Some More Hints for Translating into Predicate Logic: Pt. II

(Phil. 210, Pelletier)

Some stylistic variants of $(\exists x)Fx$: there is an F, there are F's, F's exist, at least one thing is F, some F's, an F, …

Some stylistic variants of $(\forall x)Fx$: everything is F, every x is F, each x is F, all F's…, for all F's…, no matter which F, ...

nothing is <u>not</u> the opposite of *everything*, it is instead the opposite of *something*. For example, 'Nothing is older than the pyramids' is exactly the opposite of 'Something is older than the pyramids', and is <u>not</u> the opposite of 'Everything is older than the pyramids.' *There are no F's* is the opposite of *There are F's*. The latter one gets translated as $(\exists x)Fx$ while the former gets translated as its opposite: $\sim(\exists x)Fx$. Similarly, *F's exist* gets translated as $(\exists x)Fx$, while *F's do not exist* gets translated as $\sim(\exists x)Fx$.

Often there is no explicit quantifier in a sentence which mentions a plural noun (called "bare plurals"). In these cases the sentence sometimes is to be translated with a universal quantifier (e.g., *Lobsters are mollusks*) and sometimes with an existential quantifier (e.g., *Lobsters are biting my toes*). When should you do which?? Answer: it depends upon the verb phrase. If the verb phrase is "stative" (reporting on a general characteristic of the subjects), then use the universal; if the verb phrase reports an event or a current happening, then use the existential. *Being a mollusk* is a general characteristic of lobsters, and so it is universal; *biting my toes* reports an event that lobsters are engaged in, and so gets translated with an existential. Verb phrases being used in the "progressive" (usually: involves an *-ing* on the verb, as in *are flying*) are normally reports of events....and so get the existential quantifier as a translation in a bare plural sentence.

Sometimes negations can lead to ambiguities with the stative/event verb stuff. Consider *Fish walk*. It is a bare plural with a stative verb, and so would be translated as $(\forall x)(Fx \supset Wx)$. Now consider *Fish don't walk*. You could easily understand this as the negation of the first sentence, and then you would translate it as: $\sim(\forall x)(Fx \supset Wx)$; but if you understand the verb phrase *don't walk* as itself a stative, then you would translate it: $(\forall x)(Fx \supset \sim Wx)$. These two translations are <u>not</u> equivalent: the former says that it is not true that all fish walk (but maybe some do), while the latter says of all fish that they do not walk.

Sentences with compound subjects present difficulties of interpretation. *Sam and Mary go to school* would normally be understood as a conjunction: $(Gs \& Gm)$. *Sam and Mary are married* normally means that they are married to one another: $(Msm \& Mms)$ – you need to say both conjuncts unless you have another premise that says "being married" is symmetric. But when you think about it, the English sentence doesn't <u>have</u> to say that they are married to one another....just that they are each married to someone (not necessarily to the same person, but not necessarily to different people either): $(\exists x)Msx \& (\exists x)Mmx$. Now consider plural conjoined subjects: *Men and women are persons*. 'being a person' is a stative predicate, so this is a universal statement. And since it does not state any relationship holding between men and women (unlike the being married case above), we could translate it as: $(\forall x)(Mx \supset Px)$ &

$(\forall x)(Wx \supset Px)$. This is equivalent to: $(\forall x)(\forall y)((Mx \& Wy) \supset (Px \& Py))$...note the use of different variables. It is also equivalent to: $(\forall x)((Mx \vee Wx) \supset Px)$, which uses only one variable but notice the use of '$\vee$' in the antecedent of the conditional. No one is both a man and a woman, so we can't have an *and*. (We want to talk about anyone who is *either* a man or a woman, and say that they are people). Now suppose that the verb phrase is an event designating verb, such as: *Men and women are skiing in Banff.* This calls for existential quantifiers: $(\exists x)(Mx \& Sx)$ & $(\exists x)(Wx \& Sx)$, which is equivalent to $(\exists x)(\exists y)(Mx \& Wy \& Sx \& Sy)$. But these are *not* equivalent to $(\exists x)((Mx \vee Wx) \& Sx)$. This last formula would be true if there were (say) just some men but no women skiing in Banff, but the original English would not be true in such a case.

You will recall from the sentence logic that *only* is a difficult concept. We used it with *if*, and we discovered that *only if* had the effect of making the *if* clause become a *then* clause. The word *only* can also be used as a quantifier: *Only F's are G.* It is a kind of universal quantifier, like *all*, and so it forms a type of A-statement. But it makes the F-part be the <u>consequent</u> and the G-part be the <u>antecedent</u>, unlike normal universal quantifiers. So, *Only F's are G* is a stylistic variant of *All G's are F's.* A sentence like *Only mammals suckle their young* means *Everything that suckles their young is a mammal* and is quite different from *All mammals suckle their young.* Thus:

All F's are G: $(\forall x)(Fx \supset Gx)$

Only F's are G: $(\forall x)(Gx \supset Fx)$

Just like in the sentence logic where we put *if* together with *only if* to form *if and only if*, so too here we can put *all* together with *only* to form *all and only*, and the relevant connective will be $\equiv$.

All and only F's are G: $(\forall x)(Fx \equiv Gx)$

You might note that *none but* means the same as *only.* So *None but F's are G* would also get translated as: $(\forall x)(Gx \supset Fx)$, just like *Only F's are G.*

Sometimes when *only* or *none but* gets embedded within a larger sentence, it is difficult see what is the immediate impact of these quantifiers. In such cases you might consider replacing it by *all*, translating this (which is usually clearer), and then go back and "turn the antecedent and consequent around" to capture the difference between *all* and *only.*

You might also note that sometimes the force of *only* or *none but* is restricted: *Only lions with thorns in their paws are dangerous* is really a statement about all lions, and then says that, of them, only the ones with thorns in their paws are dangerous. So it <u>should</u> be translated as

$(\forall x)(Lx \supset (Dx \supset Tx))$

and it should <u>not</u> be translated as $(\forall x)(Dx \supset (Lx \& Tx))$....which says that all the dangerous things in the world are lions with thorns in their paws.