary of English. And its technical, axiomatic usage goes back only to the nineteenth century. The mass noun *setware* is a possible and legitimate addition to English, though one that is not yet realized.

Let us add the following, independent consideration. Suppose that the semantics of mass nouns was based only upon sets. The overall semantics of English would become awkward, if not contradictory, when representing sentences that are not understood distributively. Imagine indeed that there are two pieces of furniture, a and b, and that the sentence *This furniture weighs two hundred kilos* is understood collectively, saying that the whole furniture together weighs this much. The semantics must attribute the property to weigh two hundred kilos to the set {a,b}, rather than to its members: part of the representation of the sentence will be 'Pc', where the singular constant 'c' is interpreted as the set {a,b}, and the predicate 'P' corresponds to *weigh two hundred kilos*.

Now, this would make it very difficult to integrate the semantics of mass nouns with that of plurals. Plurals, we have seen, should be represented using non-singular logic. Consider the sentence: *These pieces of furniture weigh two hundred kilos* understood collectively, its subject referring to a and b. Part of its representation will say that the predicate 'P' applies jointly to some things denoted by a non-singular constant, 'cs': 'Pcs'. We would therefore have a predicate 'P' that applies both to singular terms interpreted as sets in the case of mass nouns, and to non-singular terms interpreted as ordinary things in the case of plurals. This would be extremely odd, if not leading to contradiction.

From all this, we conclude two things. First, a semantics of mass nouns should not be based upon sets and predicate logic. Reference to a set of things should be replaced by reference to these very things, using non-singular logic. Second, a semantics of mass nouns should not be based solely upon mereology, which is not able to characterize intermediary construals. Something as expressive as sets is needed. Again, reference to several things, using non-singular logic, does the job.

This, however, does not necessarily mean that mereological sums could not play any role. A priori, one could still use non-singular logic *and* mereological sums. The thesis would be that mass nouns function as *singular terms*: a definite mass noun phrase would always refer to one thing, a certain mereological sum. Imagine that there are thousands of bottles of wine in the cellar. Then, the mass noun phrase *the wine that is in the cellar* would refer to an object t, the mereological sum of any wine that is in the cellar. Something similar would hold for any sentence of the form *The M that Qs Ps*, where *M* is a mass noun and *Qs* and *Ps* are verbal expressions.

This might be seen to fit with the fact that, under a normal usage, most mass nouns are used only in the singular, and not in the plural. However, we think that this *syntactical* fact has no semantic import. In many languages, including English, common nouns are divided into two morphosyntactic subclasses, mass nouns and count nouns (Gillon 1992). A defining characteristic of mass nouns, like *milk*, is that they are *invariable* in grammatical number, while count nouns, like *cat*, can be used in the singular and in the plural. Depending on the language, this basic morphosyntactic difference between the two types of noun is often supplemented by differences as to the determiners they can combine with. Thus, in English, mass nouns can be used with determiners like *much* and *a lot of*, but neither with *one* nor *many*. On the contrary, count nouns can be employed with numerals like *one* and determiners like *many*, but not with *much*.[9] While there are some mass nouns whose grammatical number is plural (*brains*), most mass nouns have singular grammatical number. The singular is indeed morphologically unmarked on nouns, while the plural is morphologically marked. Therefore, it is coherent to suppose that *grammatical number has no semantic import with mass nouns*. A difference between English and French confirms this. In English, mass nouns tolerate only determiners that can also be used with plurals: *some, all, any, the*. Not so in French, where the determiner must be singular: *de l'or / *des or* (some gold), *tout or / *tous or* (all / any gold), *l'or / *les or* (the gold). Like their invariability with respect to number, these data suggest that grammatical number has no semantic consequence with mass nouns.[10]

Moreover, the claim that unrestricted mereological sums exist is *ontologically extravagant*: given any number of things, why should there exist in addition another object, their hypothetical mereological sum? This charge has been levied against various systems of mereology, which guarantee the existence of scattered mereological sums. Mereologists have tried to answer this charge in various ways. Lewis (1991) is famous for claiming the following. Whenever there are some things, the ds, they have a mereological sum, t, which the ds can be said to *compose*. (This may be called the thesis of *unrestricted composition*.) However, says Lewis, this does not increase our ontological commitments, since the ds are identical with their sum. The latter claim is known as the thesis of *composition as identity*. Lewis oscillates between two versions of the thesis, a strong one, and a weak one. The strong version says that the ds are literally identical with their mereological sum. The weak version says that the ds are *so to speak* identical with their mereological sum. In other words, composition resembles identity in several respects. Lewis finally acknowledges that the strong thesis–the claim that the ds

