Running Head: Alleged Misconceptions about Fuzzy Logic

Title: On Some Alleged Misconceptions about Fuzzy Logic

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Abstract:

(Entemann 2002) defends fuzzy logic by pointing to what he calls 'misconceptions' concerning fuzzy logic. However, some of these 'misconceptions' are in fact truths, and it is Entemann who has the misconceptions. The present article points to mistakes made by Entemann in three different areas. It closes with a discussion of what sort of general considerations it would take to motivate fuzzy logic.

Keywords:

Fuzzy Logic, Proof Theory, Truth Interval Tableaux

1. Introduction

(Entemann 2002) is a forceful defense of fuzzy logic by means of an attack on (what he calls) 'misconceptions' concerning fuzzy logic. After a brief introduction to fuzzy logic and fuzzy set theory as defined in (Zadeh 1965), Entemann discusses an initial four of these alleged misconceptions. He then moves to a description of a tableaux system for fuzzy logic described in (Kenevan & Neapolitan 1992) and called "the Kenevan truth interval fuzzy logic tableaux", which sets the ground for his discussion of the final four alleged misconceptions.

Two of these final four 'misconceptions' are said to be due to me. Not surprisingly, I do not feel that these two claims of mine misdescribe any aspects of fuzzy logic, nor that I have any misconceptions about it. It is these two alleged misconceptions that I will focus my attention upon in this note, arguing that it is Entemann who has misconceptions about fuzzy logic. I will also make a few comments about the first of his eight 'misconceptions' – which is not attributed to anyone in particular and presumably is thought to be found in all the authors who are otherwise identified as having misconceptions.

At the end I will return to the general topic of fuzzy logic and argue that its supporters, and not its detractors, are the ones who refuse to take seriously what is important and unique in fuzzy logic.

2. Background

First, let me give a very brief account of fuzzy logic, so as to remind ourselves what is under discussion. As is well-known, fuzzy logic replaces the two truth values 0 (False) and 1 (True) with the non-denumerable set of real values in the interval [0..1]. Atomic sentences of fuzzy logic, that is, those sentences represented by the sentence letters, can be assigned any of these values. Given a specific set of these sentence letters, assigning a single value in [0..1] to each of the atomic sentences in that set is an *interpretation* (of the set). (Zadeh 1965) considers only complex sentences made from $\neg$, $\wedge$, and $\vee$, and this is the language that is discussed by Entemann in the present article. It will be the topic of the present note also, except for a few cases where I consider the effect of adding other connectives. The Zadeh-standard semantic interpretation of complex sentences using $\{\wedge, \vee, \neg\}$ is given by the function *t*, which will generate the truth value of any complex formula that uses these connectives, when given the truth values of the atomic components (that is, when given an interpretation):

$$t(\neg A) = 1 - t(A)$$
$$t(A \wedge B) = \min(t(A), t(B))$$
$$t(A \vee B) = \max(t(A), t(B))$$

Entemann (2002) defines a formula A of fuzzy logic to be *true* in an interpretation if it is assigned 1 in that interpretation, i.e., if $t(A)=1$. It is said to be *fuzzy true* in an interpretation if it takes a value strictly between 0.5 and 1.0, i.e., if $0.5<t(A)<1$. It is said to be *false* in an interpretation if it is assigned 0 in that interpretation, i.e., if $t(A)=0$; it is said to be *fuzzy false* in an interpretation if it takes a value strictly between 0 and 0.5, i.e., if $0<t(A)<0.5$. It is *fuzzy indeterminate* in an interpretation if $t(A)=0.5$. A formula is a *tautology* just in case it is true in *every* interpretation; it is a *fuzzy tautology* just in case in *every* interpretation it is either true or fuzzy true; it is a *contradiction* just in case it is false in *every* interpretation; and it is a *fuzzy contradiction* just in case in *every* interpretation it is either false or fuzzy false.

3. “Misconception #1: Fuzzy logic generates results that contradict classical logic.”

