

The Paradoxical Success of Fuzzy Logic

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Fuzzy logic methods have been used successfully in many real-world applications, but the foundations of fuzzy logic remain under attack. Taken together, these two facts constitute a paradox. A second paradox is that almost all of the successful fuzzy logic applications are embedded controllers, while most of the theoretical papers on fuzzy methods deal with knowledge representation and reasoning. I hope here to resolve these paradoxes by identifying which aspects of fuzzy logic render it useful in practice, and which aspects are inessential. My conclusions are based on a mathematical result, on a survey of literature on the use of fuzzy logic in heuristic control and in expert systems, and on practical experience developing expert systems.

An apparent paradox

As is natural in a research area as active as fuzzy logic, theoreticians have investigated many formal systems, and a variety of systems have been used in applications. Nevertheless, the basic intuitions have remained relatively constant. At its simplest, fuzzy logic is a generalization of standard propositional logic from two truth values, *false* and *true*, to degrees of truth between 0 and 1.

Formally, let A denote an assertion. In fuzzy logic, A is assigned a numerical value $t(A)$, called the *degree of truth* of A , such that $0 \leq t(A) \leq 1$. For a sentence composed from simple assertions and the logical connectives "and" (\wedge), "or" (\vee), and "not" (\neg) degree of truth is defined as follows:

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Definition 1: Let A and B be arbitrary assertions. Then

$$\begin{aligned}t(A \wedge B) &= \min \{t(A), t(B)\} \\t(A \vee B) &= \max \{t(A), t(B)\} \\t(\neg A) &= 1 - t(A) \\t(A) &= t(B) \text{ if } A \text{ and } B \text{ are logically equivalent.}\end{aligned}$$

Depending how the phrase "logically equivalent" is understood, Definition 1 yields different formal systems. A fuzzy logic system is intended to allow an indefinite variety of numerical truth values. However, for many notions of logical equivalence, only two different truth values are possible given the postulates of Definition 1.

Theorem 1: Given the formal system of Definition 1, if $\neg(A \wedge \neg B)$ and $B \vee (\neg A \wedge \neg B)$ are logically equivalent, then for any two assertions A and B , either $t(B) = t(A)$ or $t(B) = 1 - t(A)$.

A direct proof of Theorem 1 appears in the sidebar, but it can also be proved using similar results couched in more abstract form:^{1,2}

Proposition: Let P be a finite Boolean algebra of propositions and let τ be a truth-assignment function $P \rightarrow [0,1]$, supposedly truth-functional via continuous connectives. Then for all $p \in P$, $\tau(p) \in \{0, 1\}$

The link between Theorem 1 and this proposition is that $\neg(A \wedge \neg B) \equiv B \vee (\neg A \wedge \neg B)$ is a valid equivalence of Boolean algebra. Theorem 1 is stronger in that it relies on only one particular equivalence, while the proposition is stronger because it applies to any connectives that are truth-functional and continuous (as defined in its authors' paper).

The equivalence used in Theorem 1 is rather complicated, but it is plausible intu-

itively, and it is natural to apply it in reasoning about a set of fuzzy rules, since $\neg(A \wedge \neg B)$ and $B \vee (\neg A \wedge \neg B)$ are both reexpressions of the classical implication $A \rightarrow B$. It was chosen for this reason, but the same result can also be proved using many other ostensibly reasonable logical equivalences.

It is important to be clear on what exactly Theorem 1 says, and what it does not say. On the one hand, the theorem applies to any more general formal system that includes the four postulates listed in Definition 1. Any extension of fuzzy logic to accommodate first-order sentences, for example, collapses to two truth values if it admits the propositional fuzzy logic of Definition 1 and the equivalence used in the statement of Theorem 1 as a special case. The theorem also applies to fuzzy set theory given the equation $(A \cap B)^c = B \cup (A^c \cap B^c)$, because Definition 1 can be understood as axiomatizing degrees of membership for fuzzy set intersections, unions, and complements.

On the other hand, the theorem does not necessarily apply to versions of fuzzy logic that modify or reject any of the postulates of Definition 1 or the equivalence used in Theorem 1. However, it is possible to carry through the proof of the theorem in many variant fuzzy logic systems. In particular, the theorem remains true when negation is modeled by any operator in the Sugeno class,³ and when disjunction or conjunction are modeled by operators in the Yager classes.⁴ The theorem also does not depend on any particular definition of implication in fuzzy logic. New definitions of fuzzy implication are still being proposed as new applications of fuzzy logic are investigated.⁵

Of course, the last postulate of Definition 1 is the most controversial one. To preserve

Proof of Theorem 1

Theorem 1: Given the formal system of Definition 1, if $\neg(A \wedge \neg B)$ and $B \vee (\neg A \wedge \neg B)$ are logically equivalent, then for any two assertions A and B , either $t(B) = t(A)$ or $t(B) = 1 - t(A)$.

Proof: Given the assumed equivalence, $t(\neg(A \wedge \neg B)) = t(B \vee (\neg A \wedge \neg B))$. Now

$$\begin{aligned} t(\neg(A \wedge \neg B)) &= 1 - \min\{t(A), 1 - t(B)\} \\ &= 1 + \max\{-t(A), -1 + t(B)\} \\ &= \max\{1 - t(A), t(B)\} \end{aligned}$$

and

$$\begin{aligned} t(B \vee (\neg A \wedge \neg B)) &= \\ \max\{t(B), \min\{1 - t(A), 1 - t(B)\}\}. \end{aligned}$$

The numerical expressions above are different if

$$t(B) < 1 - t(B) < 1 - t(A),$$

that is if $t(B) < 1 - t(B)$ and $t(A) < t(B)$, which happens if $t(A) < t(B) < 0.5$. So it cannot be true that $t(A) < t(B) < 0.5$.

Now note that the sentences $\neg(A \wedge \neg B)$ and $B \vee (\neg A \wedge \neg B)$ are both reexpressions of the material implication $A \rightarrow B$. One by one, consider the seven other material implication sentences involving A and B , which are

$$\begin{aligned} \neg A \rightarrow B \\ A \rightarrow \neg B \\ \neg A \rightarrow \neg B \\ B \rightarrow A \\ \neg B \rightarrow A \\ B \rightarrow \neg A \\ \neg B \rightarrow \neg A \end{aligned}$$

By the same reasoning as before, none of the following can be true:

$$\begin{aligned} 1 - t(A) < t(B) < 0.5 \\ t(A) < 1 - t(B) < 0.5 \\ 1 - t(A) < 1 - t(B) < 0.5 \\ t(B) < t(A) < 0.5 \\ 1 - t(B) < t(A) < 0.5 \\ t(B) < 1 - t(A) < 0.5 \\ 1 - t(B) < 1 - t(A) < 0.5 \end{aligned}$$

Now let $x = \min\{t(A), 1 - t(A)\}$ and let $y = \min\{t(B), 1 - t(B)\}$. Clearly $x \leq 0.5$ and $y \leq 0.5$ so if $x \neq y$, then one of the eight inequalities derived must be satisfied. Thus $t(B) = t(A)$ or $t(B) = 1 - t(A)$. ■

a continuum of degrees of truth, one naturally wants to restrict the notion of logical equivalence. In intuitive descriptions, fuzzy logic is often characterized as arising from the rejection of the law of excluded middle: the assertion $A \vee \neg A$. Unfortunately, rejecting this law is not sufficient to avoid collapse to just two truth values. Intuitionistic logic rejects the law of excluded middle, but the formal system of Definition 1 still collapses when logical equivalence means intuitionistic equivalence.⁶ (The Gödel translations of classically equivalent sentences are intuitionistically equivalent.⁶ For any sentence, the first three postulates of Definition 1 make its degree of truth and the degree of truth of its Gödel translation equal. Thus the proof in the sidebar can be carried over directly.) Dubois and Prade note that if all the properties of a Boolean algebra are preserved except for the law of excluded middle, their proposition no longer holds.² This observation is compatible with a collapse assuming only the equivalences of intuitionistic logic, because although intuitionistic logic rejects the law of excluded middle, it admits a doubly negated version of the law, namely $\neg\neg(\neg\neg A \vee \neg A)$. Of course, collapse to two truth values is avoided if we admit only the equivalences generated by the operators minimum, maximum, and complement to one. However, these equivalences are essentially the axioms of de Morgan, which allow only restricted reasoning about collections of fuzzy assertions.

Fuzzy logic in expert systems

The basic motivation for fuzzy logic is clear: While many ideas resemble traditional assertions, they are not naturally either true or false; uncertainty of some sort is attached to them. Fuzzy logic is an attempt to capture valid reasoning patterns about uncertainty. The notion is now well accepted that there are many different types of uncertainty, vagueness, and ignorance.⁷ However, there is still debate as to what types of uncertainty are captured by fuzzy logic. Many papers have discussed (at a high level of mathematical abstraction) the question of whether fuzzy logic provides suitable laws of thought for reasoning about uncertainty — and if so, which varieties of uncertainty. The question of interest here is more empirical: whether or not fuzzy logic is in practice an adequate formalism for uncertain reasoning in knowledge-based systems.

I conducted a thorough search of the technical literature using the Inspec and Computer Articles databases of more than 1.3 million papers published since 1988. Using the abstracts as a guide, I found no published report of a deployed expert system that uses fuzzy logic as its primary formalism for reasoning under uncertainty. While many theoretical papers on fuzzy logic in expert systems have been published, and several prototype systems have been described, it is hard to find reports of fielded systems doing knowledge-intensive tasks such as diagnosis, scheduling, or design.

Recent conferences give a representative

view of the extent of fuzzy logic application in current commercial and industrial knowledge-based systems. All the systems in actual use described at the 1992 IEEE International Conference on Fuzzy Systems are controllers, as opposed to reasoning systems. At the 1993 IEEE Conference on AI for Applications, no applications of fuzzy logic in knowledge-based systems were reported. Of the 16 deployed systems described at the 1993 AAAI Conference on Innovative Applications of AI, three — the CAPE,⁸ Dodger,⁹ and DYCE¹⁰ systems — used fuzzy logic in some way. However, none of these systems uses fuzzy logic operators for reasoning about uncertainty. Input observations are assigned degrees of membership in fuzzy sets, but inference with these degrees of membership uses other formalisms.

In addition to DYCE, a team at IBM has developed and fielded several knowledge-based systems over the past five years. Some of these systems are used for software and hardware diagnosis, for data analysis, and for operator training.^{11,12} The systems have varying architectures and cope with different varieties of uncertainty. Experience with them suggests that fuzzy logic is rarely suitable in practice for reasoning about uncertainty. The basic problem is that items of uncertain knowledge must be combined carefully to avoid incorrect inferences. Fixed domain-independent operators like those of fuzzy logic do not work.

The correct propagation of certainty degrees must account for the content of the uncertain propositions being combined.

This is necessary whether the uncertain propositions constitute deep or shallow knowledge. In the case of shallow knowledge, which may be defined as knowledge that is valid only in a limited context (for example, a correlation between a symptom and a fault), how degrees of uncertainty are combined must be adjusted to account for unstated background knowledge.

A simple example illustrates the difficulty. Consider a system that reasons in a shallow way using a notion of "strength of evidence," and assume that, as in many expert systems, this notion is left primitive and not analyzed more deeply. (Certainly "strength of evidence" is an intuitively meaningful concept that may or may not be probabilistic, but it is definitely different from "degree of truth.") For concreteness, suppose the context of discourse is a collection of melons, and in this context by definition $watermelon(x) \leftrightarrow redinside(x) \wedge greenoutside(x)$. For some melon m , suppose that $t(redinside(m)) = 0.5$ and $t(greenoutside(m)) = 0.8$, meaning that the evidence that m is red internally has strength 0.5, and that m is green externally with strength of evidence 0.8. Are the rules of fuzzy logic adequate for reasoning about this particular type of uncertainty? They say that the strength of evidence that m is a watermelon is $t(watermelon(m)) = \min\{0.5, 0.8\} = 0.5$. However, implicit background knowledge in this context says that being red inside and green outside are mutually reinforcing pieces of evidence toward being a watermelon, so m is a watermelon with strength of evidence over 0.5.

Deep knowledge can be defined as knowledge that is detailed and explicit enough to be valid in multiple contexts. Deep knowledge is general purpose and usable in complex chains of reasoning. However, Theorem 1 says that if more than two different truth values are assigned to the input propositions of long inference chains using fuzzy logic rules and one plausible equivalence, then it is possible to arrive at inconsistent conclusions. Fuzzy logic cannot be used for general reasoning under uncertainty with deep knowledge.

The fundamental issue here is that a conjunction's degree of uncertainty is not in general determined uniquely by the degree of uncertainty of the assertions entering into the conjunction. There does not exist a function f such that the rule $t(A \wedge B) = f(t(A), t(B))$ is always valid, whatever the type of uncertainty represented by $t(\cdot)$. For example, in the case of probabilistic uncertainty, the rule $t(A \wedge B) = t(A) \cdot t(B)$ is valid if and only if A and B represent independent events. In general, for probabilistic uncertainty all one knows is that $\max\{0, t(A) + t(B) - 1\} \leq t(A \wedge B) \leq \min\{t(A), t(B)\}$.

Methods for reasoning about uncertain evidence are an active research area in AI, and the conclusions here are not new. However, our practical experience independently confirms previous arguments about the inadequacy of systems for reasoning about uncertainty that propagate numerical factors according only to which connectives appear in assertions.¹³

Fuzzy logic in heuristic control

The application of fuzzy logic has been most successful in heuristic control, where there is wide consensus that traditional techniques of mathematical control theory are often inadequate. The reasons for this include the reliance of traditional methods on linear models of systems to be controlled, their propensity to produce "bang-bang" control regimes, and their focus on worst-case convergence and stability rather than typical-case efficiency. Heuristic control techniques give up mathematical simplicity and performance guarantees in exchange for increased realism and better performance in practice. For example, a heuristic controller using fuzzy logic has been shown to have less overshoot and quicker settling.¹⁴

The first demonstrations that fuzzy logic could be used in heuristic controllers were published in the 1970s.^{15,16} Work continued through the 1980s, and recently there has been an explosion of industrial interest in the area.^{17,18} One reason for this recent interest in fuzzy controllers is that they can

be implemented by embedded specialized microprocessors.¹⁹

Despite industry interest, and consumer interest in Japan, fuzzy logic technology continues to meet resistance. For example, at IJCAI '91, Takeo Kanade gave a talk on computer vision, describing at length Matsushita's camcorder image stabilizing system without mentioning its use of fuzzy logic. Also, while a fuzzy logic controller is embedded in the 1994 Honda Accord's automatic transmission, the advertising brochures describe it as "grade logic."

Almost all currently deployed heuristic controllers using fuzzy logic are similar in five important aspects (a good example of this standard architecture appears in a paper by Sugeno and his colleagues²¹):

- (1) The typical fuzzy controller knowledge base consists of fewer than 100 rules; often fewer than 20 rules are used. Fuzzy controllers are orders of magnitude smaller than systems built using traditional AI formalisms.
- (2) The knowledge entering into fuzzy controllers is structurally shallow, both statically and dynamically. Conclusions produced by rules are not used as premises in other rules; statically rules are organized in a flat list, and dynamically there is no runtime chaining of inferences.
- (3) The knowledge recorded in a fuzzy controller typically reflects immediate correlations between the inputs and outputs to be controlled, as opposed to a deep, causal model of the system. The premises of rules refer to sensor observations, and rule conclusions refer to actuator settings. (Rule premises refer to qualitative or "linguistic" sensor observations, and rule conclusions refer to qualitative actuator settings, whereas outputs and inputs of sensors and actuators are typically real-valued. This means that normally two controller components map between numerical values and qualitative values. In fuzzy logic terminology, these components are said to defuzzify outputs and implement membership functions.)

- (4) In deployed fuzzy controllers, the numerical parameters of their rules and of their qualitative input and output modules are tuned in a learning process. The tuning can be done by human engineers or by learning algorithms; neural network methods have been especially successful.²² What the tuning algorithms themselves have in common is that they are gradient-descent "hill-climbing" algorithms that learn by local optimization.¹⁴
- (5) By definition, fuzzy controllers use fuzzy logic operators. Typically, minimum and maximum are used, as are explicit possibility distributions (usually trapezoidal) and some fuzzy implication operator.

The question that naturally arises is, Which of these five features are essential to the success of fuzzy controllers? It appears that the first four are vital to practical success, because they make the celebrated credit assignment problem solvable, while the use of fuzzy logic is not essential.

In a nutshell, the credit assignment problem is to improve a complex system by modifying a part of it, given only an evaluation of its overall performance. In general, solving the credit assignment problem is impossible: the task is tantamount to generating many bits of information (a change to the internals of the system) from just a few bits of information (the system's input/output performance). However, the first four shared features of fuzzy controllers can solve this problem for the following reasons.

First, since it consists of only a few rules, the knowledge base of a fuzzy controller is a small system to modify. Second, the short paths between the fuzzy controller's inputs and outputs localize the effect of a change, making it easier to discover a change with a desired effect without producing undesired consequences. Third, because of the iterative way in which fuzzy controllers are refined, many observations of input/output performance are available for system improvement. Fourth, the continuous nature of

the controller's parameters allows small quantities of performance information to be used to make small system changes.

Thus, what makes fuzzy controllers useful in practice is the combination of a rule-based formalism with numerical factors qualifying rules and the premises entering into rules. The principal advantage of rule-based formalisms is that knowledge can be acquired from experts or from experience incrementally. Individual rules and premises can be refined independently, or at least more independently than items of knowledge in other formalisms. Numerical factors have two main advantages. They allow a heuristic control system to interface smoothly with the continuous outside world, and they allow it to be tuned gradually — small changes in numerical factor values cause small changes in behavior.

None of the features contributing to the success of systems based on fuzzy logic is unique to fuzzy logic. It seems that most current fuzzy logic applications could use other numerical rule-based formalisms instead — if a human or a learning algorithm tuned numerical values for those formalisms, as is customary when using fuzzy logic. A quote from the originator of fuzzy heuristic control is relevant here:

... it should be remarked that the work on process control using fuzzy logic was inspired as much by Waterman and his approach to rule-based decision making as by Zadeh ... and his novel theory of fuzzy subsets.²³

Several knowledge representation formalisms that are rule-based and numerical have been proposed besides fuzzy logic.^{24,25} To the extent that numerical factors can be tuned in these formalisms, they should be equally useful for constructing heuristic controllers. Indeed, at least one has already been so used.²⁶

Recapitulating mainstream AI

Several research groups are attempting to scale up systems based on fuzzy logic and lift the architectural limitations of current fuzzy controllers. For example, a methodology for designing block-structured controllers with guaranteed stability

properties has been studied,²⁷ as have methodological problems in constructing models of complex systems based on deep knowledge.²⁸ Controllers with intermediate variables, thus with chaining of inferences, have also been investigated.²⁹

However, the designers of larger systems based on fuzzy logic are encountering all the problems of scale already identified in traditional knowledge-based systems. It appears that the research history of fuzzy logic is recapitulating that of other areas in AI as well, particularly those dealing with knowledge engineering and state information.

The rules in the knowledge bases of current fuzzy controllers are obtained directly by interviewing experts. Indeed, the original motivation for using fuzzy logic in building heuristic controllers was that fuzzy logic is designed to capture human statements involving vague quantifiers such as "considerable." More recently, consensus has developed around the idea that research must focus on obtaining "procedures for fuzzy controller design based on fuzzy models of the process."³⁰ Mainstream work on knowledge engineering, however, has already transcended the dichotomy between rule-based and model-based reasoning.

Expert systems with knowledge consisting of *if-then* rules have at least two disadvantages. First, maintenance of a rule base becomes complex and time-consuming as the system size increases. Second, rule-based systems tend to be brittle: If an item of knowledge is missing from a rule, the system may fail to find a solution, or worse, may draw an incorrect conclusion. The main disadvantage of model-based approaches, on the other hand, is that it is very difficult to construct sufficiently detailed and accurate models of complex systems. Moreover, the models constructed tend to be highly application-specific and not generalizable.³¹

Many recent expert systems, therefore, are neither rule-based nor model-based in the standard way.¹² For these systems, the aim of the knowledge engineering process

is not simply to acquire knowledge from human experts, but rather to develop a theory of the experts' situated performance (this is true regardless of whether the desired knowledge is correlational, as in present fuzzy controllers, or deep, as in model-based expert systems). Concretely, under this view of knowledge engineering, knowledge bases are constructed to model the beliefs and practices of experts and not "objective" truths about underlying physical processes. An important benefit of this approach is that the organization of an expert's beliefs provides an implicit organization of knowledge about the external process with which the knowledge-based system is intended to interact.

The more sophisticated view of knowledge engineering just outlined is clearly relevant to research on constructing more intricate fuzzy controllers. For a second example of relevant AI work, consider controllers that can carry state information from one moment to the next (mentioned as a topic for future research by von Altrock and colleagues²⁹). Symbolic AI formalisms for representing systems whose behavior depends on their history have been available since the 1960s. Neural networks with similar properties (called recurrent networks) have been available for several years, and have already been used in control applications.³⁵ It remains to be seen whether research from a fuzzy logic perspective will provide new solutions to the fundamental issues of AI.

Applications of fuzzy logic in heuristic control have been highly successful, despite the collapse of fuzzy logic to two-valued logic under an apparently reasonable condition, and despite the inadequacy of fuzzy logic for general inference with uncertain knowledge. These difficulties have not been harmful in practice because current fuzzy controllers are far simpler than other knowledge-based systems. Theorem 1 is not an issue for fuzzy controllers because they do not perform chains of in-

ference, and they are developed informally, with no formal reasoning about their rules that applies equivalences such as the one used in the statement of Theorem 1. Second, the knowledge recorded in a fuzzy controller is not a consistent causal model of the process being controlled, but rather an assemblage of visible correlations between sensor observations and actuator settings. Since this knowledge is not itself general-purpose, the inadequacy of fuzzy logic for general reasoning about uncertainty is not an issue. Moreover, the ability to refine the parameters of a fuzzy controller iteratively can compensate for the arbitrariness of the fuzzy logic operators as applied inside a limited domain.

The common assumption that heuristic controllers based on fuzzy logic are successful because they use fuzzy logic appears to be an instance of the *post hoc, ergo propter hoc* fallacy. The fact that using fuzzy logic is correlated with success does not entail that using fuzzy logic causes success. In the future, as fuzzy controllers are scaled up, the technical difficulties identified in this article can be expected to become important in practice.