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[9] It is of course well known that mass nouns can, in certain contexts, be used as count nouns (*You should take a hot milk with some honey*), and vice versa (*You will find a lot of rabbit around here*). One then talks of *conversion*. Conversion is a common grammatical possibility, whereby a member of a grammatical category is used in the morphosyntactic environment characteristic of another grammatical category. For instance, proper names can be used as common nouns: *The professor has two Picassos in his class* (cf. Gillon 1992). The same is observed in French (cf. Nicolas 2002).

[10] It follows from this that the singular number of the terms *it*, *this* and *something* also lacks semantic import when these terms apply to what a mass noun applies to, as in the following sentences, where *M* is a mass noun:
This is M.
John bought some M yesterday. It is in the living room.
There is something in the living room. It is some M.

are literally identical with their sum t-does not make sense, for two reasons. First, he knows of "no way to generalize the definition of ordinary one-one identity in terms of plural quantification". Second, "we do not really have a generalized principle of indiscernibility of identicals" (Lewis 1991: 87).[11]

So, the strong version of the thesis of composition as identity should be rejected. What about the weak one? Lewis lists a number of analogies between composition and identity. However, why should *analogies* tell us anything concerning the ontological commitments of a theory that accepts a principle asserting the existence of unrestricted mereological sums? Formally, the theory asserts the existence of something (the mereological sum) that is *distinct* from any of the things it is the sum of (when it is the sum of two things or more). With van Inwagen (1994) and Yi (1999), we find this formal commitment *substantive.* Considerations of ontological economy therefore militate against the postulation of unrestricted or generalized mereological sums.

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[11] We understand Lewis' first worry as follows. We grasp what it means for one thing to be identical with one thing. We can also understand a sentence like *The Ms are identical with the Ns*, where the predicate *to be identical* is flanked by two plural expressions. Just as in the case of mass nouns, the interpretation of a sentence containing a definite plural noun phrase depends on the choice of a cover of the denotation of the noun phrase (more on this in §7, where we give a non-singular semantics for plurals). When the sentence contains two definite plural noun phrases, two covers must be chosen and this gives rise to more possibilities of interpretation. For the above sentence, one construal makes sense, where each plural noun phrase is understood distributively. Under this construal, the sentence is true if each M is identical with an N and each N is identical with an M. A priori, one could also imagine that each noun phrase is understood collectively. The sentence would be true if the Ms, considered together, were collectively identical with the Ns, considered together. However, what would this claim of collective identity between the Ms and the Ns mean? We have found no plausible answer to this question. Therefore, we take it that a sentence like *The Ms are identical with the Ns* can only receive a distributive interpretation. Something similar holds for a sentence like *Some things, the ds, are identical to one thing, t.* A distributive construal makes sense, which asserts that the ds are each identical to t. But we do not know what a collective construal would mean, which would claim that the ds, taken together, are collectively identical with one thing, t. What about an army and its soldiers? Can we not say that the army is its soldiers, and that the soldiers are the army? Yes, we can, but it does not seem that, doing so, we are really claiming literal identity between the army and its soldiers. After all, the army continues to exist when one of its soldiers die. Therefore, the army cannot be identical with any number of soldiers.

This is reflected in non-singular logic, where identity between non-singular terms is naturally defined as follows:

$$xs = zs \equiv_{def} \forall y\ (y \angle xs \leftrightarrow y \angle zs)$$

Suppose that the zs comprise only one thing, t. Then:

$$xs = zs \leftrightarrow \forall y\ (y \angle xs \leftrightarrow y = t)$$

Therefore, the xs must also comprise one and only one thing, t.

Lewis's second worry (1991: 87) concerns the indiscernibility of identicals: "What's true of the many is not exactly what's true of the one. After all they are many while it is one." Consider two things, the ds, which have a mereological sum. The ds are two, says Lewis, while their sum is one. Therefore, the ds have a property (the property of being two) that their sum does not have. So, if the ds were literally identical with their sum, it could not be true that things that are identical are indiscernible, i.e. that they have exactly the same properties.