This is a puzzling ‘misconception’ for Entemann to put forward, since of course there are matters of logic on which fuzzy and classical logic disagree – Entemann has listed several of them in his article. So the 'misconception' states a truth that even Entemann agrees with. What Entemann cites is this:

> If fuzzy logic is wrong then, by definition, a proof performed using fuzzy logic must contradict that using another logic, i.e., classical logic, starting from the same premises. But if the truth values used are restricted to [the classical values 0 and 1, then the evaluation rules for , , and ¬ that are given for fuzzy logic] generate the same results as do those of classical logic….Zadeh's fuzzy logic is, then, an **extension** of classical logic. Classical logic cannot process inferences using the other truth values contained in … [0.0..1.0] so there is no possibility for conflict with fuzzy logic. (Entemann 2002: 68; his emphasis)

Most logicians think that one logic is an extension of another if it contains all the theorems of the other, plus some more. For instance, among the well-known modal logics, S_5 is an extension of S_4 in this sense: S_5 is constructed by adding a new, independent axiom to those of S_4, and this generates new theorems that were not in the original S_4. But this is not a sense in which fuzzy logic is an extension of classical logic; for, (A ¬A) is a theorem of classical logic but not of fuzzy logic. Indeed, it can be shown that there is no theorem of fuzzy logic (in , , and ¬) which is not already a theorem of classical logic. So classical logic in fact is an extension of fuzzy logic, in the usual use of the term 'extension', and not the other way around.

Now, maybe the notion of an extension is flexible and what is one person's extension is another's restriction. But surely, no matter what one's view of this matter is, it is clear that fuzzy logic violates claims of classical logic – for instance in saying that (A ¬A) is not a theorem or that (A¬A) is not a contradiction. Entemann demands that one produce a classical logic proof that contradicts a fuzzy logic proof before he will admit that fuzzy logic generates results contradicted by classical logic. And since classical logic is an extension (in the more usual sense) of fuzzy logic, there of course can be no such proof…anything that is a valid proof in fuzzy logic (in □, , and ¬) is a valid proof of classical logic. But clearly this is *not at all* to say that there is no contradiction between fuzzy logic and classical logic!! It is Entemann's condition on what it means to contradict classical logic that is the misconception here.

4. "Misconception #5: There can be no proof theory for fuzzy logic (Pelletier 1994)."

Entemann replies to my claim by constructing what he claims is a proof theory for the Zadeh logic with just the three connectives mentioned above. This construction invokes the Kenevan interval tableaux method (indeed, is essentially just that technique) and is a procedure for evaluating individual formulas of the {□, ,¬} fuzzy logic (see Kenevan & Napolitan, 1992). But the fact is that the method is *not* an adequate proof theory. Perhaps part of Entemann's belief that his method is adequate is due to his consulting only the summary (Pelletier 1994) rather than the more thorough (Morgan & Pelletier 1977), where the underlying reason is more fully discussed.

Entemann employs the Kenevan tableaux method by trying to show how to convert an argument into an individual sentence in such a way that the argument is valid just in case the sentence is a tautology. But his claim is wrong and his attempted support for it by means of his Theorem 5 is misguided, for the so-called Theorem 5 is no theorem. Entemann's 'Theorem 5' is:

> Given formulae $F_1, F_2,\ldots,F_n$ and a formula G, G is a fuzzy logical consequence of $F_1, F_2,\ldots,F_n$ if and only if the formula (F_1□…□ F_n)□ G is a *fuzzy tautology*

First let us note that there is no connective □ in the fuzzy logic under consideration here, so it is not altogether clear what is being claimed. However, in the first line of the later 'proof' Entemann explains:

Recall that: (F_1□…□ F_n) □ G ≡ ¬(F_1□…□ F_n) G

(We might also note that there is no symbol ≡ in the language either. But let us suppose Entemann means it simply as a metalinguistic claim saying that the formula-type on the left side is an abbreviation for the formula-type on the right.) In such a case Entemann's Theorem 5 becomes:

> Given formulae F_1, F_2,…,F_n and a formula G, G is a fuzzy logical consequence of F_1, F_2,…,F_n if and only if the formula ¬(F_1□…□ F_n) G is a *fuzzy tautology*

This claim is false and here is a counterexample. By definition, a formula G is a fuzzy logical consequence of a set of formulas □ if and only if G is either true or fuzzy true in any interpretation where all members of □ are either true or fuzzy true. So at the very least any formula must be a fuzzy logical consequence of itself, e.g., the G in Entemann's abstract characterization must be a fuzzy logical consequence of G itself. However, ¬G G is *not* a fuzzy tautology, as can be seen from the fact that the interpretation which assigns G the value 0.5 will make ¬G G also be assigned the value 0.5. So there is at least one interpretation in which ¬G G fails to be fuzzy true. So by definition it is not a fuzzy tautology.