Theorem 1 is a crisp demonstration of one of several deep difficulties of scale in AI: the problem of maintaining consistency in long sequences of reasoning. Other difficulties of scale can also be expected to become critical — in particular, the issue of designing learning mechanisms that can solve the credit assignment problem when the simplifying features of present controllers are absent.

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The Unique Strength of Fuzzy Logic Control

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I am pleased to see that Elkan has revised his paper based on comments from fuzzy logic experts. His reference to Dubois and Prade indicates that he has realized, finally, that his alleged "new discovery" has long been known by specialists in fuzzy and multivalued logics.

Unfortunately, the new version still contains many misunderstandings and errors. I will briefly respond to some of them, avoiding a discussion of the supposedly startling proof about the purported inconsistency of fuzzy logic, which is covered in responses by Enrique Ruspini and others. I will confine my comments primarily to a fundamental misunderstanding that is the source of many of Elkan's mistaken assertions about the use of fuzzy logic in heuristic control and expert systems.

Elkan lists a number of powerful features of fuzzy-logic control, but then erroneously concludes that none is unique to fuzzy logic. He fails to realize that the unique strength of fuzzy-logic control is its dependence on fuzzy-set theory and its representational capabilities. The small number of rules typical in these systems is not the result of mere luck, but the direct consequence of the fuzzy predicates that appear in the rules. Each of these predicates covers a wide range of state variable values while facilitating interpolation of rule consequents. Fuzzy sets provide for a general, yet compact characterization of system state that requires fewer rules.

Elkan's assertion about the shallowness of fuzzy controller knowledge is simply wrong. Recent fuzzy-logic controllers, developed for more challenging tasks, use hierarchical fuzzy control methods.¹ Examples include the helicopter control developed by Sugeno and his collaborators at the Tokyo Institute of Technology (a system that can appear trivial only to those unfamiliar with control theory), and the controller for a three-linked inverted pendulum developed at Apronix. In applications such

as these, the result of the first level of control is used in deriving control rules for the second set, and so on. These examples prove that fuzzy-logic control systems can be developed to reason with considerable depth of complexity. Similarly, the control mechanisms for the local-motion control of SRI's autonomous robot² rely on several deliberation levels to determine the relevance level of each control rule (by evaluating the operational environment characteristics); to identify current goals and their state of achievement; to activate control rules according to the current context; and to blend their control recommendations.

At any rate, the "depth" of a reasoning process as Elkan seems to understand it is not even a well-defined measure of inferential system complexity. This is seen in the fact that the two-level forward chain $A \rightarrow (B \rightarrow C)$ is often "compiled" in real-time applications (such as control systems) into the single-level rule $A \wedge B \rightarrow C$ to simplify and speed computation. This simplification mechanism, which turns what Elkan would consider "complex" into an equivalent "simple" version, is used to introduce contextual and goal-dependence considerations into the reasoning chain both in the SRI's mobile robot controller and in our own two-goal inverted pendulum.

Using fuzzy sets to describe a general linguistic variable also significantly reduces the complexity of the search process in fuzzy systems that learn from experience. Elkan correctly points out that using fewer rules simplifies the credit assignment problem, but he fails to realize that this is a consequence of using fuzzy logic rather than an indicator of its current or future applicability. This feature is desirable in any control system, as is seen in the fuzzy-logic controller developed at NASA Ames for the Space Shuttle's rendezvous and docking operations.³ This controller learns to improve itself from experience using reinforcement learning techniques,^{4,5}

a complex task that would have been very difficult, if not impossible, if other symbolic control techniques had been used.

In summary, I see two major misunderstandings in Elkan's paper. First, it relies on a theorem that is irrelevant to fuzzy logic to argue that the methodology is paradoxical. Second, it fails to note that the advantages provided by fuzzy-set constructs give fuzzy control a unique methodological strength — a fact Elkan mistakenly interprets as technological immaturity.

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Broader Issues At Stake

A Response to Elkan

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The fuzzy set approach has clearly captured the interest of many researchers around the world and has been used to build applications of various sorts, of which fuzzy control applications are currently the most prominent. The approach, however, remains controversial. While this controversy has many sources, there are relatively few places where the arguments are set out in a fashion that allows debate. It is thus useful to have both Charles Elkan's analysis of the fuzzy set approach to representing uncertainty, and his examination of which features of fuzzy set theory are responsible for the success of fuzzy control systems. In particular, I commend Elkan for making his arguments about these techniques in a nonpolemical way, letting technical arguments and results do most of the talking.

In Elkan's first argument, he claims that the axioms of fuzzy set theory, in conjunction with what appear to be a number of reasonable versions of logical equivalence between sentences, lead to a collapse of truth functions into just two values — a fate that fuzzy set theory was expressly meant to avoid.

As Elkan points out, a result similar to his collapse theorem was already known to researchers within the fuzzy set community (Dubois and Prade). My understanding is that they weren't too worried by this result, since they think that the traditional notion of logical equivalence or any of its variants should be abandoned for fuzzy sets. This response seems to me to be formally reasonable, but I think in practice it would be hard to work with a system in which logical equivalence itself is a fuzzy relation. Ultimately, we will have to see how much really interesting work is possible with this notion of fuzzy equivalence.

In the second argument, Elkan asserts that when fuzzy control systems that work well are analyzed, the real source of their success seems to be not the inferential capabilities of fuzzy set theory (derived from the theory's composition axioms) but rather a combination of things exclusive of fuzzy set axioms. Among these are the ability to represent certain things as continuous quantities rather than all-or-nothing quantities; certain heuristic techniques — that are themselves outside fuzzy set theory — to get the right parameters for the problems; and the fact that there is little complex rule-chaining going on. A number of alternatives and rivals to fuzzy set theory would work as well in those applications.

Part of Elkan's point — that the success of fuzzy control systems thus far is not really a full test or proof of the axioms and claims of fuzzy set theory — is actually an instance of a larger phenomenon in AI. I think that Elkan's point can be made against the claims of not only fuzzy control proposals, but also against a number of other proposals in AI, including the rivals of fuzzy sets, such as belief nets.

The general problem is a kind of credit allocation problem and can be stated as follows. Given some mechanism M , and some specific task T , suppose I write a program P , using M as the basis for the program. And, let us say that P does well in the task T . What conclusions can we draw about mechanism M from the success of P in tackling T ? How much credit should M get for the success of P ?

A historical perspective. In the late 1970's, rule-based expert systems were capturing the imagination of many people. Mycin and R1 were great successes. In the

above terminology, rule-based languages would be M , Mycin and R1 would be the P 's, and simple diagnosis and configuration would be the corresponding tasks, T . The success of the two programs led to claims about the power of the rule-based mechanism. Similar examples involving other mechanisms, such as belief nets and truth maintenance systems, can be constructed.

In a series of articles (such as one from 1986,¹ for example), I made the following points regarding rule-based systems as a mechanism. The specifics of the mechanism were incidental in accounting for many aspects of why the programs worked. The mechanism was computation-universal, and of course could be used to implement any other mechanism or strategy. A higher order strategy — classification in the case of Mycin, or linear sequencing of subtasks in the case of R1 — was the problem-level strategy that was responsible for the programs' performance. Not only was the rule-based mechanism not the direct cause of the good performance, but they actually hid the reasons for success: The higher level strategies were programmed in the language of the lower level mechanism. The strategies had to be brought out by analysis, rather than seen by a direct inspection of the mechanism. The limitations and success of Mycin and R1 could be more insightfully analyzed by examining the adequacy of classification for diagnosis and linear subtasking for configuration design. Clancey also analyzed Mycin as a heuristic classifier² and pointed out the power such high-level analysis brought to building diagnostic systems. In the last decade or so, there has been a decisive shift in emphasis in the field of knowledge-based systems from mechanisms at the rule level to phenomena at the task level.

Thus, given an *M-T-P* triad, it is not always easy to decide exactly what the role of *M* was in the success of *P* in achieving that version of *T*. This is not to say that *M*'s properties are irrelevant. There are several ways a given mechanism might play less of a role than is readily apparent, among them:

- *M* might simply be one among many perfectly reasonable lower level mechanisms to implement the causally more relevant higher level mechanism.
- *M* might have features which actually impede good performance for the class of problems in *T*. This might not be evident from the specific instance of *T* for which *P* was written. In this instance, the troublesome features of *M* might not have been used or their effect might be minimal. Fuzzy set theory has been successfully applied to simple versions of the control problem. As Elkan argues, however, the problematic features of the theory might start showing up as more complex versions of the control problem are encountered.
- In some cases, *M* has many more features than needed for capturing the essence of *T*. Hence, using *M* to build *P* for solving *T* calls for making commitments to details that are either irrelevant or that detract from building good *P*'s. However, when such a program is built, it takes quite a bit of analysis to tell which features of *M* are necessary. There is often a tendency, especially among those who are enthusiasts of *M* for other reasons, to ascribe the success of *P* to those features of *M* that were actually incidental to *P*'s success. Even more seriously, success with *M* might lead to its use for more complex versions of *T*, where these additional features actually make building successful *P*'s more difficult. Elkan makes a good case for this possibility as fuzzy control approaches are applied to more complex control problems.

The history of Mycin is another source of wisdom about the role of uncertainty-handling mechanisms. When Mycin came out,

much was made of the uncertainty-factor formalism. Debates raged about this formalism versus Bayesian formalism versus fuzzy set formalisms as an appropriate calculus. Cooper and Clancey got the idea of doing an experiment in which they coarsened the uncertainty factors in Mycin's knowledge base rules and examined how well the modified Mycin did in the same cases.³ The modified Mycin solved the problems as well as the original Mycin.

How could this be? Clearly the calculus as such didn't play as fundamental a role in the ability of Mycin to solve the problems. The fine structure of uncertainty didn't really matter. The knowledge base had enough knowledge to establish or reject the conclusions in a near-definitive way. None of the conclusions were based on even moderate distinctions in uncertainty between the candidates. There were multiple ways to get to or reject conclusions, and even moderate changes in the uncertainties didn't matter. The correct conclusions were very strongly established, and the incorrect conclusions were very strongly rejected. Mycin did well, not because of the fine points of its uncertainty calculus — it would have done just as well with any of a number of alternative calculi — but because of the robustness of its knowledge base. This is another instance of the allocation of credit problem.

The nature of fuzzy theory

I have followed fuzzy set theory almost from its inception. The theory's claim that all senses of uncertainty in human knowledge cannot be reduced to some version of probability has always struck me as right. One of the most useful consequences of the fuzzy set movement has been the identification of different types of uncertainty. In particular, the theory suggests that many predicates such as "bald," "most," and "large" are neither binary predicates, nor are they simply probabilistic. This also seems to me to be true. However, the specific solutions offered and claims made by fuzzy set theory, and the way they have often been applied to problems like control, are problematic for me.

A psychological theory? At the heart of fuzzy set theory is an ambiguity about the nature of the theory, and how one goes about validating it. If it is a psychological theory — that is, a theory of how humans deal with certain types of uncertainty — we would need certain kinds of evidence about human behavior in uncertainty handling. I am unconvinced that fuzzy set theory is a psychological theory. I have not done an extensive literature survey, but the work of Kempton^{4,5} raises doubts that human behavior in uncertainty handling follows the axioms of fuzzy set theory.

Even if it turns out that the theory does correspond to human behavior in this area, we must then decide what kinds of scaling and rationality properties the relevant human behavior has before it is used to make machines that make decisions.

Two relevant analogies are found in commonsense physical reasoning and reasoning about probabilistic uncertainty. We all have approximate rules about how the physical world behaves: "If we push this a little, this will move a moderate distance, while the other object would hardly move." We use such rules when we have to predict behavior in the physical world, but these rules are typically chained over a few steps. When a problem calls for many steps, these rules start accumulating large errors (to be expected), but curiously, they also start accumulating ambiguities of another sort. So many alternative possibilities are generated that we adopt all kinds of goal- and context-specific strategies to select a "future history" over other alternatives. Or, if we are physicists, we resort to a pencil and paper for more exact calculations even if what we really want are approximate answers. Clearly such approximate reasoning by humans does not scale up very well.

In the case of probability assessment behavior, human behavior is not always what an outside observer might regard as rational.⁶ Thus, in addition to the scalability problem, there is the problem of rationality of human behavior as well.

The point that I want to make with these two examples is that, in many domains, automated decision systems should not be designed to emulate human behavior. Thus, even if fuzzy set theory turns out to be a model of how humans handle a certain type of uncertainty, we need additional arguments to make the theory the basis of automatic control.

A mathematical theory? On the other hand, fuzzy set could be a theory of an abstract mathematical system whose properties model some domain of human interest. Examples of such systems are arithmetic and deductive logic. The formalization of arithmetic starts with our intuitive notions about numbers, but it is not a psychological theory. It posits a world of numbers and operations on them, and the formalization is an attempt to capture the properties of this world. We can in fact construct the abstract world, recognize its objects as the familiar numbers and perform operations on them, and then verify those operations against the predictions of the axiomatization. For example, we can multiply 2 and 3, and check if the axiom system in fact generates the number 6 for the answer.

If fuzzy set theory is a theory of an abstract world whose constituents are uncertainties of certain types, and whose operations are the sort of things we do when we combine uncertainties, then the theory has to give two kinds of evidence. First, there must be evidence that such an abstract world indeed exists. Many abstract worlds that can be postulated fail to exist because their axioms lack a certain internal coherence. Second, it must give evidence that the fuzzy set axioms capture the operations of this world. Establishing that such an abstract world exists is actually quite hard. In fact, I think it is quite possible that there is no abstract world of uncertainty combination of the type that fuzzy set theory attempts to capture. In any case, fuzzy set theory has to worry about validation of its assumptions and about the existence of an abstract calculus for combining this kind of uncertainty.

What do I mean by "such an abstract world may not exist"? Again, the analogy of qualitative physics is relevant. We know there is a real physics, whose laws relate values of some state variables to the values of other state variables. If we have an exact value for the independent variables, we can calculate, using these laws, the exact values of the dependent variables.

The equations of physics are not a psychological theory. However, consider the ordinary commonsense reasoning about the physical world that I discussed earlier. People do make qualitative predictions about the physical world in response to qualitative changes in some state of the world. As I said, the qualitative rules people have cannot be chained into long inferences: The ambiguities multiply, resulting in too many possible future histories. Which one of the histories will be realized often depends on a more exact value for some variables than we can get from qualitative rules alone. I have described elsewhere a number of strategies people use to handle such an explosion of possibilities, but almost all of the strategies depend on the problem-solving goal and context.⁷ The conclusion is not the result of applying an abstract, context-independent calculus. In short, there is no qualitative physics that is a homomorphism of the quantitative physics such that the qualitative physics gives answers that are just qualitative versions of the answers given by the quantitative physics.

With respect to uncertainty handling, many researchers seem to be looking for a similar abstract system that may not exist. They are looking for a calculus of uncertainty handling which has the following features:

- The semantics of its uncertainty terms capture the intuitive meaning of uncertainty terms that people use in their commonsense behavior.
- The operations of combination in the calculus capture human behavior when their uncertainties are combined.

This assumes that there is in fact a calculus that underlies the combining of uncertain-

ties through human common sense. What if human behavior, in combining everyday uncertainties, is really governed by a combination of goal- and context-dependent strategies that make use of a rich body of domain-specific knowledge? What if this cannot be captured by a calculus of the type that fuzzy set or other theorists are looking for? If human conclusions are robust with respect to moderate changes in the uncertainty values of the constituents — as in the Mycin experiment by Cooper and Clancey — then the real explanation of human behavior is not given by a calculus, fuzzy or otherwise, but by the complex collection of situation- and goal-specific knowledge that people bring to bear on instances of the problem.

Like the case in qualitative reasoning mentioned earlier, people might in fact avoid anything like a chain of uncertainty combination. If the conclusion seems robust with respect to moderate changes in the uncertainty values of its constituents, people feel comfortable with the conclusion. If not, they might get additional data so that a robust conclusion can be reached, postpone making a decision, or make decisions that may not in general be considered the best, but that are fine for the specific goal at hand. In other words, the same values of uncertainties for two constituent beliefs would lead to a conclusion with an uncertainty value *A* in one situation, an uncertainty value *B* in another, additional information gathering in a third, explicit use of probability models in a fourth, and simply a shrugging of shoulders and no decision at all in a fifth. If this is the case, then the search for a calculus of the type fuzzy set theorists (and many others in the research community concerned with modeling uncertainty in reasoning) are looking for is likely to be futile. The issue is illustrated well in Elkan's example of his expert system, for which neither the probability scheme nor the fuzzy set approach was appropriate.

The problem with fuzzy set theory, in

my view, is not in the mathematics of the formal system. It is clearly a mathematical system of some interest. However, a theory of this type has to be judged either as a psychological theory or as a theory that has captured an abstract calculus that underlies some type of human reasoning. As I have just argued, an abstract calculus of this type may not exist.

The problem of context. In the 1980's, my colleagues and I were faced with a similar problem with uncertainty in medical diagnosis. Physicians have to come up with an assessment of the "likelihood" of some disease for which a number of data were potentially relevant. The relation between the data and the strength of belief in the disease was of course a classic example of uncertainty. For various reasons — not the least of which was that we didn't have the data needed to use the frequency version of the probabilities for this relationship — we needed a technique to model human expertise in this area. Bayesian approaches, fuzzy set theory, Dempster-Shafer theory, and uncertainty factor calculus were all available to us. All these calculi shared one important property or assumption about human expertise — that there was a situation- and goal-independent way of combining uncertainties.

For example, if two symptoms, s_1 and s_2 , were relevant to making a decision about disease d , such calculi would provide ways in which evidence for s_1 and s_2 would be combined to give evidence about d , and additionally, that the rule of combination itself is independent of the specific labels for s_1 , s_2 , and d . If the evidence for s_1 is large, and s_2 is medium, the rule would specify what the evidence for d would be. But the rule cannot be one thing where s_1 is "bilirubin," s_2 is "alkaline phosphatase," and d is "liver disease," while another rule is used where s_1 is "cholesterol level," s_2 is "alkaline phosphatase," and d is "heart disease."

We found, however, that expert behavior in uncertainty combination in fact differed from context to context, and problem-solving goal to problem-solving goal. We had

to resist the mathematical attractions of an abstract calculus. Instead, we developed a formalism in which we could incorporate the uncertainty-combining behavior of experts,⁸ who were compiling a complex of background knowledge in such context-specific rules. It was also important to note that the chaining length was relatively small: Two or three steps were all that were used. If the problem called for much longer chaining, we took it as a sign that we were modeling the expert knowledge inaccurately, and sought additional pieces of knowledge that would shorten the chain.

Fuzzy set theory has done quite well as a formal mathematical system. Whether its theorems are interesting is a subjective opinion among mathematicians, but a large body of mathematical work exists. Where more work needs to be done is in establishing that fuzzy set theory actually captures something real and can make a pragmatic difference, for the right reasons.

I think Elkan has performed a service by initiating a debate about the properties of fuzzy set theory. I have argued that the points Elkan makes about fuzzy sets are really an instance of problems that apply to a number of other AI mechanisms and ideas, and specifically to many other proposals for subjective calculi for handling uncertainty. The issues raised are large in scope, and not only the fuzzy set community, but the AI community as a whole could benefit from giving them thought.

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A Better Path to Duplicating Human Reasoning

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The paradox that arises from Elkan's Theorem 1 is mild in comparison to some of the logical problems that lurk behind the apparently innocent equations in Definition 1. In fact, although fuzzy logic has been promoted as a way of writing programs that carry out inference in the same way a person might, the equations of Definition 1 can lead inescapably to conclusions that no human being would accept.

Consider a simple example: You know that the airplane on which John Doe was traveling has crashed in some remote location, but you have no information about whether anyone on board has survived. In this situation, you might make the following assignment: $\pi(\text{"John Doe is alive"}) = 0.5$. The equations of Definition 1 would lead you immediately to $\pi(\text{"John Doe is dead"}) = 0.5$. While this is a reasonable assignment, it would in turn lead you to $\pi(\text{"John Doe is both dead and alive"}) = 0.5$. Thus, there is an element of truth in the statement "John Doe is both dead and alive." However, any rational person will argue that it is impossible for John Doe to be both dead and alive, so that the statement "John Doe is both dead and alive" must always be false, and have a truth value of zero.

We can imagine putting a fuzzy logic system to the Turing test on the matter of John Doe's well-being:

Interrogator: Is John Doe alive ?

Respondent 1: It is half-true that John Doe is alive.

Respondent 2: I don't know.

Interrogator: Is John Doe dead ?

Respondent 1: It is half-true that John Doe is dead.

Respondent 2: I don't know.

Interrogator: Is John Doe both dead and alive?

Respondent 1: It is half-true that John Doe is both dead and alive.

Respondent 2: It is impossible for John Doe to be both dead and alive.

While there is an element of caricature in this dialogue, it serves to highlight the problem. It is clear that if A is any proposition with a non-zero truth value, the equations of Definition 1 will lead to the conclusion that the truth value of the compound statement (A and (not A)) is also non-zero. This is a very simple example of how fuzzy logic diverges from human logic. It is to be expected that this divergence will increase with the complexity of the inference process.

Of course, people have been assigning truth values between zero and one to make inferences since the time of Laplace, on the basis of probability theory. As Cox has shown,¹ using the axioms of probability theory is essentially the only way to carry out this form of inference and remain consistent with human reasoning — any other way will lead to contradictions and inconsistencies. However, proponents of fuzzy logic appear to be unaware of Cox's work and that of Jaynes² and Tribus,³ where the question of how to write programs that make inference based on incomplete knowledge is discussed.

As Cheeseman⁴ pointed out for AI in general, the bottom line is that if you want to write a program or build a machine that will perform inference in the same way as people, then you must build the basic equations of probability theory into it, or face the inevitable outcome that it will not perform as required.

Perhaps the real paradox of fuzzy logic's success is that proponents hail it as a successful technology despite the fact that it is incapable of performing as they claim it can and does.

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Partial Truth is not Uncertainty

Fuzzy Logic versus Possibilistic Logic

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Charles Elkan has questioned fuzzy logic and cast serious doubts on the reasons for its success, arguing that “fuzzy logic collapses mathematically to two-valued logic.” We completely disagree, and we especially object to two points:

- (1) Elkan’s proof uses too strong a notion of logical equivalence. The particular equivalence he considers, while valid in Boolean algebra, has nothing to do with fuzzy logic.
- (2) Elkan claims that De Morgan’s algebra “allows very little reasoning about collections of fuzzy assertions,” although he correctly states that when logical equivalence is restricted to De Morgan algebra equalities’ “collapse to two truth values is avoided.”