Wallace (ms.) suggests the following answer. Since Frege, we would know that counting depends on the identification of a concept: one would not count things *simpliciter*, but only things that fall under a certain concept. When saying that the ds are two and their sum is one, one would be employing different concepts for each count. It is not clear to us how successful this answer is. After all, what concepts would yield these two counts, whereby the ds would be counted two and their sum would be counted one? Be that as it may, we find that the first worry voiced above is decisive: we simply do not know what it could mean to say that several things taken together are literally identical to one thing.

Moreover, we see no reason why our ontology should contain objects that are so counter-intuitive. Varzi (2000, 2003) claims that psychological considerations of this sort should have no bearing on ontological issues. We disagree. Of course, it is to be expected that a scientific understanding of the world and a common sense one should differ. Still, the intuitions of common sense should be explained and, given the choice between two theories that model equally well a certain range of data, priority should be given to the one that fits common sense intuition better.

We therefore prefer an account of mass nouns that treats them as *non-singular terms*: terms that can refer to several things at once. We develop such an account in what follows, using non-singular logic.

5. A non-singular semantics for mass nouns

We want to develop a semantics that accounts for the data presented in §2 and treats mass nouns as non-singular terms, which may refer to several things at once. In order to do so, the following questions need to be examined:

- First, what are the constraints on what counts as M?
- Second, what are the truth-conditions of sentences where mass nouns appear?

5.1. What are the constraints on what counts as M?

Here are several possible answers to this question:

a) A thing that counts as M must be self-connected and "maximal" (in a sense characterized below).

b) A thing that counts as M must be self-connected.

c) What counts as M depends on various factors: self-connectedness, function, causality, context, etc.

d) It is "mereological atoms" or "simples" arranged in a certain way that count as M.

Someone like van Inwagen (1990) would presumably adopt the last position. He would argue that, when you have some wood in front of you, say two pieces of elm tree, what is really present, from a metaphysical standpoint, is many atoms arranged wood-wise. While this position is one of the options available to the metaphysician, we find that it does not do justice to our intuitions as naïve cognitive agents. When we talk of *the wood* with two pieces of elm tree in front of us, we have the impression of referring to two things, and not to many atoms arranged wood-wise. We want our semantics of mass nouns to be as close as possible to the untutored intuitions of naïve cognitive agents. Therefore, we will not consider this option any further.

Instead, we will take a mass noun to satisfy the property of *distributed reference*, which we define as follows:

(DR) $Mxs \leftrightarrow \forall y\, (y \angle xs \leftrightarrow My)$ *Distributed reference*

Some things are M just in case each of them is M.

In this bi-conditional, the implication from the right to the left guarantees that a mass noun has the property of cumulative reference mentioned in §2.1 (if each of some things is M, then these things referred to collectively are also M).

Now, the non-singularist is moved by statements like the following:

The furniture is in the truck, said when there are three pieces of furniture in the truck.

The gold is in the safe, said when there are three solid bits of gold in the safe.

The wine is in the kitchen, said when there are three bottles of wine in the kitchen.

In each of these cases, he claims, the speaker is referring to three objects at once (rather than to a scattered object, the would be mereological sum of these objects).

Therefore, one might suppose that a thing that counts as *M* must be "maximally self-connected". A solid bit of gold would count as gold, but an undetached part of it would not. When trying to characterize things in this manner, we are using two notions. First, we are using a notion of topological connection: the notion of something being *self-connected*, of being of a single piece. This notion must be strong enough, so that two objects that are merely in external contact are not deemed to be connected in the sense that is relevant here (cf. Varzi (1996), and Casati and Varzi (1999)). Then, one requirement is that any thing that is M must be self-connected:

(SC) $Mx \rightarrow$ self-connected(x) *Self-connectedness*

Second, we are using the notion of something being *maximal*. We take the relevant notion to be the following one:

M-maximal(x) $\equiv_{def} Mx \wedge \forall y\,(My \rightarrow \neg\,(x < y))$

The idea is that a thing that counts as *M* would have to be M-maximal:

(MM) $Mx \rightarrow$ M-maximal(x) *M-maximality*

Remark: We take the relation of parthood to obey the axioms of *extensional mereology* (cf. Simons (1987) and Varzi (1996)). From the notion of part ($\leq$) taken as primitive, the notions of proper part ($<$) and overlap (O) are defined in this way:

(Def) $x < y \equiv_{def} x \leq y \wedge \neg\,(x = y)$ *Proper part*

(Def) $Oxy \equiv_{def} \exists z\,(z \leq x \wedge z \leq y)$ *Overlap*

The axioms of extensional mereology are then:

(P1) $x \leq x$ *Reflexivity*

(P2) $x \leq y \wedge y \leq x \rightarrow x = y$ *Antisymmetry*

(P3) $x \leq y \wedge y \leq z \rightarrow x \leq z$ *Transitivity*

(P4) $\neg\,(y \leq x) \rightarrow \exists z\,(z \leq y \wedge \neg Ozx)$ *Strong supplementation*

However, the axiom of M-maximality stated above runs counter to our use of the word *part* together with mass nouns that designate substances. Consider the following sentence:

Part of the gold that makes up that ring was dug in France.

The following situation makes this sentence true. The gold that makes up the ring is something x that is self-connected and gold-maximal. x has something y as strict part, which is also gold, self-connected, and was dug in France. Since y is strictly part of x, y is not gold-maximal, and therefore y is not "maximally self-connected". Formally, what is relevant for us may be rendered in this way:

$\exists x\,\exists y$ (gold(x) $\wedge$ self-connected(x) $\wedge$ gold-max(x) $\wedge\ y < x\ \wedge$ gold(y) $\wedge$ self-connected(y) $\wedge\ \neg$ gold-max(y))

We observe something similar in the following case:

What is in this strangely shaped container[x] *is wine. In particular, what is in the lower half of this container* [y] *is also wine.*

We see that when a mass noun *M* names a substance, it is often the case that, if x is something self-connected and M-maximal, there is something y that is strictly part of x, self-connected, and counts as M. Since y is strictly part of x, y is not M-maximal.

One may perhaps answer that the undetached parts that we have considered do not really exist. These undetached parts would only be "conceptual parts", parts that are imagined, parts-in-waiting. When talking of parts, as above, we would be talking of possible, but not actual, divisions (cf. van Inwagen 1990). Correlatively, one may want to insist that what counts as M is highly context dependent. In typical contexts, only something that is self-connected and M-maximal would count as M. But in more specific contexts where the word *part* is used, what counts as M would change, now including some self-connected parts of something that is both self-connected and M-maximal.

This kind of position, which would be a mixture of options a) and c), may perhaps be sustained. However, we find option b) more attractive. Our intuition as a naïve speaker is that the denotation of a mass noun may contain both something that is self-connected and M-maximal, and some self-connected parts thereof (cf. the sentence *What is in the lower half of this container is also wine*). Option b) takes this at face value, requiring only that something that is M be self-connected.

5.2. The truth-conditions of various types of sentences

We ignore tense and intensionality. As remarked by Bunt (1985), these seem to pose no specific problems for mass nouns.

Our account is inspired by the semantics proposed by Gillon (1992) for plurals and mass nouns.[12] Gillon's own account is *singularist*: a definite noun phrase is taken to be a singular term, which refers to a single object, a certain mereological sum. His key idea is that the interpretation of a sentence where a mass noun appears is relative to the choice of a *cover* of the denotation of the mass noun or the mass noun phrase.[13] We modify Gillon's notion of cover in order to arrive at a semantics that treats mass nouns as non-singular terms, which may refer to several things at once.

As said above, we take a mass noun to satisfy the following principle:

(DR) $\mathrm{Mxs} \leftrightarrow \forall \mathrm{y}\, (\mathrm{y} \angle \mathrm{xs} \leftrightarrow \mathrm{My})$ *Distributed reference*

This allows us to define a simple notion of denotation (or extension). In any circumstance in which a mass noun is used, we associate to it some things, the ds, which are its denotation. These things comprise any thing that can be truly said to be M in the circumstance:

$\forall \mathrm{y}\, (\mathrm{y} \angle \mathrm{ds} \leftrightarrow \mathrm{My})$

We now need to specify what a definite mass noun phrase refers to. From the discussion in §5.1, we keep the following intuition. Although any thing that counts as M need not be M-maximal, a definite mass noun phrase should still refer to things that are maximal in an appropriate sense. For instance, when there are three solid bits of gold in a safe, someone who says *The gold in the safe comes from Grandpa* refers to these three things. Likewise, the phrase *the butter that is in the lower half of these two containers* should refer to two things, each of which is the butter in the lower half of one of the containers. The phrase should not, in addition, refer to strict parts of these two things, like the butter that is in the lower quart of one of the containers. Moreover, our account must guarantee that whenever there is some M that Qs, then one can successfully refer to *the M that Qs*.