Using the notation □|=G to mean that an argument which has the set of premises □ and conclusion G is a valid argument, what we have just illustrated is that, although G|=G, it is not the case that (¬G G) is a fuzzy tautology. So we have shown that Entemann has failed in his attempt to describe a formula the fuzzy-tautologousness of which will express the fact that some related argument is fuzzy-valid. In classical logic there *is* a formula which will express this fact, and this expressive ability is referred to as *the deduction theorem*, which can be stated as

F|=G if and only if |=(F□ G).

This means that if one has a valid argument with premise F, then there is a tautologous formula (F→G) that reports this fact. In classical logic, the ordinary conditional connective, →, performs the feat of relating being a premise of an argument to a specific formula. This is the feature of classical logic that allows for 'natural deduction' techniques of proof whereby one makes an assumption and sees where it leads, and eventually discharges the assumption. It is legal to do this because classical logic has a deduction theorem. For the particular case we have been considering, the classically valid argument G|=G corresponds to the classical tautologousness of (G→G). And of course using the classical logic equivalence of (G→G) and (¬G∨G), we could also have stated this as:

G|=G if and only if (¬G∨G) is a tautology.

It is presumably this sort of equivalence to which Entemann is adverting in his 'proof.' But the fuzzy logic connectives do not support any such equivalences.

The fact of the matter is that the {∧,∨,¬} fuzzy logic *does not have* a deduction theorem. This means that, although one can *describe* what a fuzzy valid inference would be ('in any interpretation where each premise is true or fuzzy true, the conclusion is either true or fuzzy true'), there is *no formula* of the {∧,∨,¬} fuzzy logic that expresses this. Given a valid argument $F_1, F_2, \ldots, F_n \models G$, there is *no* definable connective → of the {∧,∨,¬} fuzzy logic such that $(F_1 \wedge \ldots \wedge F_n) \rightarrow G$ is a fuzzy tautology.[1] We will consider below he possibility of adding some new primitive connective which does allow for a deduction theorem. For now, though, we simply note that there is no deduction theorem for the {∧,∨,¬} fuzzy logic. Since there is no single formula which reports that an argument is valid, any method that relies on evaluating single formulas will not serve as a proof theory. So the fact that one can use the Kenevan tableaux to determine fuzzy tautologousness is simply beside the point.

Furthermore, without a deduction theorem there can be no natural deduction version of the logic, because the essence of natural deduction is to break down the task of proving a complex formula into one where an assumption is made in a subproof, the consequences of this assumption are derived within this subproof, and finally the assumption is discharged in such a way that the complex formula can thereby be claimed to be proved. In classical logic, for example, to prove a conditional statement, one assumes the antecedent of the conditional to hold in a subproof, and in that subproof one attempts to show that the consequent of the conditional must hold. If one succeeds in this, then one can appeal to the deduction theorem to escape the subproof and to justify the claim to have proved the conditional in the outer proof level. But without a deduction theorem for the conditional this would not be a valid manner of proof. In fuzzy logic (using only $\wedge, \vee, \neg$ and therefore lacking a deduction theorem) we can never make an assumption to see where it goes (perhaps concluding that such an assumption would lead to X), and then conclude that if the assumption were true or fuzzy true then X would also be true or fuzzy true. Such is a consequence of lacking a deduction theorem.

The same flaw appears again in Entemann's 'Theorem 6', which says

> Given formulae $F_1, F_2,\ldots,F_n$ and a formula G, G is a fuzzy logical consequence of $F_1, F_2,\ldots,F_n$ if and only if the formula $(F_1\wedge\ldots\wedge F_n)\wedge\neg G$ is a fuzzy contradiction

Once again: although G is fuzzy logical consequence of G, the formula $G\wedge\neg G$ fails to be a fuzzy contradiction because it is not fuzzy false in at least one interpretation – namely, where G is assigned the value 0.5. We again see that there is no formula of $\{\wedge, \vee, \neg\}$ fuzzy logic that can express the fact that G is a fuzzy consequence of $F_1, F_2,\ldots,F_n$.