Furthermore, Elkan fails to understand the important distinction between two totally different problems that fuzzy-set-based methods address.¹ These are the handling of *gradual* (thus non-Boolean) properties whose satisfaction is a matter of degree (even when information is complete) on the one hand, and the handling of uncertainty pervading Boolean propositions, the uncertainty being induced by incomplete states of knowledge that are represented by means of fuzzy sets, on the other hand.² The first problem requires the plain use of fuzzy sets, while the second is the realm of possibility theory^{3,4} and possibilistic logic⁵. We now discuss in greater detail the points above and the distinction between truth functional fuzzy (multivalued) logic and non-fully compositional possibilistic logic.

Fuzzy logic equivalence is not classical. Elkan claims that in fuzzy logic, four requirements hold for any assertions A and B , t being a truth assignment function such that $\forall A, t(A) \in [0, 1]$:

$$\begin{aligned} t(A \wedge B) &= \min(t(A), t(B)) & (1) \\ t(A \vee B) &= \max(t(A), t(B)) & (2) \\ t(\neg A) &= 1 - t(A) & (3) \\ t(A) &= t(B) \text{ if } A \text{ and } B \text{ are logically equivalent.} & (4) \end{aligned}$$

While Equations 1–3 are indeed the basic relations governing degrees of truth in fuzzy logic (as well as fuzzy set membership degrees) as proposed by Zadeh,⁶ Equation 4 (where “logically equivalent” is understood in a stronger sense than the equivalences induced by 1–3) has never been seriously considered by any author in the fuzzy-set literature. (There are, as can be expected, a few erroneous attempts at the subject in a corpus of more than 10,000 published papers). Obviously, some classical logic equivalences still hold with fuzzy assertions obeying Equations 1–3, namely, those allowed by the De Morgan structure induced by 1–3, such as

$$\begin{aligned} A \wedge A &\equiv A ; A \vee A \equiv A \text{ (idempotency)} \\ A \wedge (B \vee C) &\equiv (A \wedge B) \vee (A \wedge C) ; \\ A \vee (B \wedge C) &\equiv (A \vee B) \wedge (A \vee C) \text{ (distributivity)} \end{aligned}$$

But other Boolean equivalences *do not* hold, for instance:

$$A \wedge \neg A \not\equiv \perp$$

since Equations 1 and 3 entail *only*

$$t(A \wedge \neg A) = \min(t(A), 1 - t(A)) \leq 1/2;$$

and

$$A \vee \neg A \not\equiv \top$$

since Equations 2 and 3 entail *only*

$$t(A \vee \neg A) = \max(t(A), 1 - t(A)) \geq 1/2$$

where $t(\perp) = 0$ and $t(\top) = 1$. Indeed, as many authors have emphasized, the failure of contradiction and excluded-middle laws is typical of fuzzy logic. This is natural with gradual properties like “tall.” For example, in a given context, somebody who is 1.75 meters high might be considered neither as completely tall (tall with degree 1) nor as completely not tall (tall with degree 0). In this case, we might have, for example, $\mu_{\text{tall}}(1.75) = 0.5 = \mu_{\neg\text{tall}}(1.75)$.

To establish the collapse of fuzzy logic to binary logic, Elkan uses the logical equivalence

$$\neg(A \wedge \neg B) \equiv B \vee (\neg A \wedge \neg B) \quad (5)$$

postulated as being “plausible intuitively.” If Equations 1–3 hold, the left-hand part of Equation 5 can be equivalently written in fuzzy logic as

$$\neg(A \wedge \neg B) \equiv \neg A \vee B$$

while the right-hand part can be equivalently written as

$$B \vee (\neg A \wedge \neg B) \equiv (\neg A \vee B) \wedge (B \vee \neg B),$$

which clearly relates to the excluded-middle law. Thus, it is expected that Equation 5 fails to hold in fuzzy logic — and indeed it can be checked, using Equations 1–3, that a counterexample to Equation 5 is provided by $t(A) = 0$, $t(B) = 0.5$, for instance. Thus, Elkan’s claim of “a paradox in fuzzy logic” relies only on faulty assumptions, or at best on a logical equivalence, the rationale of which is far from natural in the scope of fuzzy logic.

Gradual and interpolative reasoning.

Fuzzy logic is concerned with the handling of assertions like “John is tall” — assertions whose truth is a matter of degree due

to gradual predicates within them. The degree of truth of compound expressions can be easily computed using Equations 1–3. (Although we restrict ourselves here to the operators minimum, maximum, and complement to one, there is a panoply of others^{7,8} that enable us to model different kinds of AND and OR operations between properties in a multicriteria aggregation perspective.)

More than 20 years ago, R.C.T. Lee⁹ provided the basic machinery for reasoning in fuzzy logic by extending the resolution rule in accordance with Equations 1–3. He established that if all the truth values of the parent clauses are greater than 0.5, then a resolvent clause derived by the resolution principle always has a truth-value between the maximum and the minimum of those of the parent clauses.

We can also use an implication operator to model “gradual rules,”¹⁰ which express knowledge of the form “the more X is A , the more Y is B ,” such as, “the taller you are, the heavier you are.” This is captured by the implication defined by

$$t(A \rightarrow B) = 1 \text{ if } t(A) \leq t(B) \\ = 0 \text{ if } t(A) > t(B) \quad (6)$$

This implication is the natural counterpart of Zadeh’s fuzzy set inclusion defined by the pointwise inequality of the membership functions.⁶ It is also directly associated with Equations 1–3, since $A \rightarrow B \equiv T$ if and only if $A \wedge B \equiv A$. Such an implication expresses a purely gradual relationship and has nothing to do with uncertainty. Besides, Takagi and Sugeno¹¹ have proposed an interpolation mechanism between n rules with fuzzy condition parts and non-fuzzy conclusions of the form “if X is A_i and Y is B_i then $Z = c_i$,” by computing the following output when $X = x_0$ and $Y = y_0$ is observed

$$Z = \frac{\sum_i \gamma_i \cdot c_i}{\sum_i \gamma_i} \quad (7)$$

where $\gamma_i = \min(\mu_{A_i}(x_0), \mu_{B_i}(y_0))$, $i = 1, n$. Again, this kind of “inference” (which is widely used in fuzzy control) has nothing to do with uncertainty handling, since only

an interpolation between typical conclusions is performed, based on degrees of similarity between the input (x_0, y_0) and the prototypical values in the core of the fuzzy set $A_i \times B_i$. This similarity is measured by the coefficients γ_i which cannot be considered as degrees of uncertainty in any case. In spite of its apparently ad hoc nature, Equation 7 can be justified with one-premised rules using Equation 6 and viewing the rules as expressing “the more X is A_i and Y is B_i , the closer Z is to c_i ” and using appropriately shaped membership functions.¹²

As this shows, contrary to Elkan’s claim, some kinds of reasoning, as exemplified by Takagi and Sugeno’s, and Lee’s methods, can be handled in a De Morgan algebra framework.

Possibility theory and uncertainty. In

addition to modeling the gradual nature of properties, fuzzy sets can be used to represent incomplete states of knowledge. In this second use, the fuzzy set plays the role of a possibility distribution that provides a complete ordering of mutually exclusive states of the world according to their respective levels of possibility or plausibility. For instance, if we know only that “John is tall” (but not his precise height), where the meaning of “tall” is described, in context, by the membership function of a fuzzy set (that is, μ_{tall}), then the greater $\mu_{\text{tall}}(x)$ is, the greater the possibility that $\text{height}(\text{John}) = x$; the smaller $\mu_{\text{tall}}(x)$ is, the smaller this possibility.

Given a $[0, 1]$ -valued possibility distribution π describing an incomplete state of knowledge, Zadeh⁴ defines a so-called possibility measure Π such that

$$\Pi(A) = \sup\{\pi(x), x \text{ makes } A \text{ true}\} \quad (8)$$

where A is a Boolean proposition (a proposition that can only be true or false). It can be easily checked that for Boolean propositions A and B , we have

$$\Pi(A \vee B) = \max(\Pi(A), \Pi(B)) \quad (9)$$

but that we only have the inequality

$$\Pi(A \wedge B) \leq \min(\Pi(A), \Pi(B)) \quad (10)$$

in the general case (equality holds when A and B are *logically independent*). Indeed if $B \equiv \neg A$, $\Pi(A \wedge B) = \Pi(\perp) = 0$, while $\min(\Pi(A), \Pi(\neg A)) = 0$ only if the information is sufficiently complete for having either $\Pi(\neg A) = 0$ (A is true) or $\Pi(A) = 0$ (A is false). If nothing is known about A , we have $\Pi(A) = \Pi(\neg A) = 1$. By duality, a necessity measure N is associated to Π according to the relation (which can be viewed as a graded version of the relation between what is necessary and what is possible in modal logic)

$$N(A) = 1 - \Pi(\neg A) \quad (11)$$

which states that A is all the more necessarily true as $\neg A$ has a low possibility to be true. It entails

$$N(A \wedge B) = \min(N(A), N(B)) \quad (12)$$

and

$$N(A \vee B) \geq \max(N(A), N(B)). \quad (13)$$

Equations 9, 11, and 12 should not be confused with Equations 2, 3, and 1, respectively. In 9, 11, and 12 we deal with Boolean propositions pervaded with uncertainty due to incomplete information, while 1–3 pertain to non-Boolean propositions whose truth is a matter of degree (the information being assumed to be complete). Very often, discussions about fuzzy expert systems or uncertain knowledge base systems get confused because of a lack of distinction between degrees of truth and degree of uncertainty. Fuzzy logic, as understood by Elkan, is a logic where the truth status of propositions is multiple-valued; that is, there are intermediary truth values between true and false (like “very true,” “rather true,” and so on). On the contrary, degrees of uncertainty apply to all-or-nothing propositions, and do not model truth values but express the fact that the truth value (true or false) is unknown. The uncertainty degrees then try to assess which one of “true” or “false” is the most plausible truth value. This distinction was made by one of the founders of subjective probability theory — De Finetti¹³ — but with a few exceptions (including ourselves) it has

been quite forgotten by the AI community in general and by Elkan in particular. Still, we consider this distinction a crucial prerequisite in any discussion about fuzzy sets and possibility theory and their use in automated reasoning.

Observe also that neither Π nor N are fully compositional with respect to \wedge , \vee , and \neg . This is not surprising, since the only way to map a Boolean structure on $[0,1]$ by a fully compositional mapping f is to have $f(A)$ equal to 0 or to 1 for any A .¹ Truth-functionality in Equations 1–3 is preserved only by having A and B elements of a weaker structure, namely, a De Morgan algebra. Thus, logics of uncertainty cannot be fully compositional with respect to uncertainty degrees. This point is also recognized by Elkan in the case of probability measures, and dates back at least to De Finetti in the 1930s! Partial compositionality is possible, however; probabilities are compositional with respect to negation, possibilities with respect to disjunction, necessities with respect to conjunction. Based on his article, however, it seems that Elkan has not heard about possibility theory, which is another side of fuzzy sets.

Let us consider Elkan's watermelon example:

$$\begin{aligned} \text{watermelon}(x) &\equiv \\ \text{redinside}(x) \wedge \text{greenoutside}(x) \end{aligned}$$

It is supposed that "for some melon m , evidence that m is red internally has strength 0.5, and m is green externally with strength of evidence 0.8." It is not clear what Elkan means by "strength of evidence" in the light of the above comments. We shall assume they are indeed degrees of uncertainty, rather than degrees of red and degrees of green. But then the only way to anchor this discussion in the fuzzy logic debate is to interpret these degrees in possibility theory. Elkan's watermelon sentence can be understood as $N(\text{redinside}(m)) \geq 0.5$ and $N(\text{greenoutside}(m)) \geq 0.8$, expressing that the available information makes us certain to the degree of 0.5 that m is red inside, and to the degree 0.8 that it is green outside. A direct application of Equation 12 leads to

$$N(\text{watermelon}(m)) \geq \min(0.5, 0.8) = 0.5,$$

a result also obtained under an equality form by Elkan by applying Equation 1 in an inappropriate way. However, he would like to conclude that " m is a watermelon with strength of evidence over 0.5." This seems a strange requirement, and one that a probabilistic model would not satisfy either (since $\text{Prob}(A \wedge B) \leq \min(\text{Prob}(A), \text{Prob}(B))$). Indeed, we are not in a data fusion situation where two independent sources provide the same conclusion with various strengths,¹⁴ but in a situation where the logical conjunction of two conditions is required to conclude that m is a watermelon (namely the inside redness of m and its outside greenness). Note that in case we have both $N(A) \geq \alpha$ and $N(A) \geq \alpha'$ as obtained from distinct arguments, we shall conclude that $N(A) \geq \max(\alpha, \alpha')$.

Reasoning with possibility theory. In possibilistic logic, first-order logic formulas are weighted by lower bounds of necessity or possibility measures, which reflect the uncertainty of the available information. Possibilistic logic^{2,5} has been developed both at the syntactic level, where there is an inference machinery based on extended resolution and refutation (the lower bound of the resolvent clause necessity is the minimum of the lower bounds of parent clauses necessity measures), and at the semantic level, where a semantics in terms of a possibility distribution over a set of classical interpretations has been proved to be sound and complete with respect to the syntax. Due to the fact that a possibility distribution encodes a preferential ordering over a set of possible interpretations, possibilistic logic has been shown to capture an important class of nonmonotonic reasoning consequence relations¹⁵ and has capabilities for handling partial inconsistency in knowledge bases.⁵ Moreover, possibilistic assumption-based truth maintenance systems¹⁶ based on possibilistic logic have been defined for dealing with uncertain justifications and ranking environments in a label; they have been successfully applied to a data-fusion application.¹⁷

However, possibility theory offers more general applications to reasoning with uncertain, imprecise, or fuzzy pieces of information by manipulating possibility distributions explicitly. An example of these reasoning capabilities is provided by the so-called generalized modus ponens,¹⁸ which from a fuzzy fact " X is A " (represented by a possibility distribution $\pi_X = \mu_{A'}$) and a fuzzy rule "if X is A then Y is B " (also represented by a possibility distribution $\pi_{Y|X}$), enables us to infer the possibility distribution restricting the possible values of Y by combining π_X and $\pi_{Y|X}$ and projecting the result on the domain of the variable Y . According to the multiple-valued logic implication \rightarrow used to compute $\pi_{Y|X}$ from μ_A and μ_B , different kinds of fuzzy rules can be modeled. In particular, we can distinguish, for example, between the purely gradual rules already mentioned (of the form "the more X is A , the more Y is B ") and certainty rules of the form "the more X is A the more certain Y is B ." Thus, graduality can also be encountered in the expression of incomplete knowledge states pertaining to little-known relationships between variables (like the ones expressed by fuzzy rules).²

Expert systems with fuzzy rules have been designed that are not as simple as fuzzy controllers (where no chaining of rules is required, but only an interpolation between the conclusions of a parallel rules set). These expert systems, as expected by Elkan, do "knowledge-intensive tasks such as diagnosis, scheduling, or design," and include Cadiag-2,¹⁹ Taiger,²⁰ RUM,²¹ Milord,²² OPAL.²³ All these systems were or are used in applications in one of the above-mentioned fields. These systems use some form of fuzzy set or possibility-theory-based inference mechanisms that is much more sophisticated than the three formulas proposed by Zadeh in 1965 (Equations 1–3) — and to which fuzzy set and possibility theory methods cannot be reduced. There are many other important works on fuzzy set and possibility theory-based inference systems in temporal, qualitative, and abductive reasoning, that, for the sake of brevity, we do not mention here.

Fuzzy logic is not as simple as Elkan seems to believe. In this respect, the absence of any mention in Elkan's discussion of Zadeh's possibility theory and approximate reasoning approach^{4,18} is quite revealing.

In the literature, the expression "fuzzy logic" usually refers either to multiple-valued logic (as in the first part of Elkan's paper) or to fuzzy controllers. However, the two domains have very little in common, due to the fact that control engineers usually do not know about logic, and logicians do not know about control. In that sense, the first part of Elkan's article has very little relevance to his discussion on fuzzy control.

If the success of fuzzy logic is paradoxical, it is certainly not because of Elkan's collapsing property. More importantly, Zadeh's view of fuzzy logic seems to go far beyond multiple-valued logic, and is as much a framework for handling incomplete information as a methodology for capturing graduality in propositions. The concept of fuzzy truth values refers as much to the idea of a partially unknown truth value as to intermediate truth values. This is why we have emphasized the crucial distinction between the truth-functional handling of gradual properties and the possibilistic treatment of uncertainty (which is not fully compositional).

It is certainly true that the huge quantity of fuzzy set literature — whose quality is unavoidably inconsistent — does not contribute much toward helping newcomers have a synthetic, well-informed, and balanced view of the domain. Fuzzy controllers have encountered great success by providing an efficient way of implementing an interpolative mechanism, not only in small, but also in very large and complex problems. However, this should not obscure other existing applications, and the great potential of fuzzy set and possibility theory for AI applications in general.

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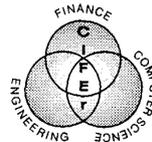
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Fuzzy Logic

An Interface Between Logic and Human Reasoning

Christian Freksa, University of Hamburg, Germany

Charles Elkan addresses two distinct areas of fuzzy logic: formal expressiveness and practical usefulness. He describes as a paradox that although the theory of fuzzy logic is not generally accepted, it is successfully used in many real-world applications. He also calls paradoxical the fact that these applications are predominantly found in the control domain.

I will not discuss here the alleged equivalence between fuzzy and two-valued logic; by choosing criteria established for the more restricted two-valued formalism, Elkan does not have a suitable framework for a meaningful comparison. To point out prerequisites for the practical usefulness of knowledge representation formalisms, I will focus on the role of fuzzy logic in linking two formally incommensurable worlds: the natural world of human perception and experience that leads to subjective cognitive concepts, and the formal world of classical logic that yields universal truth conditions.

Given the premise that there is no one-to-one mapping between human conceptual structures and the framework of classical logic, it is not important for the analysis of a formal representation structure if two logically equivalent expressions are evaluated identically; what we have to ensure is that derivations accepted in human reasoning can also be derived in our formalism.

Classical logic and human knowledge. In AI, propositions and various kinds of logic formalisms serve to represent and derive knowledge about formal or real domains. Traditionally, most effort has been put into the development of logically correct and consistent operations *within the formal representation*; however, little attention has been paid to the correspondence problem

between the structure of these propositions and operations, on one hand, and the knowledge structure they are supposed to represent, on the other. When we represent formal domains (for example, card games or mathematical theorems), establishing this correspondence may not cause major problems. However, when we represent knowledge about a real domain, the correspondence between our formalism and the represented structure becomes a major issue.

A representation system consists of:

- a represented world, and the relations and operations in it;
- a representing world, and the relations and operations in it; and
- the correspondence between the two worlds.¹

When representing knowledge about the *real* world, it is inherently impossible to prove something about the represented real-world knowledge; this part of the representation system is outside the formalism. We only can prove something within the representing formalism. Thus, the represented real world and its representation are *formally incommensurable*.

In expert systems, the knowledge engineer establishes the correspondence between the real and formal worlds, but he cannot prove its correctness; he depends on his perception and intuition to determine the equivalence between the two. Usually, a knowledge engineer relies upon assumptions to determine the validity of operations on a representation. These assumptions stem from his knowledge about formal logic, rather than from knowledge about specific properties of human reasoning. Nevertheless — as Elkan's article shows — this approach appears to be widely accepted for the treatment of human knowledge.

One of Lotfi Zadeh's main motivations for introducing the notions of fuzzy sets and fuzzy logic was his observation that real-world knowledge generally has a different structure and requires different formalization than existing formal systems. Contrary to established practice, a one-to-one correspondence between natural-language propositions and predicate calculus propositions can be shown to be inadequate. In particular, the instantaneous switch from truth to falsity can easily distinguish propositions in classical logic from those in natural language. In addition, numerous assumptions of the formally correct treatment of the propositions cannot be established in the corresponding source knowledge.

The fuzzy logic interface. Zadeh recognized the power of a formal approach to knowledge processing as well as the advantages of using soft knowledge in human reasoning. He thus took a first step in incrementally relaxing constraints imposed on existing formalisms to accommodate important properties of natural inference. This step was to generalize the classical notion of a set to the notion of a fuzzy set that allowed gradual membership. The choice of numerical degrees of membership was largely made for formal reasons: it provided a transparent way of formally treating the new notion. Using the familiar language of mathematics, the theory can easily be implemented in computer systems, while at the same time offering a better approximation to the associated human concepts.

Because human notions and concepts form the basis for reasoning in expert systems, the success of these systems depends upon the correspondence relation between human concepts and their formalization.

Studying the formal properties of the representation is insufficient.

Zadeh realized that it was much more important to have a good model of the semantics of human concepts and perform reasonable operations than to have a bad model and perform verifiably correct operations. He never insisted that his initial proposal for a fuzzy logic should be viewed as the final solution for representing human knowledge about the world; rather, he offered a model based on established notions that could easily be grasped by engineers and researchers alike as a step toward formalizing human reasoning. Because of this, Zadeh's basic notion of a fuzzy set stimulated enormous research activity in soft knowledge processing.

Zadeh's work also helped establish a radically different view of the status of expert knowledge. No longer is it viewed as a collection of absolute truths piped into an inference engine to derive all sorts of unexpected results; rather, it is now considered as a system of more or less soft constraints that are applied to specific situations to make reasonable decisions.

Soft knowledge is processed differently than logic clauses — the reasoning power is typically due to processing breadth rather than depth. The ability to use shallow processing to merge knowledge from different sources produced useful decisions. (Elkan uses the terms "deep" and "shallow" in two different senses: to distinguish general knowledge from specific knowledge, and to distinguish extensive and restricted knowledge propagation. I use the terms here in the second sense, which is the usual sense.) Elkan appears to attribute the fact that fuzzy systems employ only a few rules to the domain's simplicity. However, this fact can also be attributed to the important capability of summarizing complex knowledge into a dense and transparent description.

Success and limitations. The fuzzy set paradigm introduced a new concept of soft knowledge that helped characterize an important aspect of knowledge about complex environments. It also provides a language to bridge the gap between soft and shallow

knowledge, on the one hand, and systematic and formal methods for dealing with it, on the other. This contribution might have a much more significant impact on human thought and the role of classical logic in systems analysis than the fuzzy set notion will have on the success of expert systems.

