Numerical statements are ones that assert that there are some (finite) number of a particular type of thing. For example, "There are 3 graduate students in the class". The standard view of numerical statements is that they are to be translated by a sentence that would be true if the numerical statement is true, and false otherwise. For example, the sentence just mentioned would be translated as $\exists x \exists y \exists z (Gx \& Gy \& Gz \& x \neq y \& x \neq z \& y \neq z \& \forall w (Gw \supset (w = x \lor w = y \lor w = z)))$, at least in one of its meanings.

It is also usually said that numerical statements are ambiguous between an "at least", an "at most", and an "exactly" reading. The above translation gave the "exactly" reading of the English sentence, since it is true when but only when there are exactly three Gs (grad students in the class).

Here is a general schema for how to translate numerical statements, in all their ambiguities.

To translate "There are at least n Gs"

\[
\begin{align*}
&n \text{ translation} \\
1 & \exists x Gx \\
2 & \exists x \exists y (Gx \& Gy \& x \neq y) \\
3 & \exists x \exists y \exists z (Gx \& Gy \& Gz \& x \neq y \& x \neq z \& y \neq z)
\end{align*}
\]

and so on. You start out with existential statements of n things of type G, and then say that each of them is different from all the others. So you are guaranteed that there must be at least n of them.

To translate "There are at most n Gs"

\[
\begin{align*}
&n \text{ translation} \\
1 & \forall x \forall y ((Gx \& Gy) \supset x = y) \\
2 & \forall z \forall y \forall z ((Gx \& Gy \& Gz) \supset (x = y \lor x = z \lor y = z)) \\
3 & \forall z \forall y \forall z \forall w ((Gx \& Gy \& Gz \& Gw) \supset (x = y \lor x = z \lor y = w \lor y = z \lor y = w \lor z = w))
\end{align*}
\]

and so forth. You are asserting that for any time you think you have one extra G over and above the n, it will turn out that there is at least one identity going on.

To translate "There are exactly n Gs". You could translate "There are at least n Gs", translate "There are at most n Gs", and put an & between them. Or, you could do this slight shortcut.

\[
\begin{align*}
&n \text{ translation} \\
1 & \exists x (Gx \& \forall y (Gy \supset x = y)) \\
2 & \exists x \exists y (Gx \& Gy \& x \neq y \& \forall z (Gz \supset x = z \lor y = z)) \\
3 & \exists x \exists y \exists z (Gx \& Gy \& Gz \& x \neq y \& x \neq z \& y \neq z \& \forall w (Gw \supset (x = w \lor y = w \lor z = w)))
\end{align*}
\]
and so on. You start out translating "There are at least $n$ Gs", but before closing the parentheses, you add on that whenever you try to get another G, it turns out to be identical to one for the first $n$.

There’s an even shorter version for "exactly". It is commonly used for "exactly one", but the analogous version for the higher numbers is not so common. Here it is for "There is exactly one G".

$$\exists x \forall y (Gx \equiv x=y)$$