In order to do so, we first define:

$$\mathrm{ys} = \max[\mathrm{zs} / \mathrm{Qzs}] \equiv_{\mathrm{def}} \forall \mathrm{zs}\, \forall \mathrm{u}\, ((\mathrm{Qzs} \wedge \mathrm{u} \angle \mathrm{zs}) \rightarrow \exists \mathrm{v}\, (\mathrm{v} \angle \mathrm{ys} \wedge \mathrm{u} \leq \mathrm{v}))$$
$$\wedge\ \forall \mathrm{v}\, (\mathrm{v} \angle \mathrm{ys} \rightarrow \exists \mathrm{zs}\, (\mathrm{v} \angle \mathrm{zs} \wedge \mathrm{Qzs})) \wedge \neg(\exists \mathrm{u}\, \exists \mathrm{v}\, (\mathrm{u} \angle \mathrm{ys} \wedge \mathrm{v} \angle \mathrm{ys} \wedge \mathrm{u} \neq \mathrm{v} \wedge \mathrm{Ouv}))$$

Among all the zs such that Qzs, the ys are the maximal elements for the relation of parthood.

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[12] Schwarzschild (1996) proposes a similar account for plurals, and Chierchia (1998) is also greatly inspired by the work of Gillon.

[13] As indicated in note 6, Gillon considers only *minimal covers* (no element of the cover should be part of another element), which he calls *aggregations*. We use the more general notion of cover.

Then, we adopt the following axiom:

(MR) $\exists zs\,(Mzs \wedge Qzs) \rightarrow \exists ys\,(ys = max[zs / Mzs \wedge Qzs])$ *Maximal reference*

This axiom guarantees that, whenever there are some zs that satisfy Mzs $\wedge$ Qzs, there are some ys that satisfy M and that are maximal for the relation of parthood among all the zs such that Mzs $\wedge$ Qzs. These ys are the referents of the definite expression *the M that Qs*. Given distributed reference, we know that M will apply to the ys taken together.[14] Note, however, that Q need not always apply to the ys. Indeed, imagine a circumstance in which the silverware present in the universe of discourse consists in four sets of silverware. Three of them cost a hundred euros, while the fourth one costs fifty euros. Since there is some silverware that costs a hundred euros, the principle of maximal reference ensures that there exist some things ys to which the definite noun phrase *the silverware that costs a hundred euros* refers. In this circumstance, it would be rather peculiar if these things also cost a hundred euros all together!

Finally, we characterize a non-singularist notion of cover as follows, using a new predicate, 'among°':

$x \angle^{\circ} zss \equiv_{def} \exists ys\,(x \angle ys \wedge ys \angle zss)$ *x is among° the zss*

A cover is then a "plurality of pluralities", as characterized below.

Non-singularist notion of cover for a mass noun M

The xss are an M-cover^{n} of the zs just in case:

i) Any thing among° the xss is M

$\forall y\,(y \angle^{\circ} xss \rightarrow My)$

ii) For anything v, v overlaps some thing among° the xss just in case v overlaps one of the zs

$\forall v\,(\exists y\,(y \angle^{\circ} xss \wedge Ovy) \leftrightarrow \exists w\,(w \angle zs \wedge Ovw))$

This notion of cover allows us to specify the truth-conditions of various kinds of sentences.

This M Ps.