As the transition from crisp sets to fuzzy sets is a rather moderate step toward accounting for the nature of human concepts, we should not expect it to solve all our problems. In particular, fuzzy sets and fuzzy logic do not answer the fact that human concepts develop and are modified in an open world, while formal concepts are fixed in closed worlds, for the most part. Therefore, it is not surprising that successful applications of fuzzy logic are so far found mainly in well-defined closed domains like control problems which, to a large extent, share the properties of synthetic, formal problems. The way gradual membership is represented in fuzzy sets quite naturally suits such application domains.

The further we move from representing human knowledge about clearly delineated problems to representing concepts relating to open domains, the more we will have to overcome certain rigidities of the classical formal approaches.

Classical logic has proved extremely useful for solving formal problems specified in two-valued terms. Fuzzy logic is proving particularly useful for quasi-formal problems involving gradual transitions between various system states. For adequately formalizing less rigid domains, like the open world of human fuzzy concepts, we must relax the constraints on the formalisms even more. Specifically, numerical graduation of membership used in classical fuzzy logic is hardly justified for the representation of cognitive concepts; instead, less constraining ordering relations like partial orderings may be appropriate.

Considering the fact that it took 25 years to put fuzzy logic into wide use in the well-understood engineering domain of control,

we should not be surprised if some barriers must be removed before fuzzy logic will be widely applied to more delicate areas of fuzzy reasoning.

For judging the quality of a representation formalism, I have proposed taking a representation-theoretical viewpoint: The correspondence between the represented domain and the formalism is at least as important as the representation's formal properties taken by themselves. This viewpoint permits a high-level characterization of the overall representation problem. I have also argued that real-world knowledge and formal knowledge are formally incommensurable. As long as the laws of human reasoning are not well understood, a good model of human reasoning should be expected to preserve some paradoxes; experimentation with the model may deepen the understanding and help resolve them.

Acknowledgments

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Known Concerns About Fuzzy Logic

Oscar N. Garcia, George Washington University

I thank Charles Elkan for bringing into the open questions about fuzzy inferences that seem to bother him and others. I hope the result of this discussion will be a clearer understanding of many-valued logics in general, and fuzzy logic in particular. My comments address three topics: questions about Theorem 1, the "watermelon" example, and the issue of fuzzy logic in control.

Much of the confusion surrounding Theorem 1 stems from its rather unclear statement. I interpret the theorem to say the following:

"Let $f1(A,B) = \neg(A \text{ AND } \neg B)$ and $f2(A,B) = B \text{ OR } (\neg A \text{ AND } \neg B)$. Using Definition 1, if one were to require the following four equivalences —

- (1) $f1(A,B) \leftrightarrow f2(A,B)$ AND
- (2) $f1(\neg A,B) \leftrightarrow f2(\neg A,B)$ AND
- (3) $f1(A, \neg B) \leftrightarrow f2(A, \neg B)$ AND
- (4) $f1(\neg A, \neg B) \leftrightarrow f2(\neg A, \neg B)$

— then such a logic system would also require that $t(A) = t(B)$ or that $t(A) = 1 - t(B)$ "

I can prove this supposition or "theorem" following the valuation t of Definition 1 for values of A and B in the interval $[0,1]$. Such valuation yields validity for the first of the four equivalences above except for $0 < t(A) < t(B) < 1 - t(A)$ when $t(f1) - t(f2)$ has the value $t(B) - t(A)$ if $t(B) < 1/2$ and the value $1 - t(A) - t(B)$ if $t(B) > 1/2$. The area where the equivalence is not satisfied is an isosceles triangle in the square $[1,0] \times [1,0]$ not including the isosceles sides. Similarly, for the other three equivalences, the non-overlapping triangles where the equivalences are not satisfied would cover the whole unit square — except for the isosceles sides, which constitute the two diagonals of the square. Thus, either $t(A) = t(B)$ in one diagonal of the square, or $t(A) = 1 - t(B)$ in the other.

Where is the catch? First, the fourth line of Definition 1 in Elkan's paper indicates that each side of a "logically equivalent" formula has the same evaluation. This is not a fair imposition, and Elkan need not choose such a formula to make his point. Just consider requiring the valuation of two "logically equivalent" formulas:

$$(5) \quad t(A \text{ AND } \neg A) \leftrightarrow t(\neg(A \text{ OR } \neg A))$$

which, of course, only occurs in the bivalent case following Definition 1. Equivalences are tautologies, and while the arguments of t on each side of equivalence 5 are "logically equivalent" in classical logic, they are not so in fuzzy logic where the law of the excluded middle does not hold.

Thus, it is not surprising that the attempt to evaluate these formulas using the classical bivalent logic interpretation of "logical equivalence" would not yield sound results. It can be easily shown that the manipulation of $f1$ or $f2$ in classical logic leads to a disjunction of a variable and its complement. We should not take a tautology that supports a rule base in one logic, use it in another logic that does not support that tautology, and expect it to work — and then go on to claim that a "collapse" of one logic to another has been proved. The requirement of "logical equivalence" in Definition 1 is therefore suspect. Elkan raises the question of why it is that intuitionistic logic is capable of rejecting the law of the excluded middle while fuzzy logic is not. While this is not directly relevant to the claimed "collapse," it is clear that intuitionistic logic is not used to the extent that fuzzy logic is used in controller design.

Another issue that might be troubling Elkan — implicit in his choice of the functions called $f1$ and $f2$ in my interpretation

of his Theorem 1 — is what deductive tautologies (those involving implication, and particularly those known as the inferential implication tautology¹) should be used in fuzzy logic if limited by Definition 1. This question is worthy of investigation, and has led to multiple alternatives to Zadeh's original definition of implication; however, it is beyond my concern here. My acquaintance with expert systems applications indicates that, in practice, value sets are categorized as designated (truth-like), antidesignated (false-like), and neutral (those for which insufficient knowledge exists for the model to be useful). A typical example for the real interval $[0,1]$ would be antidesignated $A = [0,0.4)$, neutral $N = [0.4, 0.6]$, and designated $D = (0.6,1]$. (The complement of a designated value is antidesignated and vice-versa, while neutral values are the complements of other neutral values.) The object of expert systems is to mimic, as closely as possible, the reasoning of expert humans in terms of the best causality relations known to them, and to incorporate them in a knowledge-based model, often represented as a rule base.

Because fuzzy logic is known as a "normal" logic (all of its truth-assuming tautologies are included in classical logic, though the converse is not true, as can be shown in the case of equivalence 5) the puzzlement shown by Elkan is not novel. Indeed, the tautology involving the law of the excluded middle from classical bivalent logic does not hold in fuzzy logic, nor in many other normal many-valued logics. For those systems in which operations involving the designated and anti-designated defined sets coincide with those of classical logic, Shaw has enumerated the possible homomorphisms of any ordered designated system into one of 12 groups defined by the conjunction table of their designated (D), antidesignated (A), and neutral (N)

subsets.² (A deeper and more thorough algebraic approach to the theory of many-valued logics, including fuzzy, intuitionistic, and probabilistic inferencing is given by Bolc.³) For example, Shaw shows that the table for element conjunctions from these fuzzy logic subsets is

δ	A	N	D
A	A	A	A
N	A	N	N
D	A	N	D

This table is also valid for Lukasiewicz's n -valued logic, which, as one would expect, shares many analogies with fuzzy logic. The Lukasiewicz's logics are first defined in terms of negation and implication, and other operations are defined in terms of these two. The table of conjunctions above is also valid for Kleene's (strong) three-valued logic based on negation, conjunction, and disjunction operators. Kleene's logic has no true-assuming tautologies (formulas that always assume the highest truth value if more than one designated value is available). If only the operators of Definition 1 are used, and different independent operators are defined as part of fuzzy logic, then Elkan's point that only DeMorgan-like tautologies are possible in fuzzy logic is well taken, but of no great consequence as long as viable deductive laws are available. (A discussion of what those deductive laws could be and their relation to implication is treated nicely by Trillas, who characterizes a generic "modus ponens generating function."¹) As these references point out, there is not only practical but theoretical credibility to the inferences proposed for fuzzy logic well beyond the limitation to DeMorgan equivalences suggested by Elkan. Elkan's acceptance of the so-called collapse as an established fact in his conclusions could be considered disingenuous by the finality with which he considers the hypotheses of Theorem 1 to be "apparently reasonable conditions."

The watermelon problem. Elkan's version of this example is a revision of the 1993 AAAI conference publication, where the watermelon model was in error because

it was incomplete. The problem I find here is not directly related to the logic, but rather to the use and interpretation of the model. Elkan has not expressed what he calls "implicit" background knowledge in terms of rules. While he uses the equivalence operator to define *watermelon(x)*, a knowledge engineer trying to identify watermelons might have given two rules (with different logical meaning) from (*insidered(x) → watermelon(x)*) AND (*outsidegreen(x) → watermelon(x)*) to indicate that the two predicates contribute separately to the implication of *watermelon(x)*.

Alternatively, the engineer might have given the one rule with the conjoined antecedents. In the former case, in many expert systems shells the connotation that the conditions *insidered(x)* and *outsidegreen(x)* give "separate" results to argue the consequent from two different viewpoints would yield a heuristic function of the two valuations — somewhere between the values of *insidered(m)* and of *outsidegreen(m)*. If the knowledge engineer had selected the model (*insidered(x) AND outsidegreen(x) → watermelon(x)*) for this example, it would connote the necessity to satisfy "simultaneously" both related conditions, and it can be argued that the conservative answer would be the "weakest link" answer (the minimum of the two valuations).

Fuzzy logic in control. It seems reasonable that the longer a chain of implications with uncertain predicates is — whatever the definition of the approximate deductive law — the more uncertain the result at the end of the chain will be (as in computing the range of values in worst-case designs). So it seems that it would be a good thing, in general, to have short inference chains and a small number of rules whenever possible. Furthermore, the fuzzifying and defuzzifying that takes place at times reminds me of the reshaping done in the analog transmissions of digital pulses to avoid signal deterioration through consecutive repeaters to distort information.

The fact that so many applications have

been possible with short inference chains raises more interesting questions yet: Under what circumstances are long chains of inferences indispensable? How could long chains of inferences be avoided? However — make no mistake — even a set of one-layer rules requires some form of inference, and rule sets will increase their sequential complexity when hysteresis is taken into account.

Elkan repeats conventional wisdom when stating, "The basic problem is that the ways in which items of uncertain knowledge are combined must be carefully controlled to avoid incorrect inferences. Fixed, domain independent operators . . . do not work" to which I add: regardless of the logic system. We should not expect to find an exact function f such that $t(A*B) = f(t(A), t(B))$ for a logical operator $*$ unless we know either the functional relations of occurrence between A and B or, equivalently, know that they are independent (and if that were the case, an exact analytical model could be built!) It is then not surprising that knowledge engineering and incremental learning methods are used in conjunction with parameter determination to compensate for this lack of generic knowledge, not the weakness of a logic system. So, what is new? The dogma of generality versus efficiency strikes again, and knowledge engineering and machine learning are not exempted.

Elkan's ability to generate interest in both the topic of nonclassical logics for AI and the need for more general understanding of many-valued logics and basic research on how it is applied, are important contributions that should be acknowledged. It is a good thing that the relatively smooth imprecisions of natural-language semantics — when contrasted with crisp symbolic approaches — are available without excessive complexity when simpler, closed-form, and linear designs are not forthcoming. This occurs frequently around those transitional regions of system operation where decision changes interface, and points to the value of vagueness in processing natural language — usually considered in the negative — as a useful, approximate,



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Elkan Goes Wrong — Again

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Elkan's article has three basic parts: a mathematical part consisting of one definition and one theorem; a discussion of the roles played by fuzzy logic in expert systems and control systems (based upon the mathematical part); and his appraisal of the roles of fuzzy logic and its likely significance in the future. Here we discuss the major fallacies we found in the first two; due to space limitations, we will not address the third, though we disagree with almost all of the author's opinions.

As is well known, Elkan's article is a revised version of his original paper, published last year. These two versions are not fully compatible, especially in the mathematical part. Here, we point out discrepancies between the two versions and address both alternatives.

In Definition 1, Elkan introduces a particular system of fuzzy logic by choosing the standard fuzzy operators for conjunction, disjunction, and negation, and by requiring that " $t(A)=t(B)$ if A and B are logically equivalent," where $t(A)$ and $t(B)$ are, respectively, the degrees of truth of arbitrary propositions A and B . Clearly, $t(A)$ and $t(B)$ are values in $[0,1]$. In the original paper, the term "logically equivalent" is defined as "equivalent according to the rules of classical two-valued propositional calculus." This is, of course, nonsense, since one logic system (in our case, a particular system of fuzzy logic) cannot be defined in terms of logical equivalence of another system (the more restrictive classical two-valued logic).

In the revised version, the meaning of the term in Definition 1 is not explicated. It is only remarked that "depending on how the phrase 'logically equivalent' is understood, Definition 1 yields different formal systems." Since the role of Definition 1 is to characterize a system of fuzzy logic, logical equivalence in this definition must be expressed in terms of all possible truth values of fuzzy propositions, that is, in terms of all real numbers in $[0,1]$. Specifically, two expressions in fuzzy logic based on the opera-

tors of Definition 1 are logically equivalent if and only if their truth values are equal for all possible assignments of truth values in $[0,1]$ to logic variables involved.

The principal result (and the only mathematical result) of Elkan's papers, which purport to demonstrate "technical limitations of fuzzy logic," is Theorem 1. What is this result? The answer depends on which version of the paper you use. In the original version:

For any two assertions A and B , either $t(B)=t(A)$ or $t(B)=1-t(A)$.

The theorem is supposed to apply to the system of fuzzy logic introduced by Definition 1. However, as explained above, the definition is based on the logical equivalence of two-valued logic and hence it is not a definition of a fuzzy logic system. The proof of the theorem is based on the fact that the sentences $\neg(A \wedge \neg B)$ and $B \vee (\neg A \wedge \neg B)$ (and seven other pairs of sentences obtained by exchanging and complementing A and B) are logically equivalent in classical two-valued logic. However, these sentences are not equivalent in a fuzzy logic that employs the logic operators of Definition 1. Hence the theorem has no relevance to this fuzzy logic. Let us turn now to the revised version of the theorem:

Given the formal system of Definition 1, if $\neg(A \wedge \neg B)$ and $B \vee (\neg A \wedge \neg B)$ are logically equivalent, then for any two assertions A and B , either $t(B)=t(A)$ or $t(B)=1-t(A)$.

The fundamental difference between the original and revised version of the theorem reflects the difference in the two versions of Definition 1. In the revised version, the logical equivalence of $\neg(A \wedge \neg B)$ and $B \vee (\neg A \wedge \neg B)$ is employed as a condition in stating the theorem rather than a fact in proving it. If the notion of logical equivalence in the revised Definition 1 is understood as applying to all truth values in $[0,1]$, in spite of its confusing characterization by the author (as discussed above), then the revised version of Theorem 1 is relevant

to the fuzzy logic involved. However, the theorem is still stated incorrectly or, alternatively, its proof is incorrect. The proof depends on eight logical equivalencies, only one of which is included in the statement. The last paragraph of the proof is thus mathematically incorrect. It would be correct if logical equivalencies representing the seven implications listed in the paragraph were included as conditions in the statement of the theorem. Without these seven logical equivalencies as conditions, the theorem must be reformulated as follows:

Given the formal system of Definition 1, for any two assertions A and B , if $\neg(A \wedge \neg B)$ and $B \vee (\neg A \wedge \neg B)$ are logically equivalent, then the truth values $t(A)$ and $t(B)$ are constrained by the inequalities $t(A)+t(B) \geq 1$ or $t(B) \leq t(A)$.

In this case, the last paragraph of Elkan's proof is incorrect and must be excluded.

Assume that the statement of Theorem 1 and its proof are made compatible in one of the two ways we suggest. What then is the meaning of the resulting theorems — one with the single logical equivalence as a condition, and one with the eight logical equivalencies as conditions? These theorems basically show that the truth values of propositions within the system of fuzzy logic introduced by Definition 1 become appropriately constrained when additional extraneous conditions are imposed. With the eight conditions, the constraint is obviously more severe than with only one of them. If, for example, we required our system to satisfy $A \vee \neg A = 1$ then the truth values would become constrained to the set $\{0,1\}$, and the system would collapse to the classical two-valued logic.

All this is well known, and Elkan's theorem (when properly fixed) does not offer anything new. It is absurd, however, to constrain a system by extraneous requirements and then claim that the original system has "technical limitations." This is what Elkan attempts to do in his papers. The fact that every system of fuzzy logic must violate,

under the assumption of truth functionality, some properties of Boolean algebra (and, hence, the classical two-valued logic) is a simple consequence of the decision to formulate logics that can deal with propositions that are not required to be either true or false, but may be true or false to various degrees.¹

Elkan's remarks about the connection between fuzzy logic and intuitionistic logic also contain some errors. For example, it is not sufficient to characterize fuzzy logic by the rejection of the law of excluded middle. The system of fuzzy logic determines which properties of Boolean algebra are rejected. The system introduced by Definition 1, for example, rejects not only the law of excluded middle, but the law of contradiction as well.² This differs from intuitionistic logic, which rejects the law of excluded middle and the implication $\neg\neg A \rightarrow A$, but does not reject the law of contradiction and the opposite implication $A \rightarrow \neg\neg A$.^{3,4} Other systems of fuzzy logic do not reject any of the mentioned laws; instead, they reject distributivity and idempotence.² Furthermore, de Morgan's laws are valid only in some systems of fuzzy logic. Another error is to consider the logical equivalence in Definition 1 as intuitionistic equivalence. A distinctive feature of intuitionistic logic is the operator of negation upon which it is based. For any proposition A , where $t(A) \in [0,1]$, the intuitionistic negation, $\neg A$, is defined by

$$t(\neg A) = \begin{cases} 1 & \text{when } t(A) = 0 \\ 0 & \text{otherwise} \end{cases}$$

This negation is not involutive, nor is it continuous — it acts as a defuzzifier. Clearly, there is no compatibility between intuitionistic logic and the fuzzy logic introduced in Definition 1.

Fuzzy logic applications. Elkan's discussion of fuzzy logic in expert systems reveals his confusion between degrees of truth in fuzzy logic and degrees of evidence expressed in terms of some fuzzy measures (probabilities, belief measures, and so on).⁵ While the former are a matter of compatibilities of given objects with relevant fuzzy predicates, the latter result from information deficiency regarding the classification of a given (incompletely characterized) object in

relevant crisp sets. While these two areas have distinct application domains, they can be combined, resulting in statistics with imprecise probabilities⁶ or in fuzzified evidence theory,⁷ for example.

In his discussion of fuzzy controllers, Elkan's lack of understanding is again revealed. He fails to understand that most of the simple fuzzy controllers on the market (we may call them the first generation of fuzzy controllers) are not explicitly based on fuzzy logic, but rather on the approximation of relevant control functions by fuzzy numbers that represent chosen linguistic states of the variables involved. This is similar to classical control, which is also not explicitly based on classical two-valued logic. It is well established that fuzzy controllers of this kind are universal approximators.^{8,9}

While most existing fuzzy controllers are rule based, research on combining rule- and model-based approaches in designing fuzzy controllers is ongoing. Models employed in these controllers are expressed, in general, in terms of relations among relevant fuzzy variables. Hence, the use of fuzzy set theory (not necessarily fuzzy logic in the narrow sense) involves both parts of the controller — the rule-based part as well as the model-based part.

Elkan's papers do not contribute to knowledge. The mathematical part is fallacious; and, while some critical errors in the original version are corrected in the revised version, new errors are introduced and some statements become less specific. Even if we fix all the mathematical errors to help Elkan obtain his intended result, we find only that the result is trivial and well known: If one takes an axiomatic system and adds to it additional requirements, the system becomes more constrained. Given a free choice of requirements, one can constrain the system as he or she wishes. This is precisely what Elkan attempts, in an amateurish way. He tries to find requirements that would constrain a given system of fuzzy logic so severely that only two truth values are allowed. He then argues that this shows technical limitations of fuzzy logic. This sort of argumentation is absurd.

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Fuzzy Control

A Misconception of Theory and Application

E.H. Mamdani, Queen Mary & Westfield College, London

The argument in Charles Elkan's article has three steps. First, he provides a theorem that "proves" that fuzzy logic is deficient because it collapses to a two-valued logic. He then shows what makes the current applications of fuzzy logic successful, although this success may seem paradoxical. His final step shows how such a success cannot be guaranteed as applications scale up in the future — thus resolving the paradox. I expect other commentaries will deal with the misconceptions regarding the theorem. I focus attention here on the remainder of Elkan's argument.

The source of Elkan's paradox is the link between the first two steps formed by his statement that "One way to defend a calculus is to show that it succeeds in interesting applications." But first a couple of very different but relevant pointers.

Fuzzy logic control is successful because it replaces the classical PID controller. When tuned, the parameters of a PID controller affect the shape of the entire control surface. Because fuzzy logic control is a rule-based controller, the shape of the control surface can be individually manipulated for the different regions of the state space, thus limiting possible effects to neighboring regions only. Furthermore, the use of fuzzy mathematics provides interpolation between the adjoining regions, resulting in an overall smooth control surface — an important requirement in the control of continuous systems. This also suggests that fuzzy sets are an efficient way of representing continuous variables in rule-based systems.

Secondly, I have always felt that fuzzy logic has similarities with Boole's logic. That logic, originating over 150 years ago, was the first system of reasoning in the

form of a calculus. However, after reading Boole's "Laws of Thought" it is difficult to discern whether Boole is concerned with a descriptive explanation of how people actually think, or with a prescriptive model of how they ought to think. AI research workers have seldom addressed this key distinction properly.

Within AI there are three distinct areas of research: the descriptive, the prescriptive, and what I call the applicative concerns. In the first area, researchers deal with descriptive theories about cognitive processes. These theories are very hard to prove experimentally (or more specifically to disprove experimentally — if one applies the Popperian view) because the level of control in experimental studies on human cognition is far below that in the natural sciences. The second group of researchers are concerned with prescriptive models: different reasoning systems and a variety of logics. Here, the issue is one of correctness of these models, variously defined. Again, it is not possible to use natural-science methods to devise controlled experiments that demonstrate the correctness of these models; correctness can only be dealt with by means of philosophical arguments (more on this later).