The 'proofs' of Theorems 5 and 6 presented in (Entemann 2002) and the corresponding ones in (Entemann 2000)[2] are just plain wrong in their 'only if' direction. Instead of assuming that G is a fuzzy consequence of F_1, F_2,…,F_n and trying to show from this assumption that a certain formula is a fuzzy tautology, in (Entemann 2000) it is in addition illegitimately assumed that F_1, F_2, …,F_n *are* each true or fuzzy true, while in (Entemann 2002) it is instead illegitimately assumed that the conjunction of F_2, …,F_n is either true, fuzzy true, false, or fuzzy false. Neither of these types of assumptions is correct to make, since they leave out possible interpretations, and the proofs therefore fail. The fact is that although the Kenevan truth interval technique might be a correct method to investigate the truth value interval in which a given formula resides (including the tautologous interval), this is irrelevant to the issue of a proof theory because *there is no formula of this fuzzy logic that expresses whether or not a given argument is or is not valid*, unlike the case in classical logic. And since there is no such formula, the truth interval method cannot be applied to the {□, ,¬} fuzzy logic. And indeed, *no* method that works on single formulas can adequately characterize this version of fuzzy logic.

5. "Misconception #6: Fuzzy logic can never be proven to be proof theoretic complete (Pelletier 1994)."

Entemann (p. 79) distinguishes two different notions of 'completeness', and concludes that I intended the meaning where a logical system is complete just in case "any theorem that can be composed using the syntax of a theory can also be proven using the inference rules of the theory."[3] In fact what I meant was that fuzzy logic cannot have a correct proof theory, and so my claim is that there are semantically valid arguments for which there could be no proof. That is, there are arguments which obey the definition of a fuzzy-valid argument ('in any interpretation where each premise is true or fuzzy true, the conclusion is either true or fuzzy true') but for which there can *never* be *any* rules of inference that could generate a proof of this fact.

The issue is similar to, but not exactly the same as, that discussed just above under 'Misconception #5.' Let me elaborate. Let us first note the difference between the following two notions[4]:

Sentence Completeness: if $\models A$ then $\vdash A$

Argument Completeness: if □ $\models A$ then □ $\vdash A$

Both notions of completeness talk about the relation between the semantic evaluation of formulas and arguments on the one hand (using '⊨' to say 'in any interpretation where each premise is true or fuzzy true, so is the conclusion either true or fuzzy true') and the notion of having a proof on the other hand (using '⊢' to say 'the conclusion has been generated from the premises by the given set of rules of inference'). The difference between sentence and argument completeness concerns whether we are talking about arguments with no premises vs. those with premises. In the case of having no premises, the issue would be whether a formula that is a fuzzy tautology (i.e., is true or fuzzy true in all interpretations) can be proved from no premises using the rules of some system of logic; in the case of having premises, the issue is whether a semantically valid argument (i.e., in every interpretation where all the premises are either true or fuzzy true, so is the conclusion) can be proved by using those premises in some logical system. The charge I make here is that there is no non-trivial proof system that is argument complete for fuzzy logic.

As mentioned in the previous section, although in fact there is no such connective that is definable in the {□, ,¬} fuzzy logic, we could postulate a new primitive connective that obeyed the (semantic) deduction theorem (unlike the □ that Entemann defined) and add it to this language. Let's use □ for this new fuzzy logic connective that obeys the deduction theorem. Then argument completeness for arguments containing finitely many premises would be the same as sentence completeness. For, if we had finitely many premises $F_1, F_2,\ldots,F_n$, then we could move from

$$F_1, F_2,\ldots,F_n \models G$$

to

$$\models(F_1 □ (F_2 □ (\ldots □ (F_n □ G))\ldots)$$

by repeated applications of the (semantic) deduction theorem, and from this we could get

$$\vdash(F_1 □ (F_2 □ (\ldots □ (F_n □ G))\ldots)$$

by sentence completeness. And finally we could move to

$$F_1, F_2,\ldots,F_n \vdash G$$

by the nature of □ and the rules of inference that govern it. Therefore, if we had such a connective then argument completeness for finitely many premises would be true just in case sentence completeness were true.