A condition for the sentence to be truth-valuable is that *this M* have a (non-empty) denotation. The interpretation of the sentence then depends on the choice of a cover of this denotation. Let the as be the denotation of *this M* (the M demonstrated by the speaker). Let the css be the chosen M-cover^{n} of the as. The sentence is true just in case:

$\forall ys\,(ys \angle css \rightarrow Pys)$

As an example, consider the sentence: *This silverware costs a hundred euros*. Suppose that what is demonstrated consists in several pieces of furniture, the as. First, imagine that the cover has for "components" each piece of silverware in the shop. Then the sentence is true, relative to this choice of cover, if each piece of silverware costs a hundred euros. This is the distributive construal of the sentence. Second, imagine that the cover has a single "component", the as themselves. Then the sentence is true, relative to this choice of cover, if all the silverware, taken together, costs a hundred

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[14] By way of comparison, someone holding that mass nouns function as singular terms would need to make the following assumptions:

(Def) $z = \sigma[x / Qx] \equiv_{def} \forall v\,(Ovz \leftrightarrow \exists x\,(Qx \wedge Ovx))$ *Generalized mereological sum of any thing that Qs*

(GCR) $\exists x\,(Mx \wedge Qx) \rightarrow \exists! z\,(z = \sigma[x / Mx \wedge Qx] \wedge Mz)$ *Generalized cumulative reference*

In such an account, a definite mass noun phrase would refer to a certain generalized mereological sum.

euros. This is the collective construal of the sentence. Finally, imagine an intermediary case where, say, the silverware is divided into two sets, each set containing four pieces of silverware. Then the sentence is true, relative to this choice of cover, if each set costs a hundred euros. This is an intermediary construal.

The M that Qs Ps.
A condition for the sentence to be truth-valuable is that *the M that Qs* have a (non-empty) denotation. The interpretation of the sentence then depends on the choice of a cover of this denotation. Let the as be the denotation of *the M that Qs*. They satisfy:
as = max[zs / Mzs $\wedge$ Qzs]. Let the css be the chosen M-cover^n of the as. The sentence is true just in case:
$\forall$ys (ys $\angle$ css $\rightarrow$ Pys)

This yields a satisfactory result notably with the sentence: *The gold on the table weighs seven ounces*. Bunt (1985: 40) thought that this kind of example could not be dealt with by a semantics that associates to a mass noun a set of instances of M. Now, in one respect, the non-singularist semantics that we are advocating is not very far from a set-theoretic account. It associates to a mass noun, not a set of things, but some things, each of which is M. Yet, it has no problem with the above sentence. The denotation of *the gold on the table* is some bits of gold, the as, each of which is gold on the table and is maximal for the relation of parthood. The interpretation of the sentence is then relative to the choice of a cover of this denotation. The sentence says, of each "component" of this cover, that the things that make up this "component" weigh seven ounces together. Among the various possible construals, the collective one is the most salient (the other construals require much more context to become available). It is obtained when the cover contains only one "component", the as, which are the denotation of *the gold on the table*. The sentence then says that these as (e.g. the solid bits of gold that are on the table) weigh seven ounces together.

A lot of M Ps.
The sentence is true if and only if there exist xs satisfying the following conditions:
- Mxs $\wedge$ a-lot-of(xs,c), where c is the context of utterance (the interpretation of *a lot of* is contextually dependent, hence the presence of this parameter of context);
- there exists an M-cover^n of the xs, the zss, and:
- $\forall$ys (ys $\angle$ zss $\rightarrow$ Pys)

Some M Ps.
Let the ds be the denotation of the mass noun *M*. The interpretation of the sentence is relative to the choice of a cover of this denotation. Let the zss be the chosen M-cover^n of the ds. The sentence is true just in case:
$\exists$ys (ys $\angle$ zss $\rightarrow$ Pys)

All M Ps.
Let the ds be the denotation of the mass noun *M*. The interpretation of the sentence is relative to the choice of a cover of this denotation. This cover must have at least two "components" (say ys_1 and ys_2). Let the zss be the chosen M-cover^n of the ds. The sentence is true just in case:
$\forall$ys (ys $\angle$ zss $\rightarrow$ Pys)

Consider an example from Roeper: *All phosphorus is either red or black*. Roeper (1983) thought that this kind of example was problematic for set-theoretic approaches of mass nouns. In the present account, the sentence is made true by a cover that is a partition of all the phosphorus (i.e., a cover no elements of which overlap), such that the phosphorus in each "component" of this partition (a certain "plurality of things" in each case) is either (wholly) red or (wholly) black.

5.3. Substances and identity over time

We have made certain hypotheses concerning what the denotation of a mass noun contains. Do some of these hypotheses run counter to our intuitive understanding of what it is to be a substance, particularly concerning identity over time?