I belong to the third group of AI researchers, whose main concern is to build industrially successful artifacts. Such artifacts are successful in their own right, and do not owe their success to the underlying theory or a mathematical model. It is sad how many AI workers have lost the ability to distinguish between applications and well-designed controlled experiments set up to disprove a particular theory. Applications address the scientific needs of a specific domain, and cannot replace experiments conducted to test a theory. Many

features of the domain knowledge introduced in an application also contribute to its success. There is a common misconception that models are created and then applied, and that success then legitimizes a model. This view is superficial, because an application's requirements seldom match the underlying axioms of the model exactly. The fixes that are added (defuzzification in fuzzy logic control) are instrumental in the industrial success — but often sit uncomfortably in the original theory. This is true of all applications inspired by prescriptive models.

The links between these three groups (descriptive, prescriptive and applicative) must be properly understood if one is to avoid the methodological trap Elkan has fallen into. In AI, the work of each group inspires the direction of the others — but that is all they do. The results of one group can never be used to legitimize the approach of another. Weak though these links are, they still play a significant role in scientific advances. My point is not to belittle the interplay between the three areas, but to point out that a misunderstanding of their relationship is clearly the source of Elkan's perception of the paradox.

What then is the relationship between fuzzy logic control applications and fuzzy logic itself? Precisely the same as that between Boole's laws of thought (a descriptive theory?), Boolean logic (a prescriptive model?), and logic circuits (an application) — namely, an effective tool presented itself that met many, though surely not all, of the application needs. However, the widespread success of logic circuits cannot be used to legitimize Boole's logic any more than the industrial success of fuzzy logic control legitimizes the philosophical correctness of fuzzy logic. Therefore, the question of a paradox — a central idea in Elkan's paper



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— does not arise. Similarly, his argument on the philosophical deficiencies of fuzzy logic focuses on a theorem without fully discussing the assumptions and axioms it is based upon; this does nothing to argue against the adoption of fuzzy logic control. The terms “logic” in logic circuits and “fuzzy logic” in fuzzy logic control are purely incidental, and a matter of historical evolution.

The AI approach puts a much higher value on prescriptive mathematical models than they actually deserve. These models cannot be legitimized by controlled experiments or by application, nor can they be justified by some underlying descriptive theory (in spite of Boole). Prescriptive models can only be argued over at a philosophical level — an ability few AI researchers possess. Philosophical disputations about prescriptive models within informed groups such as Uncertainty in AI, have, nevertheless, helped to enlighten many difficult points. In the end, however, such disputations can never completely settle the matter.

Because AI researchers are mostly trained in mathematical skills, another frequently applied but false way of legitimizing prescriptive models is on the grounds of mathematical symmetries or some intrinsic sophistication of potential function. On rare occasions when models are abstracted from applications, the concern is no longer what led to the success of the application. Rather, the academic game of looking for the symmetries and the sophistication of the form or the soundness of the calculus begins.

Having rightly or wrongly detected a paradox, one then has to resolve it; in doing so, Elkan commits further errors. He has a lot to say about the small number of rules, the shallowness of fuzzy rule bases, and so on — implying that some beauty of the form often plays a significant role in assessing the worth of a model (and the intellectual enterprise of a researcher) rather than the content or industrial usefulness. To argue

that fuzzy logic control is not worthy of industrial consideration because of its lack of complex form and structural sophistication, as Elkan effectively does in the final part of his paper, is to subscribe to an anti-inventions culture. Accentuating form without attention to the content is like praising beauty and ignoring the brain. To use the colloquial term, the scientific mythology within AI has created a “bimbo science.”

The scenario worth keeping in mind is that since its inception, fuzzy logic has had its detractors and antagonists not least because the tag “fuzzy” is seen as debasing to the somber image of science. So incensed are some that they will clutch at any straw to rid us of fuzzy sets research, even through a paper based on mistaken interpretations and modish posturing. This scenario leaves me saddened, for reasons explained above.

It is the word “paradox” I find most baffling in Elkan's article. Science at its best is often counterintuitive; but paradoxical? Our accepted understanding of the scientific method is based on natural science and descriptive theories. But applying descriptive theories to computer science — which is dominated by prescriptive theories — cannot, in my opinion, work. New prescriptive theories often alienate many researchers, but they also inspire others to build novel applications. It may be that some of these applications are a runaway success. Rather than talking of “paradoxes,” what is required at this point is a rigorous attempt to discover the secret of that success. Because this investigation is descriptive in nature, the traditional scientific method is likely to yield dividends. In the case of fuzzy control, this process is now underway.

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Fuzzy Logic A Misplaced Appeal

Francis Jeffrey Pelletier, University of Alberta

I have long found puzzling the acceptance and apparent success of fuzzy logic. We philosophically oriented logicians have pretty much sneered at fuzzy logic ever since it was introduced with that name.* Yet what can I say when I own an excellent fuzzy logic camera? I am grateful to Elkan for his explanation of this point of tension.

Basically, Elkan explains that the notion of “fuzzy logic” as it is used in control systems has nothing to do with the term as it is used in logic. That is, it has nothing to do with fuzzy logic as a formal system with rules of formation, evaluation, and inference. Fuzzy controllers are so-called because of a certain *analogy* with fuzzy logic, but in fact they do not embody, implement, or instantiate fuzzy logic.

For Elkan, the relationship between fuzzy controllers and fuzzy logic is rather like that between on-off light switches and predicate logic: Yes, there is a certain analogy between on-off and true-false, but it's *only* an analogy, a way of looking at light switches. There is nothing in the light switch corresponding to the connectives of sentence logic nor to predicates, names, and quantifiers of predicate logic. To identify the two, or to say that light switches implement or instantiate predicate logic, would be to ignore most of predicate logic and mistakenly fixate on just one insignificant aspect. According to Elkan, we should not be surprised that critiques of fuzzy logic have no impact on fuzzy control theory; the areas of fuzzy logic that get criticized are simply not employed in the control arena (whether practical or theoretical).

Elkan's theorem shows one of the difficulties surrounding fuzzy logic as a formal

system. Supporters of fuzzy logic are without doubt tempted to respond to this by focusing on the assumption that logical equivalence in classical (or intuitionistic) logic is a warrant for formulas having the same truth value in fuzzy logic. I do not wish to enter this debate; instead, I will take this opportunity to point to some other features of a logical nature that have been used to criticize fuzzy logic and its claims of usefulness in various tasks.

Presentations of fuzzy logics have generally been semantic in nature, while the syntax — axioms and rules of inference — has generally been ignored. The basic semantic notion is that propositions can take any real value in $[0 \dots 1]$ intuitively corresponding to “degrees of truth” of the proposition. Many advocates of fuzzy logic, especially those who want to replace classical logic as the medium of representation for ordinary reasoning and the description of natural-language phenomena, would like to “use” the semantics of fuzzy logic. That is, they are not interested merely in asserting theorems, nor in the uninterpreted formulas of fuzzy logic, but rather would like to be able to claim that a proposition is true to a certain degree, that it can be compared to another proposition which is true to some different degree, and that certain conclusions can be drawn from this comparison.

For example, it might be that “Sally is wealthy” is true to degree 0.7 while “Mike is wealthy” is true to degree 0.4. Now, we might wish to draw certain conclusions from this information, such as that Sally is wealthier than Mike, or wealthier to a certain degree than Mike. To do this, we need some way to “use the semantics.” Technically speaking, we wish to have a kind of

“autodescriptivity” in the logic: a way of mirroring the semantics within the syntax. This autodescriptivity is regarded by some authors as necessary for the adequacy of *any* many-valued logic,² for without it, the apparent many-valuedness is only illusory because we cannot *say* anything in a many-valued way. There are a number of ways of accomplishing this, depending on what sorts of operators are available within the language. The direct way is to have so-called parametric operators in the language: For each k , where $0 \leq k \leq 1$, there is a unary sentence operator J_k . The truth of such sentences is evaluated thus:

$$\begin{aligned} t(J_k[\Phi]) &= 1, \text{ if } t(\Phi) = k \\ &= 0, \text{ otherwise.} \end{aligned}$$

That is, a J_k operator says that the formula it operates on takes exactly the value k . Although there are other approaches, I will adopt this direct approach — that the language being used to “express the semantics” contains the parametric operators directly.

(There are many other ways to get their effect. Some writers allow constants — symbols that denote the truth values — others have “threshold operators,” and so on. With suitable such other operators, we can indirectly define the parametric operators. Since there are innumerable truth values in the real range $[0 \dots 1]$, the methods of autodescriptivity mentioned here cannot really be applied. Instead, we must consider the fuzzy logic generated by the *rationals* in the $[0 \dots 1]$ interval. Attempts by fuzzy logicians to incorporate ever more inclusive — some would say obscure — operators indicates to me a lack of appreciation of what a logic is. For, if it can be shown that there is no algorithmic, deterministic procedure to determine the truth value of an arbitrary expression, then it is very unclear that there can be any use of the formalism.)

* It was studied by J. Lukasiewicz and A. Tarski in 1930 under the name “infinitely many-valued logics.”¹ and received intensive study by several mathematical logicians in the 1950s and early 1960s.²

Observation 1: Fuzzy propositional logic is not argument-complete. The first shortcoming of fuzzy logics concerns propositional logic (and hence any fuzzy logic, because they all contain propositional logic as a part). There is no theory of argumentation for fuzzy propositional logic such that whenever all premises of the argument are designated, then so is the conclusion. (Intuitively, some of the truth values are considered "good" or designated, while the others are undesignated. Exactly which ones are designated might vary from application to application. The point is that an argumentation theory is designed to take us from "good" premises to "good" conclusions, and never mislead us by deriving a "bad" conclusion from "good" premises. The present observation says that this cannot be done, ever; and this holds for any decision on what is designated, so long as at least one number is designated and at least one is undesignated.)

This result does not depend on there being (or lacking) any particular syntactic machinery around (other than the parametric operators); rather, there simply can be no such theory of argumentation. The proof of this is via the fact that fuzzy logics are not semantically compact. That is, it is *not* true for fuzzy logic that a set of formulas is satisfiable just in case every finite subset of it is satisfiable. For example the infinite set

$$\Gamma = \{ \neg J_k[p] \mid 0 \leq k \leq 1 \}$$

is not satisfiable, since the sentence letter p must take on one of the values $0 \leq k \leq 1$, whereas the membership condition in Γ says it doesn't. Yet any finite subset of Γ is satisfiable. Similar sets can be described using quantified sentences, such as

$$\Gamma' = \{ J_1[Fa_1], J_{1/2}[Fa_2], J_{1/4}[Fa_3], \dots, \neg J_0[\forall xFx] \}$$

Having noted this fact, it is an easy step to the conclusion that fuzzy logic, even fuzzy propositional logic, is not argument-sound, since all proofs are finite. Thus, there can be no adequate scheme for making inferences in general within fuzzy logic.

Observation 2: There is no normal form for fuzzy monadic predicate logic in which quantifiers have widest scope. The second shortcoming of fuzzy logic is found in the attempt to add quantifiers, even a simple monadic predicate logic. Fuzzy logic dictates that a universally quantified formula, such as $\forall xFx$, takes the least value of all the substitution instances for x in the formula Fx , or the greatest lower bound if there is no such least value. An existentially quantified formula takes the greatest value of all the substitution instances, or the least upper bound if there is no such greatest value.

A sentence like $J_k[\forall xFx]$ says that the greatest lower bound of the Fx 's values is exactly k . There is no formula that has any quantifier outside the scope of J_k that has the same truth value. For example, $\forall xJ_k[Fx]$ says that every individual instance of Fx has a greatest lower bound of exactly k , which is clearly wrong. $\exists xJ_k[Fx]$ says that there is some particular individual that is F to exactly degree k , which is also wrong because there might not exist an object that has the greatest lower bound value. But the lack of a normal form makes it unlikely that there can be any method to detect theoremhood in fuzzy monadic predicate logic. Certainly resolution will not work.

So, not only can we not tell when a conclusion is validly derived in fuzzy logic (Observation 1), but we cannot even tell when a formula (even of monadic predicate logic) is a theorem. Surely together these two observations should give fuzzy control theorists pause; they show that fuzzy logic as an abstract theory reduces to stating intuitive principles without any way to generalize or use them. And, since fuzzy control theory is surely committed to using *something* — it follows that what it is committed to using is *not* fuzzy logic, just as Elkan said.

Observation 3: Full fuzzy predicate logic is not recursively axiomatizable. The real underlying reason that fuzzy logic fails to be of any logical interest does not have to do with the elementary fragments of propositional fuzzy logic and monadic fuzzy predicate logic, even though it is cute to note that even these elementary parts of fuzzy logic are not usable in the desired form. Instead, it is that full predicate logic is not really a logic.

This result was proved by Scarpellini³ for infinite-valued Lukasiewicz logics, and the proof carries over to all the well-known modifications (such as adding parametric operators or various arithmetic operators) of this logic, which includes any of the fuzzy predicate logics ever described in the literature. The thrust of the proof is that the set of unprovable formulas of ordinary two-valued predicate logic can be mapped one-to-one into the set of valid (designated) formulas of fuzzy logic, for any closed or open range of values ($k \dots 1$) that we designate. But the set of unprovable formulas of ordinary predicate logic is not recursively axiomatizable, and therefore neither is the set of valid formulas of fuzzy logic. Hence, they cannot even be adequately characterized or talked about coherently, except by example. Furthermore, fuzzy control theorists do not merely wish to appeal to examples of valid formulas of fuzzy logic, but to be able to characterize them in some way or other.

Lest my message be thought entirely critical of fuzzy control theory, let me point out that I believe that everything its proponents wish to do can be adequately carried out. (My camera works!) However, their appeal to fuzzy logic is misplaced. Every fuzzy logic application has an analogue in finitely many-valued logic, and each one of these is logically well-behaved. There are correct theories of argumentation for them, there are resolution-like theories of theorem-detection for them, and they are axiomatizable.

The only apparent advantage to fuzzy logic is that it seems to be a grand generalization of all those finitely many-valued log-

ics — after all, we never know in advance which particular finite value might be needed for a specific application. However, it is an illusion to think that fuzzy logic is the correct generalization. It cannot be used and it has no reasonable logical foundation. But a “variable precision” finitely many-valued logic can do the sort of things desired. In such logics we have a superstructure of (say) three values. Having determined that some sentence takes one of these top-level values, we can then expand it to determine which of those three values it has at a lower level. For example, we grade essays as “good,” “so-so,” and “bad,” but then given that an essay has been categorized as “so-so” we can look more closely at whether it is a good, so-so, or bad example of being so-so. And this process can continue for some finite number of

times. Such a logic does not have any of the shortcomings that fuzzy logic does, and would seem to be the sort of thing that could form a logically adequate background theory for fuzzy control systems.

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On the Purportedly Paradoxical Nature of Fuzzy Logic

Enrique H. Ruspini, SRI International

Elkan's original paper purportedly showed that fuzzy logic was paradoxical in nature due to its reliance on formal bases that preclude truth values other than 0 and 1. Elkan has now modified some of his claims and arguments slightly, although he still depends on that result as his major source of insight into the technology.

We are now told, for example, that fuzzy logic is paradoxical because it is successful in many applications while its foundations remain under attack. Perplexing as this situation might be from a sociological viewpoint, it is hardly a logical self-contradiction, and describing it as a paradox is totally inappropriate.

Nor is a paradox implied by the claim that most theoretical fuzzy logic papers seem to deal with representation and reasoning methods, while most fuzzy logic applications have resulted in embedded controllers. The embedded controllers have been developed, of course, upon foundations provided by the representation and inferential methods of fuzzy logic. Elkan is not only unaware of this fact, but his overall analysis of the technology is colored by the strange notion that the depth and quality of deductive procedures in a controller are inferior to those in "sophisticated" reasoning systems.

For reasons of space, I will not discuss here Elkan's statements about application of fuzzy logic to control and other intelligent reasoning systems, but will confine my comments to the formal result (Theorem 1) that remains the major basis of his claims about the purported paradoxical nature of fuzzy logic. Other assertions about the methodology — arising in some cases from superficial analyses of relevant

literature and issues, but mostly out of ignorance or plain confusion — are appropriately addressed by other respondents.

Starting from an axiomatic characterization of fuzzy logic proposed by Gaines, and assuming that logical equivalence in fuzzy logic means equivalence in the sense of classical logic (thus implying that all classical logic theorems are also fuzzy logic theorems), Elkan shows that Gaines' axioms imply that the only possible truth values are 0 and 1: fuzzy logic collapses into conventional logic.

Anybody acquainted with fuzzy logic, however, would not have much difficulty questioning Elkan's notion of logical equivalence; it is well known that many theorems of propositional logic are not valid in fuzzy logic. Assuming otherwise immediately leads to the result that Elkan finds so paradoxical. Applying, for example, the axioms of fuzzy logic to the law of the excluded middle $\alpha \vee \neg\alpha$, which is not a theorem of fuzzy logic, leads to the equation

$$\max(t(\alpha), 1 - t(\alpha)) = 1,$$

which only has the solutions $t(\alpha) = 0$ and $t(\alpha) = 1$.

Many theorems of classical propositional logic may also be used to derive this result. Elkan's unnecessarily lengthy proof — based on the conventional propositional logic equivalence of the formulas $\neg(a \wedge \neg b)$ and $b \vee (\neg a \wedge \neg b)$ — actually assumes the validity of the law of the excluded middle (to see this, simply expand the latter and note the conjunct $b \vee \neg b$).

Elkan's "shocking" discovery has been long known, and is discussed in elementary textbooks on fuzzy and multivalued logics.¹

For example, if (C, U, I) are negation, disjunction, and conjunction operators, respectively, that is —

$$\begin{aligned} t(\neg p) &= C(t(p)), \\ t(p \vee q) &= U(t(p), t(q)), \\ t(p \wedge q) &= I(t(p), t(q)) \end{aligned}$$

that satisfy the laws of excluded middle and contradiction, then the corresponding logics can be neither idempotent nor distributive. If Elkan had probed further, he could have proved that all continuous truth-functional multivalued logics "collapse" as well.

The definition of equivalence that Elkan describes as "apparently reasonable" is, therefore, patently unreasonable. The supposedly shocking result is just a well-known fact of little relevance to the practice of fuzzy logic. Simply stated, Elkan has found that fuzzy logic and the classical propositional calculus are different logical systems.

Not much is gained either by looking into seemingly more congenial quarters for alternative definitions. Elkan turns, for example, to intuitionistic propositional calculus (IPC) as another place to borrow notions of equivalence, feeling that his result is strengthened by the fact that the law of the excluded middle — a previous source of trouble — fails for both IPC and fuzzy logic. IPC is, however, based on a negation operator with different semantics than that of fuzzy logic (one is involutive while the other is not). Once again, one does not need a proof as extensive as Elkan's. The Gödel translation $\neg(\neg\alpha \wedge \neg\neg\alpha)$ of the law of the excluded middle is a theorem in IPC but not in fuzzy logic. Assuming that it is leads once again to the same incorrect conclusion: Fuzzy logic collapses. Elkan's theorem is, therefore,

just as true for IPC equivalence as it was for classical equivalence, but it is also just as meaningless as before: All that has been proven is that fuzzy logic is neither classical nor intuitionistic logic.

This explanation, however, still does not answer a basic question: What is the meaning of the word *equivalent* in Gaines' Axiom 4:

$t(a) = t(b)$ if a and b are logically equivalent?

In classical logic, logical equivalence between two formulas α and β may be defined either as the validity of the formula $\alpha \leftrightarrow \beta$, or as the equality of the truth values of the formulas α and β for all possible assignments of truth values to their constituent propositional symbols. A quick inspection of the truth table of the \leftrightarrow connective shows that these definitions are equivalent.

While this is very reasonable, something seems to be amiss here. How can we consider Axiom 4 before we even define logical equivalence? If equivalence means that the truth value of α is always equal to that of β , why do we need an axiom to state that this should be the case?

In multivalued logics, equivalence in the sense of the validity of $\alpha \leftrightarrow \beta$ is not the same as equivalence in the sense of equality of the truth values of α and β . For example, these notions yield the same relation in the Lukasiewicz L_3 logic, but not in the 3-valued logic of Bochvar (where, if α and β have the third value $1/2$, then $\alpha \leftrightarrow \beta$ also has the third value $1/2$). In these logics it is possible to consider several characterizations of the notion of logical equivalence, each having different formal properties.²

In multivalued logics in general, and fuzzy logic in particular, equivalence is usually defined in terms of the semantics of the \rightarrow connective. Several such definitions have been proposed, notably by Zadeh, and by Trillas and Valverde.³ Seeking a wide characterization of fuzzy logics, Gaines chose not to specify a particular semantics for the implication operator, instead requiring only the use of a reasonable notion of equivalence compatible with equality of truth values.

Those who have read Elkan's original paper wondered at the time why he had to seek definitions in other logics rather than proceeding along the lines I have sketched here. In the present article, Elkan at last considers a definition based on the semantics of the negation, disjunction, and conjunction operators, but not on that of the implication connective (see his last paragraph in the section on paradox). He concludes, however, that this leads to an extremely weak system where the only equivalences are the De Morgan axioms.

This statement, unlike previous claims, is not only irrelevant but false and misleading. Simple application of fuzzy logic operators for disjunction, conjunction, and negation immediately shows that the following laws of propositional logic also hold in fuzzy logic: commutativity of disjunction and conjunction; associativity of disjunction and conjunction; distributivity of disjunction (conjunction) with respect to conjunction (disjunction); idempotence of disjunction and conjunction; identity with respect to \top and \perp ; absorption with respect to disjunction and conjunction; absorption by \top and \perp ; involution; and, surely enough, the De Morgan laws.

All these properties give fuzzy logic considerable strength as a reasoning formalism, but their consideration alone — in the absence of definitions for the implication connective \rightarrow and for the deductive rules of fuzzy logic (such as the generalized modus ponens) — cannot be the bases of any substantive argument, either pro or con, regarding the adequacy and correctness of fuzzy logic as a deductive methodology. Curiously, Elkan does not seem to feel that there is any need to discuss these matters, interpreting the independence of his theorem from any notion of implication as a sign of its universality and strength rather than as yet another indicator of its lack of relevance.

Elkan's arguments, arising from a meaningless result and a superficial and confused evaluation of the state of the art in fuzzy logic, do not provide any substantial insights into the methodology, its advantages, or its shortcomings. Given the weakness of his arguments, one can only be astonished at his conclusion that proponents of fuzzy logic are guilty of fallacious *non-sequitur* thinking (*post hoc, ergo propter hoc*). Those who propound the technology found their claims on solid theoretical results and on thousands of examples of its successful application. All that Elkan produces, on the other hand, is an irrelevant theorem and a rather shallow and mistaken discussion of a minor segment of the literature.