However, my claim being discussed in the present 'misconception' is that *even with such a connective* – or indeed, with *any* other connectives that one might wish to add to the language – propositional fuzzy logic is not argument complete. That is, *any* formulation of a proof theory for fuzzy logic is able to capture only a subset of the semantically valid arguments, namely only ones with a finite number of premises. In classical logic this is not a shortcoming because an argument with an infinite number of premises □ to the conclusion G is valid just in case there is a valid argument from some finite subset of □ to the conclusion G. (Classical logic is *semantically compact*, as this property is called). But this is not the case in fuzzy logic, where there can be (for example) an unsatisfiable infinite set □ where every finite subset is satisfiable. (Just imagine that each sentence of □ says 'Sentence *p* does not take the value *i*', for all the different *i*'s. With an infinite number of sentences to express these facts, we could in effect claim that *p* does not take *any* of the values between 0 and 1. But that is impossible, since every sentence must take one or another of the values. Note though, that every finite subset of □ *is* satisfiable merely by allowing *p* to take one of the values not mentioned by any of the formulas in the finite subset.[5]) This means that there are valid arguments to a conclusion G using infinite sets □ of premises but for which there is no valid argument to G using any of the finite subsets of □. For example, if □ were not satisfiable then an arbitrary formula X would follow (because 'in any interpretation where all the premises are either true or fuzzy true, so is the conclusion' would be vacuously true–there being no such interpretations), but it would not follow using any of the finite subsets of □. So fuzzy logic using {□, ,¬} is *not* semantically compact. And indeed, fuzzy logic, *no matter what connectives it has*, is not semantically compact. But since all proofs are by definition finite, there can therefore be *no general proof theory for fuzzy logic*. In passing I remark that this is true even of the propositional fuzzy logic: there are no special features of predicate logic and undecidable relational properties that are being appealed to.

6. General Remarks

The point of inventing a logic is twofold. There is first the aesthetic matter of describing a new abstract structure that can be studied, evaluated, and admired. Secondly there is the more practical issue of being able to describe some class of phenomena in a particularly elegant and enlightening manner. Proponents of fuzzy logic often single out engineering applications and allege that their successes validate fuzzy logic in this second way. But this is a rather treacherous and puzzling path for them to take in the present context, because the systems that are employed in these applications are so meager that they do not represent fuzzy logic in any essential way. Consider for instance the claim that 'we are not interested in arguments with infinitely many premises, and so the failure of semantic compactness does not tell against fuzzy logic.' Behind this claim lurks a refusal to tap into the full power of fuzzy logic; instead, we are forced to consider only finite sublogics of fuzzy logic. And it would be wrong to say that this is 'merely a restriction of fuzzy logic's power to the requirements of the case at hand', or that 'fuzzy logic is the underlying theory in the background which unifies all these particular applications', because *none* of these applications use what is unique to fuzzy logic. In (Morgan & Pelletier 1977: 91f) it is argued that all the applications invoked for fuzzy logic could be done in a finitely-many-valued logic, and that the full fuzzy logic is never needed.[6] And there never has been any specific application that has been shown to require the full power of fuzzy logic.

Those of us who are unconvinced by fuzzy logic hype will find Entemann's claims quite puzzling indeed. Throughout the article, indeed as crucial presuppositions of various of his proofs, it is assumed that there *are* fuzzy tautologies and fuzzy contradictions. But as a matter of fact, there are none. *No* formula of this language (fuzzy logic in □, , and ¬, where these are interpreted as min, max, and 1-minus) is a fuzzy tautology or fuzzy contradiction. (Every proof cited in discussing 'Misconceptions 5 and 6' are wrong for this reason, and I'm afraid these mistakes carry over to his earlier (Entemann 2000) on which much of his current paper is based).

There are two ways to strengthen fuzzy logic so as to have fuzzy tautologies (and fuzzy contradictions). One way is to add further connectives into the language. There are, after all, a non-denumerable number of further connectives that cannot be defined in terms of {□, ,¬}, as I remarked in footnote 3 above. Many of these connectives are quite interesting in their own right and are implicitly appealed to in informal discussions about fuzzy logic…including Entemann's (2002: 72-73) discussion of one glass being more full of water than another glass. Suppose p=0.6 and q=0.8; without further connectives there is no mechanism to show or infer or even express the idea that q is ‘more true than’ p. So there is no way to express such relational facts as ‘John is taller than Bill’ or ‘this glass is more full than that glass’ when given the degree of truth of the atomic sentences, using only the connectives {□, ,¬}. And Entemann’s claims about the alleged superiority of fuzzy logic in this application are simply wrong. What is needed are connectives that directly ‘talk about’ the truth value of a proposition. In (Morgan & Pelletier 1977) these are introduced as two-valued J-operators: J_i□=1 if *t*(□)=i and =0 otherwise. Intuitively, J_i□ says ‘□ has the value *i*’. Such *parametric* operators (as they are often called…see (Hájek 1998)) can be introduced in various ways, and are essential for the actual use of fuzzy logic in any interesting application. Yet the proponents of fuzzy logic rarely mention them and their logical properties, apparently assuming that the ability to say, in the metalanguage, that 0.8 is greater than 0.6 thereby confers some similar meaning onto ‘this glass is full’ and ‘that glass is full’.