The notion of substance (or matter, or stuff) is linked to that of an ordinary object (on various debates concerning their relationship, see for instance Cartwright (1965), Zimmerman (1995) and Steen (2005)). Gold, silver and clay are paradigmatic examples of substances. Oil, water and wine are liquids, which are also typically seen as substances. Ordinary objects include artifacts like chairs, statues and rings, organisms like persons, cats and dolphins, and material bodies like rocks, mountains and moons. In some cases, an ordinary object may be said to be made up, or constituted by, some or several substances. Thus, a chair may be made of wood, a statue of clay, and a ring of gold. A ring might also be made up of both gold and silver. An organism, however, will usually not be said to be made up of one or several substances. Rather, it will be said to be made up of smaller units like cells.

As stressed notably by Cartwright (1965), we can talk of the identity over time of some substance, in a variety of ways:

The gold that John dug now makes up those rings.

The coffee that was in that cup is the coffee that is now on the floor.

The clay that made up those three bowls is identical with the clay that now makes up these two statues.

These sentences express the identity over time of some M. Let us see how this is captured in the present account. The mass noun phrase *the clay that made up those three bowls* refers to three things that existed in the past (each of them was the clay making up a bowl). Call them the a_1s. *The clay that now makes up these two statues* refers to two things that presently exist (each of them is the clay making up a statue). Call them the a_2s. The sentence contains two definite mass noun phrases. Its interpretation is therefore relative to the choice of a cover on the denotation of each phrase. For each i, let the $c_i$ss be the chosen M-cover^n of the a_is. Then the sentence is true just in case each thing that was among the $c_1$ss is identical to some thing that is among the $c_2$ss, and each thing that is among the $c_2$ss is identical to some thing that was among the $c_2$ss. In other words, the sentence is true just in case the a_1s were composed of the $c_1$ss, each of which has continued to exist up to the present moment, and the $c_1$ss now compose the a_2s.

6. Conclusion

A dilemma put forward by Schein (1993, 2006) and Rayo (2002, 2006) suggests that, in order to characterize the semantics of plurals, we should not use predicate logic, but non-singular logic, a formal language whose terms may refer to several things at once. In §4, we showed that a similar dilemma applies to mass nouns. If we use predicate logic and sets, we arrive at a Russellian paradox when characterizing the semantics of mass nouns, just as we do when characterizing the semantics of plurals with the same means. Likewise, a semantics of mass nouns based upon predicate logic and mereological sums is too weak, since it cannot characterize the intermediary construals that sentences containing mass nouns may receive.

We then developed a non-singularist account, where mass nouns are treated as non-singular terms, which may refer to several things at once. In one respect, this treatment resembles set-theoretic approaches of mass nouns, which would assign a set of things as the denotation of a demonstrative or definite mass noun phrase. The non-singular semantics we have proposed assigns some things (rather than a set of things) as the denotation of such phrases. We therefore showed that some of the main criticisms that have been levied against set-theoretic approaches could be satisfactorily answered by the non-singularist. The following three kinds of sentences have been seen as presenting difficulties for set theories:

The gold on the table weighs seven ounces.

All phosphorus is either red or black.

The clay that made up those three bowls is identical with the clay that now makes up these two statues.

We provided a semantics for each in §5. In each case, the key to resolving the difficulty lied in employing an appropriate notion of *cover*. A notion of this kind was used by Gillon (1992) to develop a semantics of mass nouns that treats them as singular terms. We generalized the notion so that it may apply to the case of things that do not have a mereological sum. This allowed us to devise a semantics where mass nouns may refer to several things at once, while dealing satisfactorily with the three difficulties just mentioned. This semantics is faithful to the intuition that, if there are eight pieces of silverware on a table, the speaker refers to eight things at once when he says: *The silverware that is on the table comes from Italy.*

7. Appendix: a non-singular semantics for plurals

In this appendix, we propose a non-singular semantics for plurals. As in the case of mass nouns, the account proposed for plurals is partly inspired by Gillon (1992). As we noted in §2, there are parallelisms in the semantic behavior of mass nouns and plurals. Hence, accounts of the semantics of each will resemble each other. There are, however, some differences with respect to the structure of the denotation of mass nouns and count nouns, and concerning the notion of cover used in each case.

To any count noun *N*, we associate a predicate N that applies only to terms that are singular:

Nx $\equiv_{def}$ The object denoted by 'x' is such that *This is an N* is true with the demonstrative *this* referring to that object.