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Semantic Uncertainty of the Fuzzified Laws of Logic

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The confusion surrounding Charles Elkan's article is generated by a lack of clear understanding of the four levels of knowledge representation: linguistic, metalinguistic, propositional, and computational. When we attempt to convert knowledge expressed in natural language into computable knowledge, at least three significant transformations occur between these four levels.

Linguistic expressions. *Linguistic expressions* are natural language expressions, such as

"inventory is low and demand is high,"
 "inventory is low or demand is high," or
 "inventory is not low,"

where "inventory" and "demand" are nouns, and "low" and "high" are adjectives. In the terminology of fuzzy set theory,¹ the nouns are *linguistic variables*, the adjectives are *linguistic values*, and "and," "or," and "not" are *linguistic connectives* that generate interval-valued fuzzy sets.^{1,2}

A *metalinguistic* expression is a mapping from natural language to a symbolic language. For example, the metalinguistic forms of the linguistic expressions above are: " X_1 is A AND X_2 is B," " X_1 is A OR X_2 is B," and " X_1 is NOT A," where X_1 and X_2 are the metalinguistic representations of the linguistic variables, A and B are the metalinguistic representations of the linguistic values, and AND, OR, and NOT are the metalinguistic representations of the linguistic connectives. In short form, these metalinguistic expressions are represented as "A AND B," "A OR B," and "NOT A."

Propositional expressions. In the classical two-valued logic, there are at least two approaches that generate *propositional ex-*

pressions (normal forms) for every metalinguistic expression. The first is to assign the symbols \cap , \cup , and c to the basic metalinguistic connectives AND, OR, and NOT, respectively. Next, we form the canonical expressions of the basic metalinguistic expressions as $A \cap B$, $A \cup B$, and A^c . Then we derive all other propositional expressions with an application of $A \cap B$, $A \cup B$, or A^c , subject to the particular interpretations.

In the second approach, we first give an interpretation to a metalinguistic expression and define its meaning with a truth table. We then determine its normal forms from the truth table by the application of the "canonical form" generation algorithm. In this approach, two distinct but equivalent canonical forms are generated: the disjunctive normal form (DNF) and the conjunctive normal form (CNF). For example, DNF and CNF for "A AND B" are

$$\text{DNF}(A \text{ AND } B) = A \cap B =$$

$$\text{CNF}(A \text{ AND } B) = (A \cup B) \cap (A \cup B^c) \cap (A^c \cup B)$$

Fuzzy normal forms. It has been shown that fuzzy normal forms can be generated from the fuzzy truth table directly.² Depending on the set of axioms we impose, we get at least three different classes of fuzzy logics with their corresponding normal forms:

(1) If we assume $(\cap, \cup, ^c)$ is a De Morgan logic such that only boundary and monotonicity conditions together with the involutive complementation are imposed, then we have the following FDNF and FCNF expressions for the first-level fuzzy logics:

$$\text{FDNF}^{(1)}(A \text{ AND } B) = (B \cap A) \cup (A \cap B)$$

$$\text{FCNF}^{(1)}(A \text{ AND } B) = (B \cup A) \cap (B^c \cup A) \cap (B \cup A^c) \cap (A \cup B) \cap (A^c \cup B) \cap (A \cup B^c)$$

(2) If we assume $(\cap, \cup, ^c)$ is a De Morgan logic such that boundary, monotonicity, associativity, and commutativity conditions together with the involutive complementation are imposed, then we have FDNF and FCNF for the second-level fuzzy logics:

$$\text{FDNF}^{(2)}(A \text{ AND } B) = (A \cap B) \cup (A \cap B)$$

$$\text{FCNF}^{(2)}(A \text{ AND } B) = (A \cup B) \cap (A \cup B^c) \cap (A^c \cup B) \cap (A \cup B) \cap (A \cup B^c) \cap (A^c \cup B)$$

(3) If we assume $(\cap, \cup, ^c)$ is a De Morgan logic such that boundary, monotonicity, associativity, commutativity, and idempotency conditions together with the involutive complementation are imposed, then we have FDNF and FCNF which are equivalent to the fuzzified extensions of the classical normal forms:¹

$$\text{FDNF}^{(3)}(A \text{ AND } B) = A \cap B = \text{DNF}(A \text{ AND } B)$$

$$\text{FCNF}^{(3)}(A \text{ AND } B) = (A \cup B) \cap (A \cup B^c) \cap (A^c \cup B) = \text{CNF}(A \text{ AND } B)$$

In particular, it has been shown¹ that

$$\text{FDNF}^{(3)}(A \text{ AND } B) \subseteq \text{FCNF}^{(3)}(A \text{ AND } B)$$

In a similar manner, we can obtain FDNF and FCNF for the three classes of fuzzy logics and for all other metalinguistic expressions.

Computational expressions. At this level, symbolic elements of sets are assigned numeric values, and conjunction, disjunction, and complement operators are chosen. In Aristotle's logic, the assignments are

$$\mu_A : X_1 \rightarrow \{0,1\}, \text{ and } \mu_B : X_2 \rightarrow \{0,1\}$$

In Zadeh's fuzzy set theory and its logic, the assignments are

$$\mu_A : X_1 \rightarrow [0,1], \text{ and } \mu_B : X_2 \rightarrow [0,1]$$

Furthermore, for Zadeh's logic, the computational expression of the metalinguistic expression "A AND B" at the third level are

$$\mu_{\text{FDFNF}(A \text{ AND } B)}(a, b) = a \wedge b$$

$$\mu_{\text{FCNF}(A \text{ AND } B)}(a, b) = (a \vee b) \wedge (a \vee N(b)) \wedge (N(a) \vee b)$$

where $a \in A, b \in B$ are elements of fuzzy sets A, B — that is, $a = \mu_A(x_1)$ and $b = \mu_B(x_2)$ — and $\wedge = \min, \vee = \max$, and $N(\cdot)$ is the standard complement.

Interpretations

We can now reinvestigate and reinterpret the law of excluded middle for both the idempotent and nonidempotent operators as examples of our classification discussed above. For the idempotent class, an example is the min-max-standard complement triple. For the excluded middle expression, $\text{FDFNF}^{(3)}$ and $\text{FCNF}^{(3)}$ are computed to be

$$(a \wedge N(a)) \vee (a \wedge a) \vee (N(a) \wedge N(a)) \leq a \vee N(a)$$

which results in

$$0.5 \leq a \vee N(a) \leq 1.0 \text{ for } a \in [0,1]$$

This is a type-II semantic uncertainty, that is, $\mu(\mu_A(x)) = a \vee N(a)$. However, it reduces to singletons as opposed to intervals. It is clear that in Zadeh's fuzzy logic we ought not to state that the excluded middle expression holds or does not hold. We ought to instead state that it is satisfied to the degree specified by $a \vee N(a)$ for $a \in [0,1]$.

For the nonidempotent class, consider the bold intersection-union-standard complement De Morgan logic where $T_B(a, b) = \max(0, a+b-1)$, $S_B(a, b) = \min(1, a+b)$, and $N(a) = 1-a$. For this case we obtain

$$0.0 \leq \mu_{\text{FDFNF}^{(2)}(A \text{ OR NOT } A)} \leq \mu_{\text{FCNF}^{(2)}(A \text{ OR NOT } A)} = 1.0$$

and have an interval of graded values where the excluded middle expression is satisfied to a continuum of degrees in a

subinterval of $[0,1]$ that is bound by its lower bound $\mu_{\text{FDFNF}^{(2)}(A \text{ OR NOT } A)} \in [0,1]$, and its upper bound $\mu_{\text{FCNF}^{(2)}(A \text{ OR NOT } A)} = 1$.

Conclusions

I have demonstrated that there are three basic transformations between four levels of knowledge representation. Each metalinguistic expression is transformed to at least two propositional expressions known as the fuzzy disjunctive and conjunctive norms forms: FDFNF and FCNF, respectively.

A consequence of this $\text{FDFNF}(\cdot)$, $\text{FCNF}(\cdot)$ bounds is that classical expressions such as "excluded middle," "contradiction," and "equivalence," and any combination of two or more vague evidences, must be reinterpreted. The type-I fuzzy representation of linguistic expressions provides only a myopic interpretation of these expressions. These interpretations need to be restated: The fuzzified versions of the laws of classical logic hold to the degree specified by a type-II, second-order, semantic uncertainty computed by the membership of the membership grades, that is, $\mu(\mu_A(x)) = \mu^2_A(x)$. Thus, we cannot state, for example, that the law of excluded middle is satisfied or not. We can, however, state that the excluded middle expression is satisfied to a degree contained in the interval specified by:

$$[\mu_{\text{FDFNF}(A \text{ OR NOT } A)}(a, N(a)), \mu_{\text{FCNF}(A \text{ OR NOT } A)}(a, N(a))]$$

Reinterpretations for contradiction, equivalence, and so on can be stated in a similar manner. In fact, this is the source of controversy surrounding Elkan's paper.

The essence of fuzzy set theory is that all vague statements should at least be interpreted first with type-I semantic uncertainty at the primary, elemental level. But when two or more vague concepts are combined with a linguistic connective, then we are confronted with a type-II, second-order, semantic uncertainty. This generates an interval where the location of a specific degree of membership is nonspecific in that interval.

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The Promising Future of Fuzzy Logic

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As Charles Elkan points out in his article, the foundation of fuzzy logic is the notion of partial truth and degrees of truth in any proposition stating facts about real-world objects, whether these objects are entities, events, relations, algorithms, systems, or machines. Facts and propositions are uncertain, ambiguous, and incomplete — and more importantly, they are goal-oriented, intentional, and subjective to the observer's perceptual capabilities, mental constructs, and meaning systems. In this philosophical view, the universe is seen as holistic, dynamic, and chaotic.

Fuzzy logic is basically a theory of human perception and cognition. It is concerned with the marvelous paradigm and methodology discovered by evolution and realized in our brains to cope with complexity, holism, dynamism, and chaos in the world around us.

The goal of a fuzzy expert system is to take in subjective, partially true facts that are randomly distributed over a sample space, and build a knowledge-based expert system that will apply certain reasoning and aggregation strategies to make useful decisions. These decisions are again approximate, and have partial degrees of truth and likelihood; the decisions and derived facts are reliable to the best of our available knowledge.

The important fact about these systems is that decisions made by them can be iteratively and adaptively improved, and as more such random/fuzzy facts accumulate, the results will converge to real precise facts. In this view of reality, no proposition is always 100% true for 100% of the observers and experts. Absolute certainty, absolute truth, and absolute objectivity are impossible because they require infinite pieces of information, infinite number of samples, and infinitely many observers.

Fuzzy logic-based models are actually efforts in building our perceptual models and maps of reality, and not the reality itself.

Fuzzy logical equivalence. Like any other notion in fuzzy logic, the notion of logical equivalence is based on degrees of truth. The fourth postulate in Elkan's Definition 1 is not necessary, and can be replaced by classical implication relations. For example,

$$t(A = B) = t\{(A \rightarrow B) \text{ AND } (B \rightarrow A)\} = \min\{t(A \rightarrow B), t(B \rightarrow A)\}.$$

This expression can be used as the definition for degree of equivalence in fuzzy logic. For the special case where A and B have truth values of 1 or 0, the degree of logical equivalence is equal to 1 for the case of $t(A) = t(B)$. Consider the following special case. Using the classical implication relation that was generalized by Zadeh for fuzzy logic,^{1,2} we have

$$t(A \rightarrow B) = \max\{\min\{t(A), t(B)\}, 1 - t(A)\} \\ t(B \rightarrow A) = \max\{\min\{t(B), t(A)\}, 1 - t(B)\}$$

for $t(A) = 0$, $t(A = B) = 1 - t(B)$. For $t(B)$ being a number between 0 and 1, the degree of equivalence will be in a range from 0 to 1.

As seen in the above equations, for the case where $t(A) = t(B)$, $t(A = B)$ is always greater than 0.5 in fuzzy logic, which means a strong logical equivalence. Two propositions could be logically equivalent in a fuzzy sense without $t(A) = t(B)$.

We agree with Elkan's point that the last postulate of Definition 1 is the most controversial piece. He has in fact provided his own answer for preserving the continuum of degrees of truth.

Fuzzy expert systems. The types of uncertainty captured by fuzzy logic are vagueness, incompleteness, and ignorance. An example of this is the fuzzy expert systems

developed for Japan's Stock Exchange Market in Tokyo. The Nikkei average has been reportedly gone consistently higher using fuzzy logic.³ However, real applications of fuzzy expert systems have, for the most part, been kept out of the public eye because much of the work is proprietary.

As far as the domain independence of fuzzy operators is concerned, it is well known that max-min operations are default operations, and there are many different definitions suggested by the fuzzy logic research community for "and," "or," and implication operation.^{1,2} Of course, aggregation operators are important and context dependent, but they can be a part of the knowledge to be learned and gathered from the expert.

Consider Elkan's watermelon example about the context dependency of the "and" aggregators: If being red inside and green outside are believed to be mutually reinforcing pieces of evidence toward being a watermelon, then the logical proposition could read:

If X is *red* inside and X is *green* outside, then X is a watermelon is *very true*.

In this example, Elkan is using the fourth postulate to reach an intuitively incorrect conclusion. Based on the definition of the fuzzy logical "and" operation, $t(\text{red inside and green outside})$ is simply the degree to which an object is "red inside" and "green outside" and does not have anything to do with being a watermelon. The degree of being a watermelon depends on the other circumstantial information as well as the degree of being red inside and green outside. This "other piece of information" is the degree of logical equivalence that must be provided by the expert.

Fuzzy expert systems have been used in many applications. For example, Parkinson & Duerre have used both expert systems and fuzzy expert systems to choose the most suitable new "technology" for oil

recovery.⁴ In the case of classical expert systems, the sharpness of the boundaries of crisp variables involved in this application led to wrong conclusions based on the *if-then* rules. The fuzzy expert system took care of all the limiting (worst-case) problems and made natural conclusions. Although these worst-case problems are not the most common for this application (in which the recovery technologies are somewhat outdated), their occurrence will become the rule rather than the exception in future years. As it is now, major oil reserves in the US cannot be recovered by the old technologies.⁴

Fuzzy control. Most of the current applications of fuzzy logic are fuzzy expert control systems. Fuzzy controllers are expert control systems that smoothly interpolate between otherwise crisp (or predicate logic-based) rules. Rules fire to continuous degrees and the multiple resultant actions are combined into an interpolated result. The basis of fuzzy control is provided by processing uncertain information and saving energy through the use of commonsense rules and natural-language statements.

The use of sensor data in practical control systems involves several tasks that are usually done by a person, such as an astronaut adjusting the position of a satellite or putting it in the proper orbit, or a driver adjusting a car's air conditioning unit, and so on. All such tasks must be performed based on an evaluation of the data according to a set of rules that the person has learned from experience or has been trained in. Often, if not most of the time, these rules are not crisp (based on binary logic), that is, they involve common sense and human judgment in the decision making process. Such problems can be addressed by a set of fuzzy variables and rules that, if calculated and executed properly, can make expert decisions.

Fuzzy logic has given a new definition to the causality in dynamic systems. Fuzzy relational equations^{1,2} are indications of the notion of degree of causality between input and output variables in a dynamical sys-

tem. Like any other notion, causality is not a matter of black or white, or yes or no; instead, the cause-and-effect relation itself is a matter of degree. As Elkan correctly observes, the advent of the fuzzy chip, which came on the market in 1987, is a major force behind the spread of industrial applications of fuzzy logic control.

In reference to the use of words such as "image stabilization" for fuzzy logic camcorder image stabilizing systems, or "grade logic" for fuzzy logic, Elkan brings out the common difficulties that English-speaking Western communities have with this new technology, and with the innocent word "fuzzy." It is not surprising in light of this bias that manufacturers chose alternative words in their advertisements in the US, and to a lesser extent, in other English-speaking countries.

As far as the standard architectures of fuzzy control are concerned, a small number of rules are an advantage for fuzzy control systems. This is evidently a result of interpolative reasoning and the ability to aggregate the overlapping pieces of fuzzy information. Elkan brings up the point indicated by Sugeno and his colleagues — that the knowledge recorded in a fuzzy controller typically reflects immediate relations between the inputs and outputs of the system to be controlled, as opposed to a deep causal model of the system.⁵ Although this point of view is accurate, it is also true that this is the exact manner in which human experts summarized their expertise — by capturing the causal links between the inputs and outputs of the systems and putting them in the form of a set of linguistic rules. The expert might have deep knowledge of the system's causal relationships, but it is hard to access that type of knowledge in the form of linguistic protocols. For example, the knowledge of an operator with 20 years of experience at an electric power substation cannot be tapped in a few simple linguistic rules to offer a deep knowledge about the transience and stability of a power system.

Short development times have been a big

advantage of fuzzy logic in control systems. To achieve quick design periods, simple rules have been used thus far to put the designer in the ball park, and although approximate and crude, through tuning and adaptation the rules are fine tuned for better performance of the overall system. It is true that most current applications of fuzzy logic could use other rule-based formalisms, but these come with costs in terms of memory, efficiency, development times, and longer compilation of vague linguistic types of knowledge. For example, consider the following type of proposition:

Most experts believe if X is A , then Y is B is very true and fairly likely.

There are techniques that can handle this type of vague logical proposition that have elements of both probability and possibility.^{1,2} Elkan brings up the brittleness of rule-based systems caused by a missing piece of information. This is not the case for fuzzy rule-based expert systems. As mentioned earlier, this is due to the interpolative capabilities of fuzzy logic's continuous aggregation of the rules and elastic semantics assigned to the symbols, as defined by the membership functions.

Fuzzy control, as we mentioned earlier, constitutes a major application area of fuzzy logic. With most control systems, based on some real data from certain sensors, some decision must be made through a decision process. Fuzzy controllers are nonlinear controllers that provide rather reasonable robustness and adaptiveness with the changing environment — be it unmodelled dynamics in the system, external disturbance, or simply a lack of precise knowledge about the plant that is being controlled.

The subjectivity in fuzzy modeling is a blessing rather than a curse. The subjectivity in the definition of the terms is compensated for by the subjectivity of the conditional rules used by an expert. Because the set of variables and their meanings, as represented by corresponding membership functions, are compatible and consistent with the set of conditional rules used, the overall outcome turns out to be objective, meaningful, and reliable. Fuzzy mathemat-

ical tools and the calculus of fuzzy *if-then* rules opened the way for the automation and use of a huge body of human expertise that has gone untapped for years in industry. Fuzzy logic has provided a mechanism to share, communicate, and transfer a wealth of human technical expertise into computers. This has reversed the trend of machine tyranny: We are now forcing computers to think like people, rather than the other way around. This is the beginning of a new era in the applications of AI, neural networks, fuzzy logic, genetic algorithms, and probabilistic reasoning within a bigger picture called *soft computing*.⁶ The fundamental issues of AI can only be solved with an orchestrated application of fuzzy logic, neural networks, genetic algorithms, and probabilistic reasoning.

Elkan is distinguished from most critics of fuzzy logic because he seems to have sincerely studied the subject from both a theoretical and applied point of view. It seems to us, however, that Elkan's primary contact with fuzzy logic has been through open literature rather than industrial applications and the tremendous activity across the industrial world.

Some of the shortcomings that Elkan attributes to applied fuzzy logic are due to the gap that exists between theory and application, despite the revolution in the industrial use of fuzzy logic. We believe, however, that it is too soon to scrutinize this gap. For example, at the University of New Mexico's CAD Laboratory for Intelligent and Robotic Systems, fuzzy logic technology is being put on a chip to be embedded in a new generation of controllers with large industrial and technology transfer implications.⁷ We are trying to introduce the next generation of fuzzy expert systems capable of handling truth qualifiers, quantifiers, rule interaction, chaining, and hierarchical rule structures.

By starting to think in terms of a holistic, relativistic, probabilistic, and possibilistic knowledge structures, we believe scientific thinking is entering its new major stage of maturity. Crisp, binary, deterministic, first-

principle-based approaches in modeling the real world belong to the childhood years of science. The scientific thought that began with Aristotelian logic and was followed by Laplacian determinism has reached its limitations — particularly when it comes to understanding human systems. In the last hundred years, we have witnessed the development of quantum mechanics, and with it, probabilistic notions of microcosm, relativistic mechanics for macrocosm, and more recently, fuzzy logic and chaos theories. The emergence of these theories have a philosophical implication that points toward a probabilistic and possibilistic picture of reality.

Fuzzy logic — with the help of probability theory — will provide yet another powerful tool in an engineer's or scientist's toolbox for coping with complexity and nonlinearity in real-world systems. It will also furnish answers that are never 100% accurate and certain, but are acceptable within the constant constraints of real time, energy, memory, and resources.

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Toward a Framework for Fuzzy Dynamic Systems

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Fuzzy logic, according to Lotfi Zadeh, can be broadly considered as the union of fuzzified crisp logics. Its primary aim is to provide a formal, computationally oriented system of concepts and techniques for dealing with modes of reasoning that are approximate rather than exact. Charles Elkan's claims are derived mainly from entangled interpretations of fuzzy logic stemming from his mathematical approach to the formal system and intuitionistic approach to the practical system. Here we examine the mathematical structures of classical and fuzzy logic, and then point out that Elkan's view of the standard version of fuzzy logic is not valid. We then attempt to envisage fuzzy logic, in its practical aspect, as a dynamic system that will enhance control and expert systems.

Mathematical aspect of fuzzy logic. It is well known that the fundamental rules of classical logic are governed by the structure of a Boolean algebra, defined as follows:

Definition 1. A Boolean algebra $(B, \vee, \wedge, ^c, o, i)$ is a system consisting of a non-empty set B together with two binary operations \vee and \wedge , a unary operation c , and two nullary operations o and i on B , that satisfies the following axioms for any elements $a, b, c \in B$:

- (1) Commutative laws:
 $a \vee b = b \vee a$
 $a \wedge b = b \wedge a$
- (2) Associative laws:
 $(a \vee b) \vee c = a \vee (b \vee c)$
 $(a \wedge b) \wedge c = a \wedge (b \wedge c)$
- (3) Absorption laws:
 $(a \vee b) \wedge b = b$
 $(a \wedge b) \vee b = b$
- (4) Idempotent laws:
 $a \vee a = a$
 $a \wedge a = a$

- (5) Distributive laws:
 $(a \vee b) \wedge c = (a \wedge c) \vee (b \wedge c)$
 $(a \wedge b) \vee c = (a \vee c) \wedge (b \vee c)$
- (6) Involution law:
 $(a^c)^c = a$
- (7) De Morgan's laws:
 $(a \vee b)^c = a^c \wedge b^c$
 $(a \wedge b)^c = a^c \vee b^c$
- (8) Laws of Excluded Middle:
 $a \vee a^c = i$
 $a \wedge a^c = o$

Here, the nullary operations o and i are commonly known as the least element and the greatest element of the Boolean algebra.