Another way to introduce fuzzy tautologies and contradictions into fuzzy logic is to remove 0.5 from the list of possible semantic values. Entemann (2002: 79) suggests this as a way to incorporate the notion of proof by contradiction, and he even suggests that this removal is required in order to have *any* notion of logical consequence at all. (He notes the similarity of the resulting logical system to Zadeh’s (1988) ‘dispositional logic.’) Eliminating the value 0.5 from the course of semantic values would be to say that fuzzy logic tolerates no fuzzy indeterminate propositions. But then there would be no real point to fuzzy logic, for all the values above 0.5 are treated simply as True and their negations (all the values less than 0.5) are treated simply as False.[7] Surely this variant on fuzzy logic is just two-valued logic with a lot of logically uninteresting and unimportant fillips.

Acknowledgements

I gratefully acknowledge the assistance of the Canadian NSERC Discovery Grant #5525, which over the years has supported my research into logic, language, and automated theorem proving.

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[1] Putting it more generally, there is no formula X that uses only the connectives □, , and ¬, and sentence letters from $F_1, F_2,\ldots,F_n$ and G, such that X is a fuzzy tautology if and only if $F_1, F_2,\ldots,F_n \models G$ is a valid argument.

[2] The relevant theorems in (Entemann 2000) are his 'Theorem A', p. 172. Theorem A is the 'only if' direction of his 'Theorem 4.1' in that article, and corresponds to Theorem 5 of (Entemann 2002). Theorem 6 of (Entemann 2002) corresponds to Theorem 4.2 of (Entemann 2000), pp. 173-174.

[3] He correctly rejects the idea that I might have understood 'completeness' as (what is usually called) 'decidability.' Morgan & Pelletier (1977: Appendix) give a decision procedure for a fuzzy propositional logic that is an extension of the {□, ,¬} logic. So it is clear that I believe some (propositional) fuzzy logics to be complete in this sense. Another sense of completeness is that every truth function can be expressed by a formula of the logic. Since there are non-denumerably many truth functions from n-tuples of [0..1] into [0..1], it is clear that fuzzy logic must not be complete in this sense. (It is not 'functionally complete').

[4] I continue to use $\models$ to indicate semantic validity, and use $\vdash$ to say that there is a correct proof using the relevant rules of inference and axioms.

[5] A more formal expression of this, for a slightly different language than Zadeh's, is given in (Morgan & Pelletier 1977).

[6] The applications under consideration there were linguistic and cognitive.

[7] The idea of eliminating the 0.5 value had already been pursued by (Lee 1972) in the arena of automated theorem proving. He showed that all and only the theorems of classical logic were fuzzy tautologies, and so he concluded that this version of fuzzy logic was just classical logic in disguise.

References

Entemann, C. (2000). A Fuzzy Logic with Interval Truth Values. *International Journal of Fuzzy Sets and Systems* **113:** 161-183.

Entemann, C. (2002). Fuzzy Logic: Misconceptions and Clarifications. *Artificial Intelligence Review* **17:** 65-84.

Hájek, P. (1998). *Metamathematics of Fuzzy Logic*. Kluwer Academic Press: Dordrecht.

Kenevan, J. & Neapolitan, R. (1992). A Model Theoretic Approach to Propositional Fuzzy Logic Using Beth Tableaux. In L. Zadeh & J. Kacprzyk (eds.) *Fuzzy Logic for the Management of Uncertainty*, 141-157. John Wiley & Sons: New York.

Lee, R. (1972). Fuzzy Logic and the Resolution Principle. *Journal of the ACM* **19:** 109-119.

Morgan, C. & Pelletier, F.J. (1977). Some Notes Concerning Fuzzy Logic. *Linguistics and Philosophy* **1:** 79-97.

Pelletier, F.J. (1994). Fuzzy Logic: A Misplaced Appeal. *IEEE Expert* **9:** 29-31.

Zadeh, L. (1965). Fuzzy Sets. *Information and Control* **8:** 338-353.

Zadeh, L. (1988). *Dispositional Logic and Commonsense Reasoning*. Tech Report CSLI-88-117, Center for the Study of Language and Information, Stanford University.