In any circumstance in which a count noun is used, we associate to it some things, the ds, which form its denotation (or extension). These things comprise any thing that is an N (and so, they comprise any thing x that satisfies Nx).

From the predicate N, it is useful to define another predicate, N':

$N'xs \equiv_{def} \forall y\,(y \angle xs \rightarrow Ny)$

We also define a notion of *conjunction*, which will allow us to characterize the denotation of a plural definite expression:

(Def) $zs = \&[xs \,/\, Qxs] \equiv_{def} \forall xs\,(Qxs \rightarrow \forall v\,(v \angle xs \rightarrow v \angle zs)) \wedge \forall v\,(v \angle zs \rightarrow \exists ys\,(v \angle ys \wedge Qys))$ *Conjunction of any things that Q*

The axiom schema of non-singular comprehension, repeated below, entails the property of *conjunctive comprehension*:

(NSC) $\exists x\, (Qx) \rightarrow \exists zs\, (\forall y\, (y \angle zs \leftrightarrow Qy))$ *Non-singular comprehension*

(CC) $\exists vs\, (Pvs) \rightarrow \exists zs\, (zs = \&[vs / Pvs])$ *Conjunctive comprehension*

If some things P, then there are some things that are the conjunction of any things that P.[15]

The denotation of the expression *the Ns that Q* is then some things, the as, which satisfy: as = &[xs / N'xs ∧ Qxs].

We use the following notion of cover for plurals. The zss are a *cover*^{P} of the xs just in case:

$\forall y\, (y \angle^{\circ} zss \leftrightarrow y \angle xs)$

For anything y, y is among° the zss if and only if y is one of the xs.

Finally, here are the truth-conditions of various types of sentences.

These Ns P.

A condition for the sentence to be truth-valuable is that *these Ns* have a (non-empty) denotation, the as. The interpretation of the sentence depends on the choice of a cover^{P} of the as. Let the css be the chosen cover. The sentence is true just in case:

$\forall ys\, (ys \angle css \rightarrow Pys)$

The Ns that Q Ps.

A condition for the sentence to be truth-valuable is that *the Ns that Q* have a (non-empty) denotation, the as. They satisfy: as = &[xs / N'xs ∧ Qxs]. The interpretation of the sentence depends on the choice of a cover^{P} of the as. Let the css be the chosen cover. The sentence is true just in case:

$\forall ys\, (ys \angle css \rightarrow Pys)$

A lot of Ns P.

The sentence is true if and only if there exist xs satisfying the following conditions:

- Nxs ∧ a-lot-of(xs,c), where c is the context of utterance
- there exists a cover^{P} of the xs, the zss
- $\forall ys\, (ys \angle zss \rightarrow Pys)$

Some Ns P.

Let the ds be the denotation of the count noun *N* in the circumstance. The interpretation of the sentence is relative to the choice of a cover^{P} of this denotation. Let the zss be the chosen cover of the ds. The sentence is true just in case:

$\exists ys\, (ys \angle zss \rightarrow Pys)$

[15] *Proof of the entailment*: Let 'P' be any predicate that can apply to non-singular variables. Let 'Rx' abbreviate this:

$Rx \equiv_{def} \exists vs\, (Pvs \wedge x \angle vs)$

Applying (NSC) to the formula 'R', we get:

$\exists x\, (Rx) \rightarrow \exists zs\, (\forall y\, (y \angle zs \leftrightarrow Ry))$, i.e.:

$\exists x\, (\exists vs\, (Pvs \wedge x \angle vs)) \rightarrow \exists zs\, (\forall y\, (y \angle zs \leftrightarrow \exists vs\, (Pvs \wedge y \angle vs)))$

This is equivalent to conjunctive comprehension (CC).

All Ns P.

Let the ds be the denotation of the count noun *N* in the circumstance. The interpretation of the sentence is relative to the choice of a cover of this denotation. This cover^{p} must have at least two "components" (say ys_1 and ys_2). Let the zss be the chosen cover of the ds. The sentence is true just in case:

$\forall ys\ (ys \angle zss \rightarrow Pys)$

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Institut Jean Nicod
CNRS
1 bis, avenue de Lowendal
75007 Paris
France
E-mail: david.nicolas@m4x.org