Due to the pointwise definition of the operations used in the theory of two-valued logic, we can consider classical logic as a theory that is based upon the particular Boolean algebra $(\{0,1\}, \vee, \wedge, \neg, 0, 1)$ where 1 and 0 represent respectively the *true* and *false* of a certain statement or an assertion, and the operations \vee, \wedge , and \neg are defined according to the usual rules of the logical connectives *or*, *and*, and *not*, respectively. Zadeh's innovation of fuzzy logic, on the other hand, is an attempt to generalize the classical two-valued logic. Instead of the two values *true* and *false* represented by the set $\{0, 1\}$, he considered the interval $[0,1]$ to be the range of the truth value of any assertion, and replaced the binary operations \vee and \wedge on $\{0,1\}$ by the binary operations \max and \min on $[0,1]$. The unary operation \neg is also replaced by c where $c(\alpha)$ is defined to be $1-\alpha$ for any $\alpha \in [0,1]$. Under these operations, the system $([0,1], \max, \min, c, 0, 1)$ satisfies all the axioms of a Boolean algebra except the laws of excluded middle. Such a system is known as a soft algebra defined as follows:

Definition 2. A soft algebra $(S, \vee, \wedge, ^c, o, i)$ is a system consisting of a nonempty set S together with two binary operations \vee and \wedge ,

a unary operation c , and the nullary operations o and i on S , that satisfies the axioms 1 to 7 stated in Definition 1.

Thus, fuzzy logic theory can be seen as a theory based on the structure of a soft algebra. It is clear that every Boolean algebra is a soft algebra, but not vice versa. Hence, soft algebra is a more general system than Boolean algebra. Consequently, propositions that are valid in classical logic may not be valid in fuzzy logic. For example, if we view each assertion A as a set in a universe U and identify the truth value $t(A)$ of the assertion A by its characteristic function $\mu_A : U \rightarrow \{0,1\}$, then the two compound statements $(A \wedge B^c)^c$ and $B \vee (A^c \wedge B^c)$ are logically equivalent according to the rules of classical two-valued propositional calculus; however, in the context of fuzzy logic, these two statements with truth value in $[0,1]$ are not equivalent. (For example, take $t(A) = 0.3$ and $t(B) = 0.6$, then $t((A \wedge B^c)^c) = 0.7$, whereas $t(B \vee (A^c \wedge B^c)) = 0.6$.)

In his article, Elkan views a standard version of fuzzy logic as a system that satisfies the four postulates given in the following definition:

Definition 3. Let A and B be arbitrary assertions. Then

$$t(A \wedge B) = \min \{t(A), t(B)\} \quad (1)$$

$$t(A \vee B) = \max \{t(A), t(B)\} \quad (2)$$

$$t(\neg A) = 1 - t(A) \quad (3)$$

$$t(A) = t(B) \text{ if } A \text{ and } B \text{ are logically equivalent,} \quad (4)$$

where "logically equivalent" means equivalent according to the rules of classical two-valued propositional calculus.

Certainly, under these postulates, one can prove that for any assertions A and B , either $t(A) = t(B)$ or $t(A) = 1 - t(B)$. However, the main issue here is that postulate 4 is gener-

ally not valid in the realm of fuzzy logic, as shown in the above example. Thus, as a rigorous mathematical system, postulate 4 should not be included as a postulate in the formal system of fuzzy logic. Although Elkan has stated that in fuzzy logic applications it is unclear whether or not postulate 4 is assumed, and that in theoretical work it is often used explicitly, he still imposes this postulate in the formal system of fuzzy logic and then claims that the standard version of fuzzy logic collapses mathematically to two-valued logic. This type of condemnation, in our view, is an impediment to the growth of our knowledge.

Practical aspect of fuzzy logic. At the present stage, fuzzy mathematics is viewed in two ways. First, it is a theory that conforms with the precise and rigorous principles of mathematics to deal with fuzzy objects. In other words, it is a strictly mathematical theory to study the objects in a fuzzy environment. Second, fuzzy mathematics is a metamathematical theory that involves fuzzy proof techniques and fuzzy theorems with their applications. This latter status has yet to be fully developed. We have already seen many successful applications of fuzzy logic that the use of conventional mathematics could not achieve. We envisage that as fuzzy mathematics develops further, applications will be even more convincing and prominent.

Elkan and other researching technologists perhaps view fuzzy logic as fuzzy mathematics in its second status, and this might explain why Elkan has been unable to find a real-world expert system that uses fuzzy logic as its primary formalism for reasoning under uncertainty.

Fuzzy logic, in the practical aspect, deals mainly with fuzzy quantization, its meaning and means. In this respect, Elkan regards those operators in fuzzy logic as fixed and domain-independent. In fact, fuzzy quantization is introduced precisely for the purpose of generating domain-specific quantities. The numerous forms of fuzzy operations suggested in the literature were created to cater to the domain-specific

needs.^{1,2} Other semantically dependent formulations of fuzzy operations and inference relations have also been proposed.^{3,4}

Elkan's paper does bring up some valid points in the discussion of the status quo of fuzzy control. Indeed, the present fuzzy controllers are mostly structurally shallow, and in most cases, the controllers simply deal with no more than a simple static fuzzy mapping of the sensory and actuation signals. However, this is not the whole picture of fuzzy control. In fact, the success of Apronix's simulation of the two-stage inverted pendulum using a fuzzy controller is a fuzzy logic application that is not structurally shallow. When fuzzy logic is used as a way of quantization, it can serve as our quantity basis for modeling dynamic systems in the real world. This leads to the notion of fuzzy dynamic systems. Obviously, fuzzy dynamic systems are more complex, as they describe dynamic evolution of certain fuzzy quantities, not simple points or numbers. Undoubtedly, in the light of such a theory, many important issues such as stability, controllability, and observability can properly be addressed, and it may also serve to bring the seemingly diverging model-based or rule-based methodologies into a unifying framework.

An appropriate theory for fuzzy systems has not yet been developed in fuzzy control. The main task is to establish a framework in which fuzzy controllers of deeper structures can be described properly and handled with ease. Elkan has predicted a tough time ahead for fuzzy logic in general, and for fuzzy control in particular. We, too, predict a tough time ahead in working out a meaningful and acceptable framework for fuzzy-based dynamic system theory. However, we remain optimistic. We believe that such a framework will emerge.

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Misrepresentations and Challenges

A Response to Elkan

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The comments made by Charles Elkan can be classified into three categories. The first are those that are technically incorrect and should not have been allowed to pass an unbiased review process. The second are those that are truly challenging, and in point of fact help show the great representative power of fuzzy subsets. The third category are those pertaining to the practical application of fuzzy control. Some of these latter comments are quite reasonable — though not as damning as Elkan tries to make them. I shall address these issues in turn.

The first category — those that are completely incorrect — is dominated by Theorem 1. Rather than wasting considerable space on addressing this “theorem” when I am certain that other respondents will effectively show its complete absurdity, I shall make only a few comments. The key issue here is of course the last premise,

$t(A) = t(B)$ if A and B are logically equivalent.

In most texts on logic,¹ the definition of logical equivalence is specified the other way. Usually, one says that A and B are logically equivalent if propositions A and B attain the same truth value for all models of the constituent atoms. However, in this case, rather than defining the *undefined* concept of logical equivalence in terms of a well-defined idea of attaining the same truth value, Elkan tries to define the idea of attaining the same truth value from the undefined concept of logical equivalence. Once having made this error, the author then compounds it by imposing a requirement that is completely antithetical to the idea of fuzzy logic:

$$t(\neg(A \wedge \neg B)) = t(B \vee (\neg A \wedge \neg B)).$$

First we note that

$$t(\neg(A \wedge \neg B)) = t(\neg A \vee B),$$

and hence

$$\begin{aligned} t(A \vee (\neg \neg A \vee \neg B)) & \\ &= t((\neg A \vee B) \wedge (B \vee \neg B)) \\ &= \min[t(\neg A \vee B), t(B \vee \neg B)]. \end{aligned}$$

Thus we have the requirement

$$t(\neg A \vee B) = \min[t(\neg A \vee B), t(B \vee \neg B)].$$

For this condition to hold for every B requires that

$$t(B \vee \neg B) = 1$$

for all B . However, this condition is the law of the excluded middle and is exactly what fuzzy logic was constructed not to support. In fact, I have suggested a measure of fuzziness based upon the lack of satisfaction of this condition.² Furthermore, the condition $t(B \vee \neg B) = 1$ implies that

$$\max[t(B), 1 - t(B)] = 1.$$

Hence, either

$$t(B) = 1 \text{ or } t(B) = 0 \text{ (} 1 - t(B) = 1 \text{)}.$$

Thus, Elkan has essentially assumed that the logic is a binary logic.

As to the second category, Elkan raises the issue of defining the concept of watermelon in terms of the constituent concepts of redness on the inside and greenness on the outside. His basic contention is that the definition of watermelon in terms of these constituents should exhibit a characteristic of reinforcement. Essentially, he correctly requires that multiple confirmations to the constituents' criteria should reinforce each other, while disconfirmations of the constituents' criteria should also reinforce each other in the other way. The issue raised here is an interesting and challenging question. However, rather than showing the limitations of fuzzy logic, this problem illustrates the power of fuzzy logic to model sophisticated aggregation requirements.

It is fundamental to a comprehensive understanding of the agenda of fuzzy logic

to appreciate the pervasive nature of its ability to model continuity and graduality in all concepts. In using fuzzy logic, we are not confined to only using the idea of fuzziness (graduality) in the definition of the predicates (*redness/greenness*), but we can also apply the concept of fuzziness to the operators used to connect the predicates. In addressing this important issue, we must call upon fuzzy logic's ability to provide connectives lying between the logical *and* and logical *or*. Consider the definition of watermelon suggested by Elkan,

$$\text{watermelon}(x) = \text{redinside}(x) \perp \text{greenoutside}(x).$$

Elkan correctly shows that if we interpret \perp as a pure logical *and*, defined as the $\min(\wedge)$ we end up with a result that doesn't provide the appropriate property of reinforcement. Similarly, using a pure logical *or*, defined as the $\max(\vee)$ also leads to unsatisfactory results. The key point is that in fuzzy logic we are not restricted to these two extremes as we are in binary logic.

Recently my colleagues and I introduced a new class of fuzzy connectives, called uninorms,³ that provide the exact type of aggregation postulated as being required by Elkan. Consider the situation that for some object m we have $t(\text{redinside}(m)) = a$, and $t(\text{greenoutside}(m)) = b$. Our problem is to provide an aggregation operator \perp to implement the connection between these values. Formally, letting $d = t(\text{watermelon}(m))$ we require some aggregation R such that

$$d = R(a, b).$$

The question is, what form should R take to capture the type of reinforcement desired by Elkan? As I will show, uninorms provide the appropriate aggregation. These uninorms, which generalize the idea of t -norms (*and* operators) and t -conorms (*or* operators) and lie between these extremes,

do exactly what Elkan requires.

A uninorm is a mapping³

$$R: [0, 1] \times [0, 1] \rightarrow [0, 1]$$

having the properties:

- (1) Commutativity: $R(a, b) = R(b, a)$
- (2) Monotonicity: $R(a, b) \geq R(c, d)$ if $a \geq c$ and $b \geq d$
- (3) Associativity: $R(R(a, b), c) = R(a, R(b, c))$
- (4) There exists an identity element $\delta \in [0, 1]$ such that for all a , $R(a, \delta) = a$.

If $\delta = 1$, this reduces to the t -norms that are essentially pure and aggregations that include the min operator, while if $\delta = 0$, it reduces to the t -conorms which are essentially pure or aggregations and include the max operator. Thus, the logical and and or are extremes of this class.

Of particular interest is a property of these uninorms called the *upward reinforcement characteristic*. For the uninorm, the upward reinforcement characteristic is captured in the following:

$$\begin{array}{ll} R(a, b) \geq a & \text{if } b > \delta \\ R(a, b) \leq a & \text{if } b < \delta \end{array}$$

We select a value δ to be our neutral point; values above δ are considered as confirming and those below δ as disconfirming. (Actually, δ can be a range; however, for the present purpose we'll consider δ as a point.) Now, assume that both a and b are above δ . In this case we have

$$\begin{array}{l} a \leq R(a, b) \\ b \leq R(b, a) = R(a, b) \end{array}$$

and thus

$$R(a, b) \geq \max[a, b].$$

Hence there is a reinforcement in the positive direction when both criteria are "confirmed."

Now assume that both a and b are below δ . In this case we have that

$$\begin{array}{l} a \geq R(a, b) \\ b \geq R(b, a) = R(a, b) \end{array}$$

and thus

$$R(a, b) \leq \min(a, b).$$

Hence there is negative reinforcement if

both are below the neutral value.

Finally, consider the case where one is below and one above, $a \leq \delta$ and $b \geq \delta$. In this situation we see

$$\begin{array}{l} a \leq R(a, b) \\ b \geq R(b, a) = R(a, b) \end{array}$$

and hence we get

$$a \leq R(a, b) \leq b$$

and thus there is no reinforcement.

With the aid of these uninorm fuzzy aggregation operators, we can capture the type of aggregation Elkan desires.

Finally, Elkan's comments on the use of fuzzy logic in heuristic control — while in some points are quite valid — manifest a type of "fuzzy bashing" that is all too common in the AI community. For example, Honda's choice of the term "grade logic" has much less to do with their concern for any scientific resistance to fuzzy logic methodology than to the simple marketing expedient that "fuzzy" is not the type of word that sells cars.

In a recent book on fuzzy modeling and control,⁴ we look carefully at the process of building fuzzy logic controllers. The reasons we found for the success of these controllers are not in complete agreement with those Elkan suggests.

First of all, the fact that most fuzzy controllers are built with a small number of rules should be seen as one of the powers of this technology. An essential feature of the fuzzy approach is the ability to generalize — in a way, to reduce the necessity for detail.

Elkan fails to mention a feature I think is essential to the success of the fuzzy modeling approach: the partitioning of the input variable space into regions that allow a simplification of the modeling process.

Closely related to this is the idea of partial matching, which lets us smoothly combine solutions from different regions as we get near the boundary.

Elkan correctly observes that most fuzzy controllers are shallow (requiring no chaining between the rules) and usually directly connect the input to the output. I think it is here that these systems might have trouble in the future. However, the reason for these potential problems is not found in the paradigm of fuzzy modeling, but in the choice

of the implication operative.

To me, Elkan's reference to the 1980 comment by Mamdani and Sembi⁵ is most disturbing. Rather than seeing these remarks as I believe Mamdani meant them — as a statement of the power of the symbiotic relationship between the paradigms of AI (in this case rule-based systems) and the knowledge-representation capability of fuzzy logic — Elkan has chosen to interpret this as a sign of the weakness of fuzzy logic. However, if we discard the obvious misrepresentations, Elkan's paper can serve as a challenge to fuzzy researchers to continue improving the valuable tool of fuzzy logic.

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Why the Success of Fuzzy Logic is not Paradoxical

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Elkan's paper consists of two almost unrelated parts. In the first section, Elkan arrives at the conclusion that an apparently reasonable version of fuzzy logic collapses mathematically to two-valued logic. In the second section, he questions the value of fuzzy logic in control applications and concludes that fuzzy logic does not provide an effective tool for dealing with the problem of uncertainty in knowledge-based systems. As I see it, the first conclusion is based on faulty reasoning, while the second reflects a misconception of what fuzzy logic is and a misunderstanding of the role it plays in control and knowledge-based systems applications.

It is easy to show why Elkan's mathematical analysis is faulty. What he really shows is that fuzzy logic is not consistent with the law of the excluded middle. This, of course, applies in general to multivalued logical systems.

The law of the excluded middle asserts that the truth value of any logical expression of the form $B \vee \neg B$ is T (true). The law of contradiction asserts that the truth value of any logical expression of the form $B \wedge \neg B$ is F (false).

Immediate consequences of these laws in two-valued propositional calculus are as follows:

If p is logically equivalent to q then p is also logically equivalent to $q \wedge (B \vee \neg B)$.

If p is logically equivalent to q then p is also logically equivalent to $q \vee (B \wedge \neg B)$.

If p is logically equivalent to q then $p \wedge (B \vee \neg B)$ is logically equivalent to $q \vee (B \wedge \neg B)$.

Now let us consider Elkan's Theorem 1. Starting with the valid equivalence

$$\neg(A \wedge \neg B) = \neg A \vee B$$

which is an expression of De Morgan's law, we can replace the right-hand member

with $(\neg A \vee B) \wedge (B \vee \neg B)$, which is in turn equivalent to

$$B \vee (\neg A \wedge \neg B).$$

Consequently, we can assert the logical equivalence

$$\neg(A \wedge \neg B) \equiv B \vee (\neg A \wedge \neg B), \quad (1)$$

which is the example used in Elkan's proof.

What we see, then, is that Elkan's example uses a disguised form of the law of the excluded middle. As should be expected, Equation 1 is not a logical equivalence in multivalued logic because the law of the excluded middle does not hold, in general, in multivalued logic. In sum, what Elkan shows in a roundabout way is that the law of the excluded middle does not hold in multivalued logic. There is no justification whatsoever for jumping from this obvious fact to the conclusion that fuzzy logic collapses to two-valued logic.

Turning to his analysis of fuzzy logic applications, Elkan's conclusion reflects a misunderstanding of what fuzzy logic is, and a faulty analysis of the reasons for its success. First, it must be clarified that the term "fuzzy logic" is used in two different senses. In its narrow sense, fuzzy logic is a logical system that is an extension of multivalued logic and serves as a foundation for approximate reasoning. What is important to note is that even in its narrow sense, the agenda of fuzzy logic is quite different from that of traditional multivalued systems.

In its wider sense — the sense in which it is predominantly used today — fuzzy logic is a much broader theory that is fuzzily synonymous with "fuzzy set theory," that is, the theory of classes with unsharp boundaries. In this perspective, fuzzy logic in the narrow sense is one of the many branches of fuzzy logic, among which are fuzzy arithmetic, fuzzy probability theory, possibility theory, fuzzy relations, and so on. It should be noted that

fuzzy logic in the narrow sense plays a very minor role in fuzzy control, just as classical logic plays a very minor role in classical control theory.

In his article, Elkan fails to differentiate between fuzzy logic in the narrow sense and fuzzy logic. In the first part, he interprets fuzzy logic in its narrow sense. But in the second part, he interprets fuzzy logic in its wide sense, since most applications of fuzzy logic — especially in the realm of control — do not involve fuzzy logic in the narrow sense. However, narrow fuzzy logic plays an essential role in the management of uncertainty in expert systems.¹ In what follows, fuzzy logic will be used in its wide sense.

What are the reasons for the rapid growth in the number, variety, and visibility of fuzzy logic applications? The reasons are not those given in Elkan's article. What fuzzy logic offers, above all, is a methodology for representing and analyzing dependencies that are approximate rather than exact. In this methodology, the key concepts are:

- a linguistic variable, whose values are words rather than numbers;
- a canonical form, which expresses the meaning of a proposition as an elastic constraint on a variable;
- a fuzzy if-then rule and rule qualification, in particular probability qualification and possibility qualification;
- interpolative reasoning; and
- a fuzzy graph.

Through the use of techniques based on these concepts, fuzzy logic makes it possible to exploit the tolerance for imprecision and uncertainty. In so doing, fuzzy logic has proved to be successful where traditional approaches have failed or yielded inferior results.

Most fuzzy logic applications involve the use of what might be called the *calculus of fuzzy rules*.²⁻⁴ The use of fuzzy rules

in conjunction with interpolative reasoning greatly reduces the number of rules that are needed to describe imprecise dependencies, and makes it much easier for humans to articulate them. Consider, for example, the rules that people use (consciously or subconsciously) in parking a car, filling a tub with hot water, crossing a traffic intersection, or riding a bicycle. How would Elkan describe the rules that govern human behavior in these and similar instances?

What is actually used in most control applications is a subset of the calculus of fuzzy rules, which can be called the *calculus of fuzzy graphs*.³⁻⁵ In this calculus, a function $f: U \rightarrow V$ is approximated to by a fuzzy graph f^* , which is a disjunction of Cartesian products of the form

$$f^* = \sum_i A_i \times B_i$$

where A_i and B_i , $i=1, \dots, n$, are values of linguistic variables, and \sum_i represents the disjunction (union) of Cartesian products $A_i \times B_i$. For example, a fuzzy graph of a function may be expressed in a coarse way as

$$f^* = \text{small} \times \text{small} + \text{medium} \times \text{large} + \text{large} \times \text{small}$$

which is equivalent to the set of rules

If X is small then Y is small.
 If X is medium then Y is large.
 If X is large then Y is small.

The use of fuzzy graphs results in data compression, which is one of the key — though perhaps not widely recognized — advantages of using fuzzy rules. Elkan's analysis makes no reference to this point, and fails to identify the use of the fuzzy graph concept as one of the principal tools in the application of fuzzy logic to control.

Today, fuzzy logic applications in control and consumer products are far more visible than fuzzy logic applications in knowledge-based systems. Does this mean, as Elkan surmises, that fuzzy logic is limited in its applicability to simple systems? Not at all. What it means is that fuzzy logic can be applied easily and effectively to the conception and design of "high machine IQ" control systems and consumer products — applications that in most cases involve replacing a trained operator or an

experienced user with a fuzzy rule-based system. In the case of knowledge-based systems, what has to be replaced is an expert rather than an operator. This is an inherently more complex problem, no matter what approach is used.

Basically, what differentiates control applications from knowledge-based systems applications is that in control the main problem that has to be addressed is that of imprecision. By contrast, in the case of knowledge-based systems, one has to come to grips with both imprecision and uncertainty.

In applying fuzzy logic to control systems, it is generally sufficient to employ categorical rules — rules that involve no quantifiers, probabilities, or possibilities. In the realm of control, the calculi of fuzzy rules and fuzzy graphs provide the necessary tools for exploiting the tolerance for imprecision and lead to systems that are simpler, more robust and have higher machine IQ than systems designed by conventional methods. Recently published books⁶⁻⁸ provide easily understandable accounts of the methodology of fuzzy logic control and explain why the applications of fuzzy control are growing rapidly in visibility, variety, and number. It is very likely that it will not be long before familiarity with fuzzy control will be an essential qualification for control engineers and system designers.

In the case of knowledge-based systems, two sources of difficulty are that the rules are frequently probability-qualified, and that the qualifying probabilities are not compositional. More specifically, assume that we have two rules of the form

$$\begin{aligned} \text{If } p_1 \text{ then } q_1 & \quad (P_1) \\ \text{If } p_2 \text{ then } q_2 & \quad (P_2) \end{aligned}$$

where p_1 , q_1 , p_2 , and q_2 are propositions, and P_1 and P_2 are qualifying probabilities. Assume that we wish to compute the qualifying probability, P , in the combined rule

$$\text{If } (p_1 \text{ and } p_2) \text{ then } (q_1 \text{ and } q_2). \quad (P)$$

The problem is that P cannot be computed as a function of P_1 and P_2 without making some assumptions about conditional independence or, equivalently, invoking the maximum entropy principle. Such assumptions tend to be ad hoc and hard to justify.

What this implies is that the problem of inference from probability-qualified propositions may not have a satisfactory solution within the framework of classical probability theory.

In this connection, it should be noted that Elkan gives the impression that there are many expert systems that do not employ fuzzy logic and that provide effective ways of dealing with uncertainty and imprecision. This is not the case. As a test, which of the systems that he has in mind could provide an answer to the following question:

If X is small then it is very likely that Z is large.

If X is large then it is not likely that Z is large.

What is the probability that Z is large if X is medium?

What this example points to is that the conventional approaches to the management of uncertainty in expert systems fail in four important respects:

- (1) They do not provide the means for dealing with the fuzziness of antecedents and consequents.
- (2) They assume that probabilities can be estimated as crisp numbers.
- (3) They do not offer a mechanism for inference from rules in which the qualifying probabilities are fuzzy.
- (4) The rules for composition of probabilities depend on unsupported assumptions about conditional independence.

Fuzzy logic addresses some — but not all — of these problems.¹ More specifically, fuzzy logic allows the antecedents and/or consequents and/or qualifying probabilities to be fuzzy. Furthermore, fuzzy logic makes it possible to estimate probabilities as fuzzy rather than crisp numbers. There remain, however, two problems. First, the composition of qualifying probabilities can lead to fuzzy probabilities that are insufficiently specific or, equivalently, insufficiently informative. Second, inference in fuzzy logic reduces, in general, to the solution of a nonlinear program. Standard techniques for the solution of such programs may be computa-

tionally expensive. What we do not have as yet are approximate, inexpensive techniques for inference from fuzzy-probability-qualified fuzzy if-then rules. However, we do have an effective method of inference from possibility-qualified rules within a branch of fuzzy logic known as possibilistic logic.⁹

There are many statements in Elkan's articles that relate to ill-posed questions or reflect a misunderstanding of what fuzzy logic is, as well as an inadequate familiarity with its literature. I will comment here on just a few of these statements.

In the section on fuzzy logic in expert systems, Elkan states, "there is still debate as to what types of uncertainty are captured by fuzzy logic." Obviously if the boundaries of what constitutes fuzzy logic are not defined, this is not a well-posed issue. In this context, what is important to realize is that any theory X can be fuzzified by generalizing the concept of a crisp set in X to a fuzzy set, leading to a theory that can be called fuzzy X . For example, classical probability theory can be generalized to fuzzy-probability theory; topology to fuzzy topology; neural network theory to fuzzy neural network theory; control to fuzzy control; arithmetic to fuzzy arithmetic; modal logic to fuzzy modal logic; resolution to fuzzy resolution; temporal logic to temporal fuzzy logic; Mycin to fuzzy Mycin; chaos to fuzzy chaos, and so on. Many such generalizations have already been described in the literature and many more will be made in the future. What is gained from fuzzification is greater generality and better approximation to reality.

Given that any theory can be fuzzified, the question of what types of uncertainty are captured by fuzzy logic loses much of its meaning. For example, when probability theory is fuzzified, it becomes a part of fuzzy logic. In this broad perspective, then, fuzzy probabilistic uncertainties fall within the scope of fuzzy logic. The same applies to any type of uncertainty that I can think of.

In the same section, Elkan reports that his search of the literature revealed no published reports of an expert system that uses fuzzy logic as its primary formalism. This is somewhat surprising, since there are, in fact, many such examples. Among them is

Cadiag-2, the well-known large-scale medical diagnostic system.¹⁰⁻¹³ Another well-known and commercially available system is FRIL,¹⁴ which is Prolog-based and has a highly sophisticated system for the management of uncertainty. Still another example is the Yamaichi Securities Fund, and there are many more (see Table 1 on page 46).¹⁵⁻¹⁷

Elkan also seems to suggest that expert systems that combine grades of membership using operators other than max and min are not valid examples of the use of fuzzy logic. This position is hard to understand since the use of t -norms, t -conorms, and other connectives is now a standard part of fuzzy logic.¹⁸

The issue of the management of uncertainty in expert systems presents many complex and difficult problems. There is no system at present that is free of serious shortcomings, and it would be unrealistic to expect that such systems will be developed in the foreseeable future. But Elkan's statement that "experience shows that fuzzy logic is rarely suitable in practice for reasoning about uncertainty" reflects inexperience in the use of fuzzy logic. I advise Elkan to study with care the extensive literature on the management of uncertainty in expert systems based on the use of fuzzy logic. A good starting point would be the treatises by Dubois-Prade on possibility theory and approximate reasoning, and the books on fuzzy expert systems.^{16,19} There is little doubt that, in coming years, the growth in familiarity with fuzzy logic will lead to its wide acceptance as a key component of information systems and knowledge engineering methodologies.

I compliment Elkan on writing a provocative article that is likely to contribute to further discussion of the strengths and limitations of fuzzy logic. Fuzzy logic has been and still is somewhat controversial. With the passage of time, however, the controversies will abate and fuzzy logic is likely to become a standard tool for the conception and design of intelligent systems. Indeed, it would not be surprising if, in retro-

spect, the skeptics will find it hard to understand why they failed to realize that fuzzy logic is a phase in a natural evolution of science — an evolution brought about by the need to find an accommodation with the pervasive imprecision of the real world.

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Table 1. Fuzzy logic tools and products.
(Source: Sammy Wong and Nelson Wong, Computer Science Dept., Chinese University of Hong Kong.)

COMPANY	PRODUCT	DESCRIPTION
American Neuralogix	NLX 230 fuzzy microcontroller	Has 8 digital inputs, 8 digital outputs, 16 fuzzifiers; holds 64 rules. Evaluates 30M rules/sec.
	ADS230 fuzzy microcontroller development system	PC-compatible system uses NLX 230 with analog and digital I/O.
	NLX 110 fuzzy pattern correlator	Correlates eight 1-Mbit patterns; expandable to 256 n-bit patterns
	NLX 112 fuzzy data correlator	Performs pattern matching on serial data streams.
Apronix	Fide (Fuzzy Interference Development Environment)	Runs under MS Windows on 386/486 PCs. Supports development, fuzzy simulation, debug tracing, and 3D display of control surfaces. Real-time code generation for microcontrollers. Software implementation of fuzzy logic in C. Complete tutorial information and phone support.
Byte Craft	Fuzzy-C	Preprocessor translates fuzzy source code into C source code.
Fuzzy Systems Engineering	Manifold editor	Runs under MS Windows 3.1 on 386/486 PCs. Edits rules in a matrix display. Lets users view fuzzy sets graphically.
	Manifold graphics editor	Runs under MS Windows 3.1 on 386/486 PCs. Color graphics display of rules and fuzzy sets. Lets users view designs in 3-D map and slice formats.
Hitachi American Hyperlogic	H8/300 and H8/500	Microcontrollers
	Cubicalc	Software for developing fuzzy-logic applications. Runs under MS-Windows with 286 or higher processor. Simulates fuzzy and nonfuzzy systems.
	Cubicalc-RTC	A superset of Cubicalc. Provides runtime compiler support and libraries for linking. Compatible with Microsoft C and Borland C.
	Cubicalc runtime source code	Generates C source code for use in compiling to a specific processor.
	Cubicard	Includes Cubicalc-RTC and PC-based hardware for analog and digital I/O.
Inform Software	Fuzzytech Explorer Edition	Introductory fuzzy-logic system. Software runs under MS Windows. Accepts two inputs, one output, five fuzzy membership sets per variable, and 125 rules. Includes tutorial.
	Fuzzytech MCS-96 Edition	Full fuzzy development system for MCS-96 microcontrollers. Generates optimized assembly code.
	Fuzzytech Online Edition	Lets users debug and modify fuzzy-logic systems while they are running. Generates C source code.
Integrated Systems	RT/Fuzzy Module	Simulation and code generation of fuzzy logic for real-time systems.
Metus Systems Group	Metus	Fuzzy-logic development and simulation system. Runs under MS DOS. Provides high-level modeling and low-level development for embedded applications.
Modico	Fuzzle 1.8	PC-based fuzzy-logic shell. Generates source code for C and Fortran.
Motorola	Fuzzy-logic kernel for microcontrollers	Fuzzy processing kernels for 68HC05 and 68HC11 microcontrollers. Includes fuzzy knowledge-base generator to create code for kernel.
	Fuzzy-logic educational kit	Interactive training tool provides good introduction for understanding and using fuzzy logic. Runs under MS Windows. Includes demonstration version of Fide (from Apronix).
Togai Infralogic	TILShell + fuzzy C development system	Complete fuzzy development system generates C code and includes debug, fuzzy-simulation, and graphical-analysis tools. Tutorial included.
	Microcontroller evaluation packages	Fuzzy development systems for Hitachi H8/300, H8/500, and HMCS400; Intel 8051; and Mitsubishi 37450.
	Microcontroller production licenses	Unlimited production license.
	FC 110 FC 110 development system	Digital fuzzy-logic processor (1C) Hardware and software development system for FC 110. Versions support IBM PC/AT bus, Sbus, and VMEbus.

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Elkan's Reply

The Paradoxical Controversy over Fuzzy Logic

The responses to my article provide an exceptionally wide range of perspectives on the current state of research on fuzzy logic and its applications. Overall, I find that with most commentators I agree more than I disagree. I shall try here to steer a middle course between simply repeating points of agreement and narrowly countering points of disagreement.

The foundations of fuzzy logic. Some commentators take a more extreme position than I do concerning the coherence of fuzzy logic. I do not agree with Attikiouzel that "if one wishes to write a program or build a machine that will perform inference in the same way as human beings, then one must build the basic equations of probability theory into it, or face the inevitable outcome that it will not perform as required" Neither humans nor machines always require formal rigor to act successfully in the world, nor is success always guaranteed by rigor. Successful controllers and expert systems can use heuristic, shallow knowledge and therefore they can use arbitrary reasoning formalisms such as certainty factors or fuzzy logic. I also do not agree that "Proponents of fuzzy logic appear to be unaware of Cox's work and that of Jaynes and Tribus"; for evidence see the debate in a recent issue of *IEEE Transactions on Fuzzy Systems*.¹

However, I am uncomfortable with the dogmatism evinced by many of the advocates of fuzzy logic or some of its many variants. For example, Dubois, Prade, and Smets say that I fail "to understand the important distinction between ... properties whose satisfaction is a matter of degree" and uncertainty "induced by incomplete states of knowledge." Later they write that the AI community has forgotten this distinction. It appears to me that the AI community has not forgotten this very binary distinction, but rather has implicitly rejected the claim that it is a uniquely important distinction. A particular concern

that I have is whether the distinction is really well defined. On the one hand, there may be multiple types of imprecision and vagueness. Is the domain-independent imprecision involved in "around 1.80m" the same as the human-specific imprecision involved in "tall"? On the other hand, it may be possible to model some types of imprecision probabilistically. For example, the degree of truth of the assertion "1.80m is tall" might be modeled as the probability that an individual with height 1.80m would be labeled as tall given incomplete knowledge, that is, given no other information on the individual.

Overall, I am wary of the enterprise of even making an attempt to classify the types of uncertainty. A complete and consistent analysis of all the many varieties of uncertainty involved in human thinking and revealed in human language is a philosophical goal that we should not expect to achieve soon. Moreover, this aspiration is a variant of the quest for formal rigor criticized above as neither necessary nor sufficient for engineering success. As Freksa points out, it is always the case that "the represented real world and its representation are *formally* incommensurable." Therefore, however ideal the logics that one has at hand, knowledge engineering is always a tentative activity that can never succeed completely.

More varieties of uncertainty may well exist in the case of shallow knowledge than in the case of deep knowledge, because shallow knowledge is intrinsically domain-specific and of restricted generality. As Garcia points out, the reasoning in my watermelon example relies on important background knowledge that is not expressed in terms of rules. But it is not a fair reply to the example to call for this implicit background knowledge to be made explicit. The deep knowledge that underlies a given fragment of shallow knowledge may often be impossible or too expensive to make explicit. It is precisely then that the deep

knowledge becomes implicit background knowledge that must be used tacitly in tuning the allowed interactions between the items of explicit shallow knowledge. To quote Garcia, "The dogma of generality versus efficiency strikes again, and knowledge engineering and machine learning are not exempted."

Fuzzy logic in expert systems. Only three of the responses give references in an attempt to dispute the claim that there are very few deployed expert systems that actually use fuzzy logic as their principal formalism for reasoning about uncertainty. Moreover, most of the references given actually support this claim.

Before I discuss these references one by one, it is worth emphasizing that I use the term "expert system" to designate a reasoning system that applies a large base of explicit knowledge to perform a task requiring complex inference, such as diagnosis, scheduling, or design. A fuzzy controller is a knowledge-based system of a different nature. If a fuzzy controller is called an expert system, this blurs some important distinctions. As Zadeh writes, "what differentiates applications to control from applications to [general] knowledge-based systems is that in control the main problem which has to be addressed is that of imprecision. By contrast, in the case of knowledge-based systems, one has to come to grips with both imprecision and uncertainty."

As I discussed in my paper, another important difference is that most controllers do not have to remember and reason about the history of the portion of the outside world that they deal with. Most fuzzy controllers have no internal state, while expert systems retain considerable state information.

Dubois, Prade, and Smets give five references, the latest of which is five years old. The Cadiag work of Adlassnig and his colleagues is indeed impressive.² However, it is especially difficult to deploy medical expert systems in the real world, in compar-

ison, say, to applications in manufacturing. Both the cited paper and more recent papers on Cadiag-2^{3,4} state only that Cadiag-2 systems are undergoing clinical trials.

Similarly, the paper on Taiger⁵ does not claim that the system has been deployed, and I could not find any further papers on this system. The cited paper on RUM⁶ states it is a "development environment," and the only published application built using it is described as a "prototype."⁷ Finally, OPAL⁸ is described in the cited paper as "under development," and Milord⁹ is said to be a "shell." More recent versions of Milord use finite multiple-valued logics rather than fuzzy logic.¹⁰

Vadiei and Jamshidi say that "The Nikkei average has reportedly gone consistently higher using fuzzy logic." This statement is difficult to understand, let alone to believe; the only citation is to the authors' own unpublished course notes. The other application they mention is a system for choosing oil recovery methods. According to the journal paper on this system it uses the Clips shell, which is not founded on fuzzy logic.¹¹

Zadeh gives three examples of expert systems using fuzzy logic as their primary formalism for reasoning about uncertainty: Cadiag-2 again, FRIL,¹² and a system for securities trading with no citation. Recent papers indicate that FRIL is a "programming language"¹³ and that the trading system has only been "tested."¹⁴ Zadeh also cites papers on systems for acupuncture diagnosis and pavement maintenance from the 1993 International Fuzzy Systems Association World Congress, but I do not have access to these papers.

The theorem. Except for Klir and Yuan, no commentators dispute the mathematical validity of the theorem given in my paper, but several commentators disagree with the assumptions made in its statement. Dubois, Prade, and Smets say it relies "at best on a logical equivalence the rationale of which is far from natural in the scope of fuzzy logic." In my opinion, the opposite is true. The equivalence between $\neg(A \wedge \neg B)$ and $B \vee (\neg A \wedge \neg B)$ is a natural one to use (perhaps inadvertently) in compiling a knowl-

edge base of fuzzy logic sentences, and compilation into single-level rules "to simplify and speed computation" is mentioned by several commentators, Berenji in particular.

As Garcia and other commentators point out, the theorem can also be proved by considering much simpler equivalences such as $A \wedge \neg A = \neg(A \vee \neg A)$ or $A \wedge \neg A = B \wedge \neg B$. The reason the proof given uses a more complicated equivalence is that, as just mentioned, it is more natural in some intuitive sense. Intuitively speaking, in $A \wedge \neg A = B \wedge \neg B$ the two sides are irrelevant to each other, and $A \wedge \neg A = \neg(A \vee \neg A)$ is obviously similar to the law of excluded middle.

The phrase "obviously similar" in the statement above is vague. One interpretation of the theorem is that if we reject the law of excluded middle, then we must also reject many other equivalences that are not obviously similar to this law, but that are nevertheless interchangeable with the law using only the first three postulates of Definition 1. When Yager gives a derivation of the law of excluded middle from $t(\neg(A \wedge \neg B)) = t(B \vee (\neg A \wedge \neg B))$, this is an alternative statement of the theorem, not a demonstration that the theorem is absurd.

Overall, I am saddened by the hostility visible in the comments by Yager and by Klir and Yuan. I will refrain from responding line by line to their remarks on the different versions of my theorem and its proof. It is quite usual in the history of mathematics for a theorem that attracts interest to be restated and reinterpreted over time, and for similarities with previous results to be noticed later. For a similar but friendly exegesis of the development of the statement and proof of a far deeper and more important theorem the reader can consult *Proofs and Refutations* by Imre Lakatos.¹⁵

The theorem is technically correct as stated and proved both here and in my AAAI '93 paper. Klir and Yuan say that either the statement or the proof of the theorem is incorrect, because the "proof depends on eight logical equivalencies, only one of which is included in the statement." This claim is based on a misreading of the statement of the theorem, where the condition "if $\neg(A \wedge \neg B)$ and $B \vee (\neg A \wedge \neg B)$ are logically equivalent" must be understood

as asking for a schema of logical equivalences, in which A and B may be replaced by any assertions, including assertions of the form $\neg C$.

The success of fuzzy control. Perhaps the most important contention of my paper is that the success of fuzzy controllers has little to do with the theory of fuzzy logic or fuzzy sets. Several commentators confirm this. For example, Klir and Yuan say that "most of the simple fuzzy controllers on the market ... are not explicitly based on fuzzy logic." Dubois, Prade, and Smets write that "Takagi and Sugeno have proposed an interpolation mechanism ... this kind of 'inference' (which is widely used in fuzzy control) has nothing to do with uncertainty handling," and Pelletier writes that "those areas of fuzzy logic that get criticized are simply not employed in the control arena."

It is a general property of systems that use only shallow knowledge that numerical uncertainty values can be tuned, if necessary, to overcome arbitrariness in the operators used for combining uncertainty values. Alternatively, within reason, the operators can be adjusted to match given numerical values. As Chandrasekaran reminds us concerning Mycin, a system based on shallow medical knowledge: "The fine structure of uncertainty didn't really matter." Several commentators support my specific contention that this property is one reason for the success of heuristic controllers using fuzzy logic. For example, Wang, Tan, and Tan write that "...numerous forms of fuzzy operations ... were created to cater to the domain-specific needs."

I do not agree with Ruspini that the term "paradox" should only be used to mean "logical self-contradiction," so I believe that it is fair to call the lack of connection in fuzzy systems between theory and practice an apparent paradox. All paradoxes have the property that once resolved, they no longer appear paradoxical. To paraphrase a statement by Türksen, there are no paradoxes, only limited or partial understanding. The paradox that fuzzy controllers have had real industrial success, while fuzzy logic itself is still under attack mathematically, is resolved

by understanding the distinction between a scientific experiment designed to confirm or disconfirm a theory and an engineering application of the theory. Fuzzy controllers are applications, not experiments that could validate theoretical claims about fuzzy logic. On this point I agree with Mamdani: "There is a common misconception that models are created and then applied and the success then legitimizes a model."

Overall, the response by Mamdani is particularly trenchant and thought-provoking. Where we disagree, I think the cause is a misunderstanding. I do not argue that fuzzy control "is not worthy of industrial consideration because of its lack of complex form and structural sophistication." Rather, I argue that this simplicity is vital to the industrial success of the current generation of fuzzy controllers, but that fuzzy controllers for more complex applications will run into the same problems of complexity that other knowledge-based systems do today. It is the case that the "philosophical deficiencies of fuzzy logic" do something "to argue against the adoption of fuzzy logic control": These deficiencies are what makes scaling-up difficult.

Many research teams are actively working on scaling-up fuzzy controllers. A common feature of the research prototypes developed by these teams is the use of ideas for organizing large intelligent systems first proposed by mainstream AI researchers. For example, the SRI autonomous robot mentioned by Berenji uses "several deliberation levels to determine the relevance level of each control rule ...; to identify current goals and their state of achievement; to activate control rules according to the current context; and to blend their control recommendations." The main novelty here compared to classical robot architectures is the idea of interpolating smoothly between different suggested actions — but this idea is also found in other AI work, such as that of Brooks.¹⁶

The ability to interpolate between the conclusions of several rules is an important advantage of fuzzy control methodologies. As Yager writes, "the fact that most fuzzy controllers are built with a small number of rules should be seen as one of the powers of this technology," and as Berenji writes,

"Fuzzy sets provide for a general yet compact characterization of system state that requires fewer rules." However, interpolation is a purely local operation, where the conclusions of a few rules describing responses to nearby input parameter configurations are blended. It is therefore difficult to see how interpolation could reduce the amount of knowledge needed to capture a complex, multidimensional input/output mapping by more than one order of magnitude compared to other approaches.

Klir and Yuan write that "... fuzzy controllers of this kind [that do interpolation] are universal approximators." This fact is true, but less significant than it may appear at first sight. Given suitable smoothness constraints, many mathematical formalisms can be used as universal approximators of multidimensional input/output mappings. For example, any continuous function can be approximated to any desired degree of accuracy by a polynomial of sufficiently high order. Neural networks with hidden layers are also universal approximators.¹⁷ The important question is how complex an approximation must be allowed to be to achieve a given level of precision. As recognized by Kosko and Isaka,¹⁸ the number of rules required by a fuzzy controller — which is the number of patches used to approximate its control surface — grows exponentially with the dimensionality of the controller and the level of precision demanded. From a formal point of view, fuzzy controllers thus do not enjoy a clear advantage over other formalisms for approximating smooth functions. Of course they are still pragmatically very useful.

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